Minimalism in Modified Gravity

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Based on collaborations with
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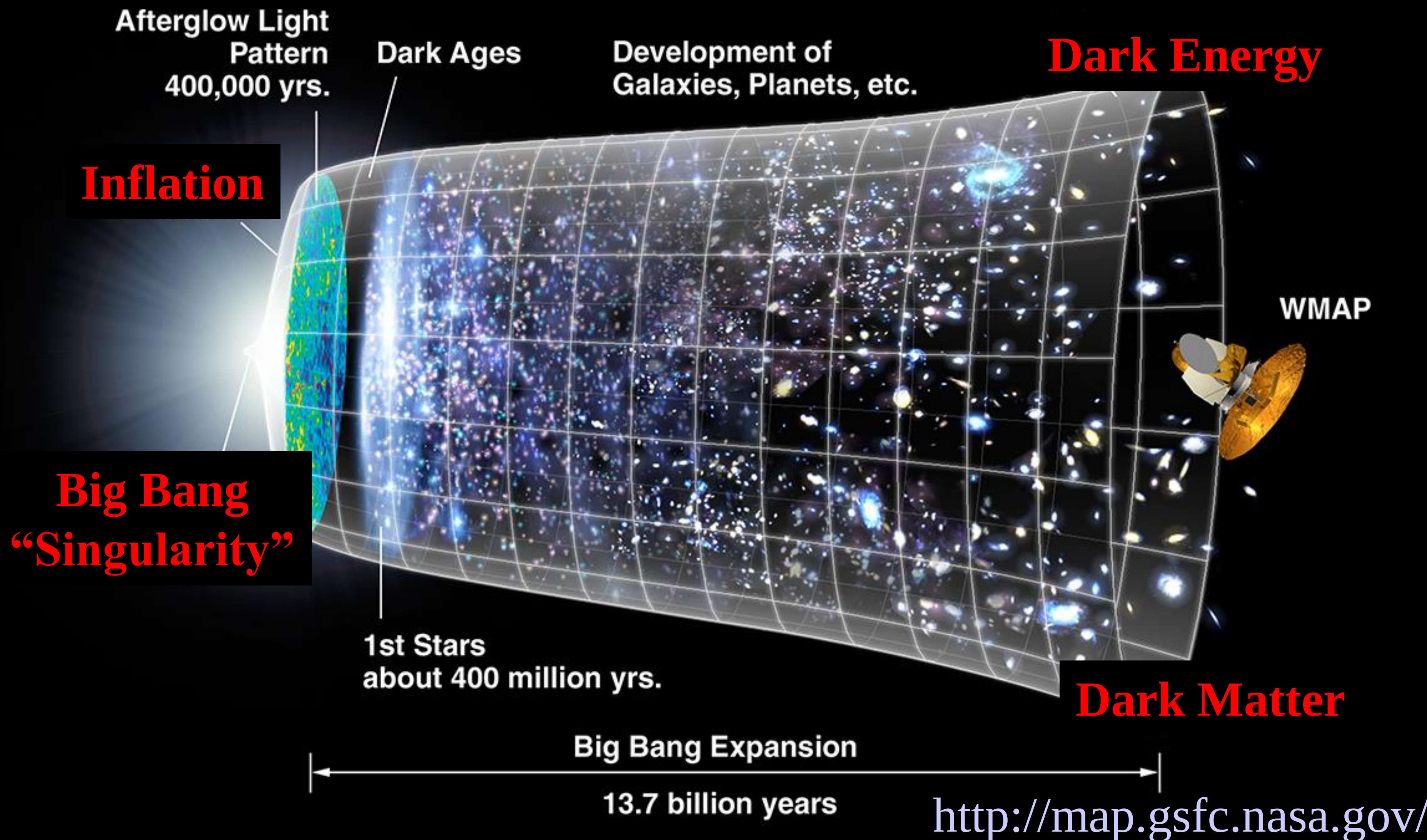
Why modified gravity theories?



A motivation for IR modification

- Gravity at long distances

  Flattening galaxy rotation curves

  extra gravity

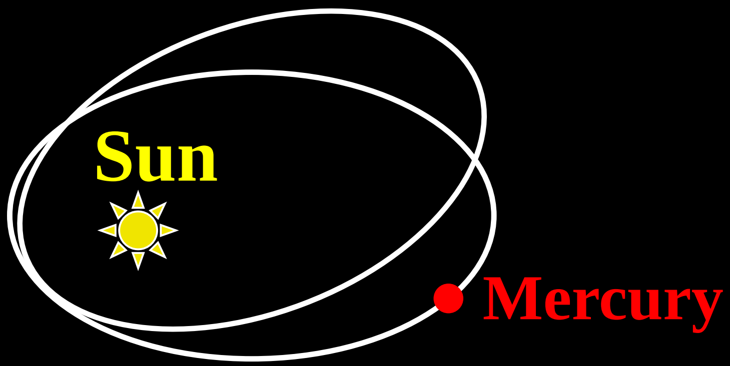
  Dimming supernovae

  accelerating universe

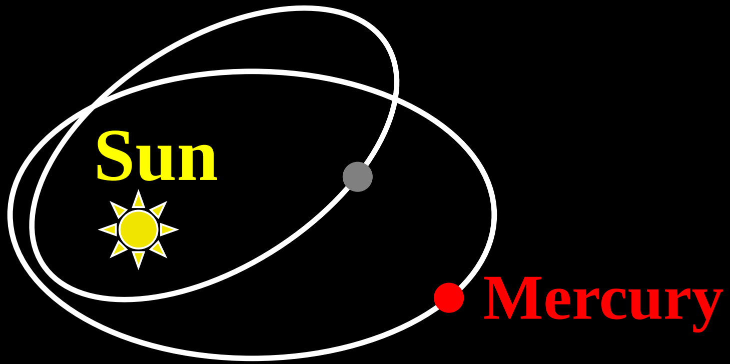
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,

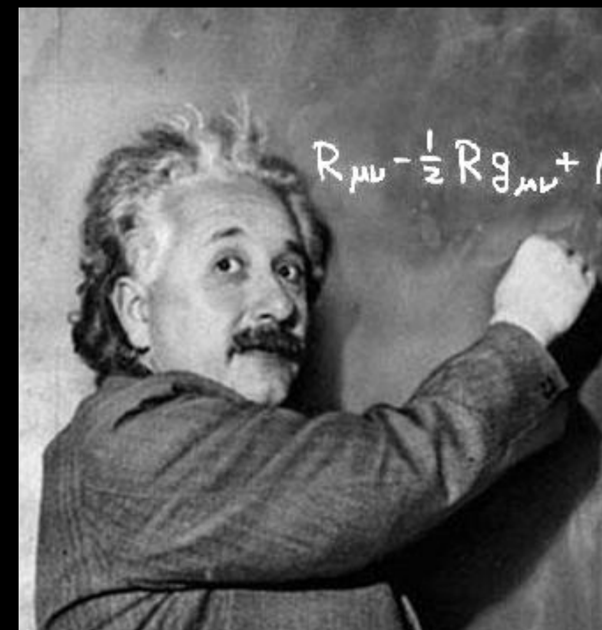


But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

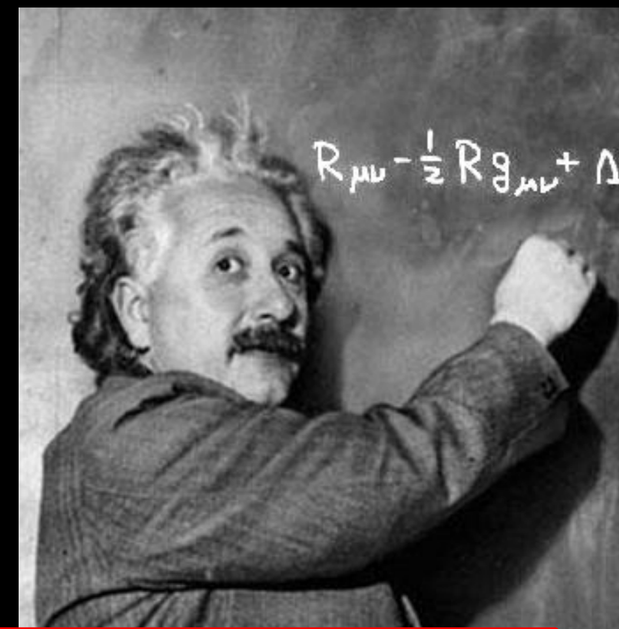
Why modified gravity?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.

How to unify Quantum Theory with General Relativity?



How to unify Quantum Theory with General Relativity?



Probably we need to modify GR at short distances

Why modified gravity?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- …

of d.o.f. in general relativity

- 10 metric components → 20-dim phase space @ each point
- Einstein-Hilbert action does not contain time derivatives of N & N^i → $\pi_N = 0$ & $\pi_i = 0$

ADM decomposition

- Lapse N, shift N^i, 3d metric h_{ij}

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Einstein-Hilbert action

$$I = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g}\, {}^{(4)}R$$

$$= \frac{M_{\rm Pl}^2}{2} \int dt d^3\vec{x} N \sqrt{h} \left[K^{ij} K_{ij} - K^2 + {}^{(3)}R \right]$$

- Extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\partial_t h_{ij} - D_i N_j - D_j N_i)$$

of d.o.f. in general relativity

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of d.o.f. in general relativity

- 10 metric components → 20-dim phase space @ each point
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  All constraints are independent of N & N^i → π_N & π_i "commute with" all constraints → 1^{st}-class

1^{st}-class vs 2^{nd}-class

- **2^{nd}-class constraint S**
  $\{ S , C_i \} \not\approx 0$ for $^{\exists} i$
  Reduces 1 phase space dimension
- **1^{st}-class constraint F**
  $\{ F , C_i \} \approx 0$ for $^{\forall} i$
  Reduces 2 phase space dimensions
  Generates a symmetry
  Equivalent to a pair of 2^{nd}-class constraints

$\{ C_i \mid i = 1,2,\ldots \}$: complete set of independent constraints

$A \approx B \Longleftrightarrow A = B$ when all constraints are imposed

(weak equality)

of d.o.f. in general relativity

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  All constraints are independent of N & N^i → π_N & π_i "commute with" all constraints → 1st-class
- 4 generators of 4d-diffeo: 1st-class constraints
- 20 – (4+4) x 2 = 4 → 4-dim physical phase space @ each point → 2 local physical d.o.f.

Minimal # of d.o.f. in modified gravity = 2

of d.o.f. in general relativity

- 10 metric components → 20-dim phase space @ each point
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Minimal # of d.o.f. in modified gravity = 2

Can this be saturated?

Is general relativity unique?

- Lovelock theorem says **"yes"** if we assume:
  (i) 4-dimensions; (ii) diffeo invariance; (iii) metric only; (iv) up to 2nd-order eom's of the form $E_{ab}=0$.
- Effective field theory (derivative expansion) says **"yes"** at low energy if we assume:
  (i) 4-dimensions; (ii) diffeo invariance; (iii) metric only.
- However, cosmological backgrounds break 4d-diffeo while keeping 3d-diffeo.
- A metric theory with 3d-diffeo but with broken 4d-diffeo typically has 3 local physical d.o.f. (e.g. scalar-tensor theory, EFT of inflation/dark energy, Horava-Lifshitz gravity)

Example: simple scalar-tensor theory

- Covariant action

$$I = \frac{1}{2}\int d^4x\sqrt{-g}\left[\Omega^2(\phi)^{(4)}R + P(X,\phi)\right] \qquad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- ADM decomposition

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N^i}{N^2} \\ \frac{N^j}{N^2} & h^{ij} - \frac{N^iN^j}{N^2} \end{pmatrix}$$

- Unitary gauge

$$\phi = t \quad \Rightarrow \quad X = \frac{1}{2}\frac{1}{N^2}$$

This is a good gauge iff derivative of ϕ is timelike.

- Action in unitary gauge

$$I = \int dt d^3\vec{x} N\sqrt{h}\left\{f_1(t)\left[K^{ij}K_{ij} - K^2 + {}^{(3)}R\right] + \frac{2}{N}\dot{f}_1(t)K + f_2(N,t)\right\}$$

$$\Omega^2(\phi) = f_1(t) \qquad P(X,\phi) = f_2(N,t)$$

Is general relativity unique?

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- A metric theory with 3d-diffeo but with broken 4d-diffeo typically has 3 local physical d.o.f. (e.g. scalar-tensor theory, EFT of inflation/dark energy, Horava-Lifshitz gravity)
- Is GR unique when we assume: (i) 4-dimensions; (ii) 3d-diffeo invariance; (iii) metric only; (iv) 2 local physical d.o.f. (= 2 polarizations of TT gravitational waves)?

A class of minimally modified gravity

Chushan Lin and SM, JCAP1710 (2017), 033

- 4d theories invariant under 3d-diffeo: $x^i \rightarrow x^i + \xi^i(t,\mathbf{x})$
- ADM decomposition

$$ds^2 = -N^2dt^2 + h_{ij}\,(dx^i+N^idt)\,(dx^j+N^jdt)$$

- Ansatz: actions linear in the lapse function N

$$S = \int dt d^3x \sqrt{h} N F\left(K_{ij}, R_{ij}, \nabla_i, h^{ij}, t\right)$$

$$K_{ij} = (\partial_t h_{ij} - \nabla_i N_j - \nabla_j N_j)/(2N)$$

- For simplicity, exclude mixed-derivative terms, i.e. those that contain spatial derivatives acted on K_{ij}
- Relation between K_{ij} and π^{ij} (momenta conjugate to h_{ij}) assumed to be invertible

$$\det\left(\frac{\partial^2 F}{\partial K_{ij}\partial K_{kl}}\right) \neq 0$$

- Seek theories with 2 local physical d.o.f.!

What we expect/need

- 10 metric components → 20-dim phase space @ each point
- $\pi_N = 0$ & $\pi_i = 0$: 1st-class constraints
- 3 generators of 3d spatial diffeo : 1st-class constraints
- If there is no other constraint then
  20 – (4+3) x 2 = 6 → 6-dim physical phase space @ each point → 3 local physical d.o.f.
- We thus need a 1st-class constraint or a pair of 2nd-class constraints to find theories with 2 local physical d.o.f.

What we found

- The necessary and sufficient condition under which a theory in this class has 2 or less local physical degrees of freedom.
- Simple examples with 2 local physical degrees of freedom

An example of MMG: square-root gravity

- Action

$$S=\int d^4x\sqrt{h}N\left[\xi M(t)^4\sqrt{\left(1+\frac{c_1(t)}{M(t)^2}\mathcal{K}\right)\left(1+\frac{c_2(t)}{M(t)^2}R\right)}-\Lambda(t)\right]$$

$$\mathcal{K}=K_{ij}K^{ij}-K^2,\ K=K^i{}_i,\ \xi=\pm 1$$

- In the weak gravity limit,

$$S\simeq\int d^4x\sqrt{h}N\left[\xi M^4-\Lambda+\frac{\xi}{2}M^2(c_1\mathcal{K}+c_2R)+\ldots\right]$$

GR with $M_p^2=\xi c_1 M^2\,,\quad c_g^2=\frac{c_2}{c_1}\,,\quad \Lambda_{\rm eff}=\frac{\Lambda-\xi M^4}{\xi c_1 M^2}$ is recovered.

- Flat FLRW with a canonical scalar $\qquad \xi=1$

$$S=\int dx^3\int dta^3\left[M^4\sqrt{N^2-\frac{6c_1}{M^2}\frac{\dot{a}^2}{a^2}}-N\Lambda+\frac{1}{2N}\dot{\phi}^2-NV(\phi)\right]$$

$$1-6c_1\frac{H^2}{M^2}=\frac{M^8}{(\Lambda+\rho_m)^2}$$

$$\boxed{H^2\to\frac{1}{6c_1^2}M_p^2\quad \text{as}\quad \rho_m\to\infty}$$

H remains finite

What we found

- The necessary and sufficient condition under which a theory in this class has 2 or less local physical degrees of freedom.
- Simple examples with 2 local physical degrees of freedom

- However, it was not clear how to couple matter to gravity in a consistent way…

Matter coupling in scalar tensor theory

- Jordan (or matter) frame

$$I = \frac{1}{2}\int d^4x\sqrt{-g^{\mathrm{J}}}\,\left[\Omega^2(\phi)\,R[g^{\mathrm{J}}] + \cdots\right] + I_{\mathrm{matter}}[g^{\mathrm{J}}_{\mu\nu};\mathrm{matter}]$$

- Einstein-frame $\quad g^{\mathrm{E}}_{\mu\nu} = \Omega^2(\phi)g^{\mathrm{J}}_{\mu\nu}$ K.Maeda (1989)

$$I = \frac{1}{2}\int d^4x\sqrt{-g^{\mathrm{E}}}\,\left[R[g^{\mathrm{E}}] + \cdots\right] + I_{\mathrm{matter}}[\Omega^{-2}(\phi)g^{\mathrm{E}}_{\mu\nu};\mathrm{matter}]$$

- Do we call this GR? No. This is a modified gravity because of **non-trivial matter coupling → <u>type-I</u>**
- There are more general scalar tensor theories where there is **no Einstein frame → <u>type-II</u>**

Type-I & type-II modified gravity

- **Type-I:**
  There exists an Einstein frame
  Can be recast as GR + extra d.o.f. **+ matter, which couple(s) non-trivially**, by change of variables
- **Type-II:**
  No Einstein frame
  Cannot be recast as GR + extra d.o.f. + matter by change of variables

Type-I minimally modified gravity (MMG)

Katsuki Aoki, Chunshan Lin and SM, arXiv:1804.03902, to appear in PRD

- **# of local physical d.o.f. = 2**
- There exists an Einstein frame
- Can be recast as GR **+ matter, which couple(s) non-trivially**, by change of variables
- **The most general change of variables = canonical tr.**
- Matter coupling just after canonical tr. → breaks diffeo → 1st-class constraint downgraded to 2nd-class → leads to extra d.o.f. in phase space → inconsistent
- Gauge-fixing after canonical tr. → splits 1st-class constraint into pair of 2nd-class constraints
- **Matter coupling after canonical tr. + gauge-fixing → a pair of 2nd-class constraints remain → consistent**

Simple example of type-I MMG

Katsuki Aoki, Chunshan Lin and SM, arXiv:1804.03902, to appear in PRD

- Start with the Hamiltonian of GR
  phase space: (N, N^i, Γ_{ij}) & (π_N, π_i, Π^{ij})
- **Simple canonical tr. $(\Gamma_{ij}, \Pi^{ij}) \rightarrow (\gamma_{ij}, \pi^{ij})$**

$$\Gamma_{ij} = -\frac{\delta F}{\delta \Pi^{ij}} \qquad \pi^{ij} = -\frac{\delta F}{\delta \gamma_{ij}} \qquad F = -\int d^3x \sqrt{\gamma} f(\tilde{\Pi}) \qquad \tilde{\Pi} := \Pi^{ij}\gamma_{ij}/\sqrt{\gamma}$$

- Gauge-fixing $\mathcal{G} \approx 0$

$$\boxed{\{\mathcal{G}, \mathscr{H}_0\} \not\approx 0} \qquad \{\mathcal{G}, \pi_N\} \approx 0 \qquad \{\mathcal{G}, \pi_i\} \approx 0 \qquad \{\mathcal{G}, \mathscr{H}_i\} \approx 0$$

- Lagrangian for $g^J_{\mu\nu} = (N, N^i, \gamma_{ij})$

$$\sqrt{-g^J}\mathcal{L} = \dot{\gamma}_{ij}\pi^{ij} - \mathcal{H}^{\mathrm{GF}}_{\mathrm{tot}}$$

$\mathcal{H}^{\mathrm{GF}}_{\mathrm{tot}}$: gauge-fixed total Hamiltonian density

- Adding matter

$$I_{\mathrm{matter}}[g^{\mathrm{J}}_{\mu\nu}; \mathrm{matter}]$$

c.f. Carballo-Rubio, Di Filippo & Liberati (2018) argued that the square-root gravity should be of type-I but did not find a consistent matter coupling.

More general example of type-I MMG & phenomenology

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM and Michele Oliosi, arXiv: 1810.01047

- Original phase space: (M, N^i, Γ_{ij}) & (Π_M, π_i, Π^{ij})
- **Canonical tr. $(\mathcal{N}, \Gamma_{ij}, \Pi_{\mathcal{N}}, \Pi^{ij}) \rightarrow (N, \gamma_{ij}, \pi_N, \pi^{ij})$**

$$\mathcal{N} = -\frac{\delta F}{\delta \Pi_{\mathcal{N}}} \quad \Gamma_{ij} = -\frac{\delta F}{\delta \Pi^{ij}} \quad \pi_N = -\frac{\delta F}{\delta N} \quad \pi^{ij} = -\frac{\delta F}{\delta \gamma_{ij}}$$

$$F = -\int d^3x \left(M^2 \sqrt{\gamma} f(\tilde{\Pi}, \tilde{\mathcal{H}}) + N^i \Pi_i\right) \qquad \tilde{\Pi} = \frac{1}{M^2\sqrt{\gamma}} \Pi^{ij}\gamma_{ij}$$

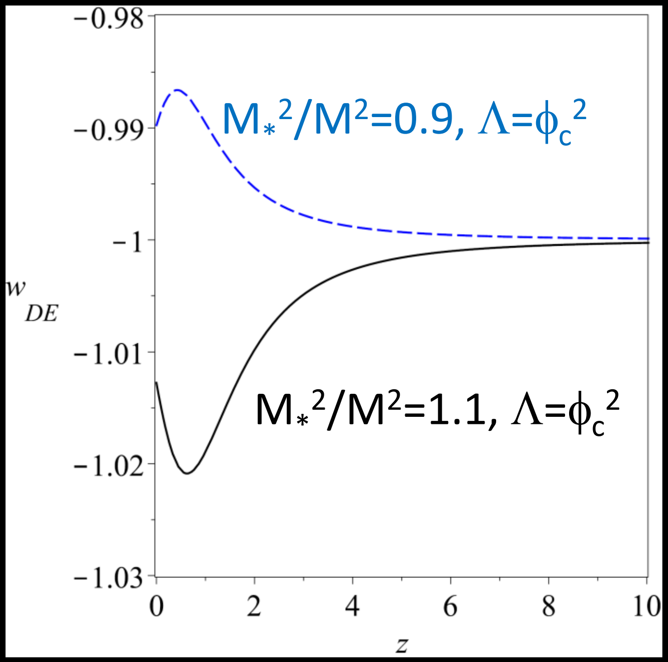
$$f(\phi, \psi) = f_0(\phi) + f_1(\phi)\psi + \mathcal{O}(\psi^2) \qquad \tilde{\mathcal{H}} = \frac{1}{M^2\sqrt{\gamma}} \Pi_{\mathcal{N}} N$$

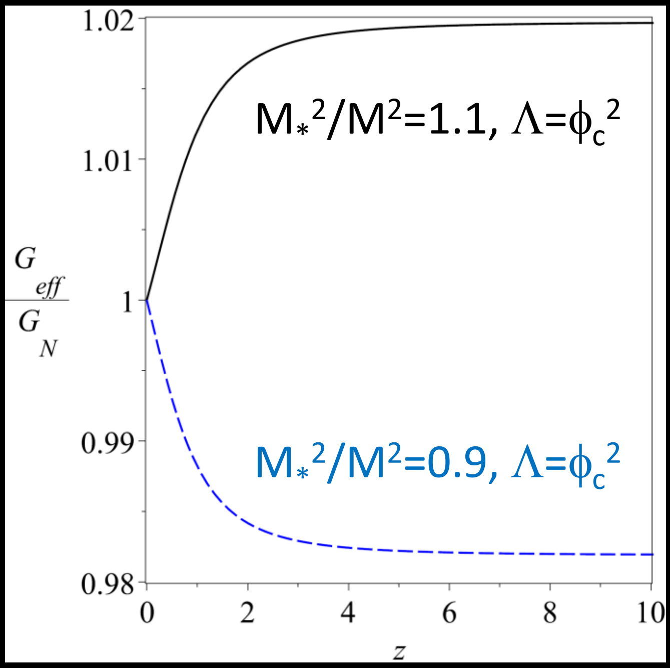
- Same sign for $\mathcal{N}$ & N, Γ_{ij} & γ_{ij} → $f_0 > 0$, $f_1 > 0$
- $c_T^2 = f_1^2/f_0'$ → $f_0' = f_1^2$
- $w_{DE} \neq -1$ in general (without dynamical DE)
- $G_{eff}/G = 1/f_0' \neq 1$ in general while $\Psi/\Phi = 1$

Example with $w_{DE} \neq -1$ & $G_{eff}/G \neq 1$

- $\Lambda \neq 0$ before canonical tr.
- $c_T^2 = f_1^2/f_0' \rightarrow f_0' = f_1^2$
- A choice of f_0

$$f_0' = \frac{(M_*/M_{\rm pl})^2 + (\phi/\phi_c)^2}{1 + (\phi/\phi_c)^2}$$





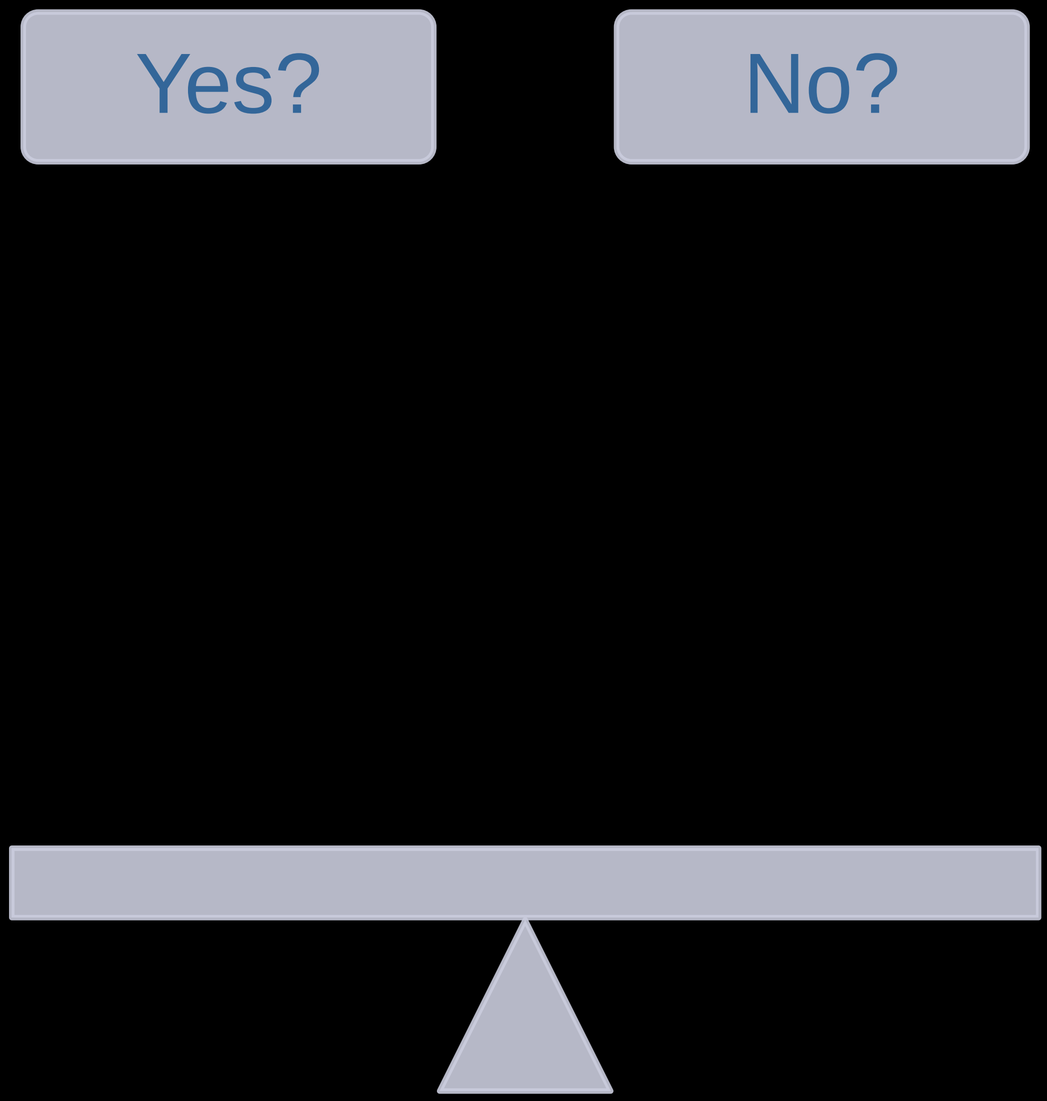
Type-II minimally modified gravity (MMG)

- **# of local physical d.o.f. = 2**
- **No Einstein frame**
- Cannot be recast as GR + matter by change of variables
- **Is there such a theory? Yes!**
- Example: Minimal theory of massive gravity
  [Antonio De Felice and SM, PLB752 (2016) 302; JCAP1604 (2016) 028; PRL118 (2017) 091104]
- Another example? : Ghost-free nonlocal gravity (if extended to nonlinear level?)

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



Massive gravity in a nutshell

Simple question: Can graviton have mass?

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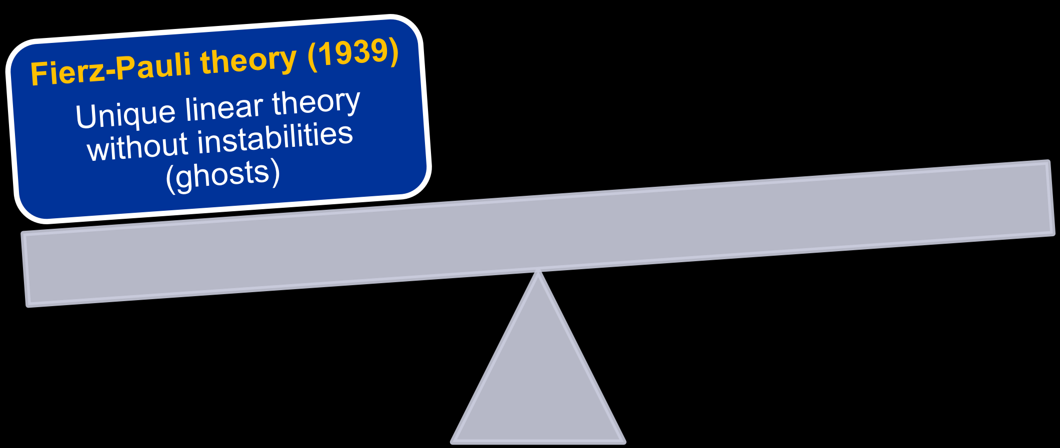
Yes? No?



Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)

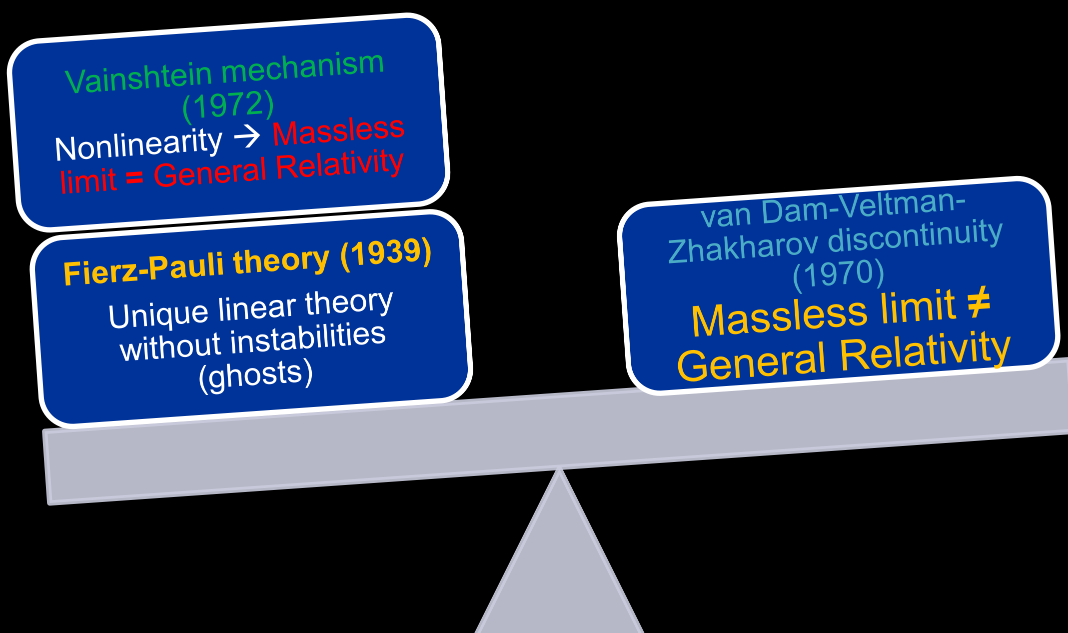
Massless limit ≠ General Relativity

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes? No?



Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Vainshtein mechanism (1972)

Nonlinearity → Massless limit = General Relativity

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)

6th d.o.f.@Nonlinear level → Instability (ghost)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

Massive gravity in a nutshell

Simple question: Can graviton have mass?

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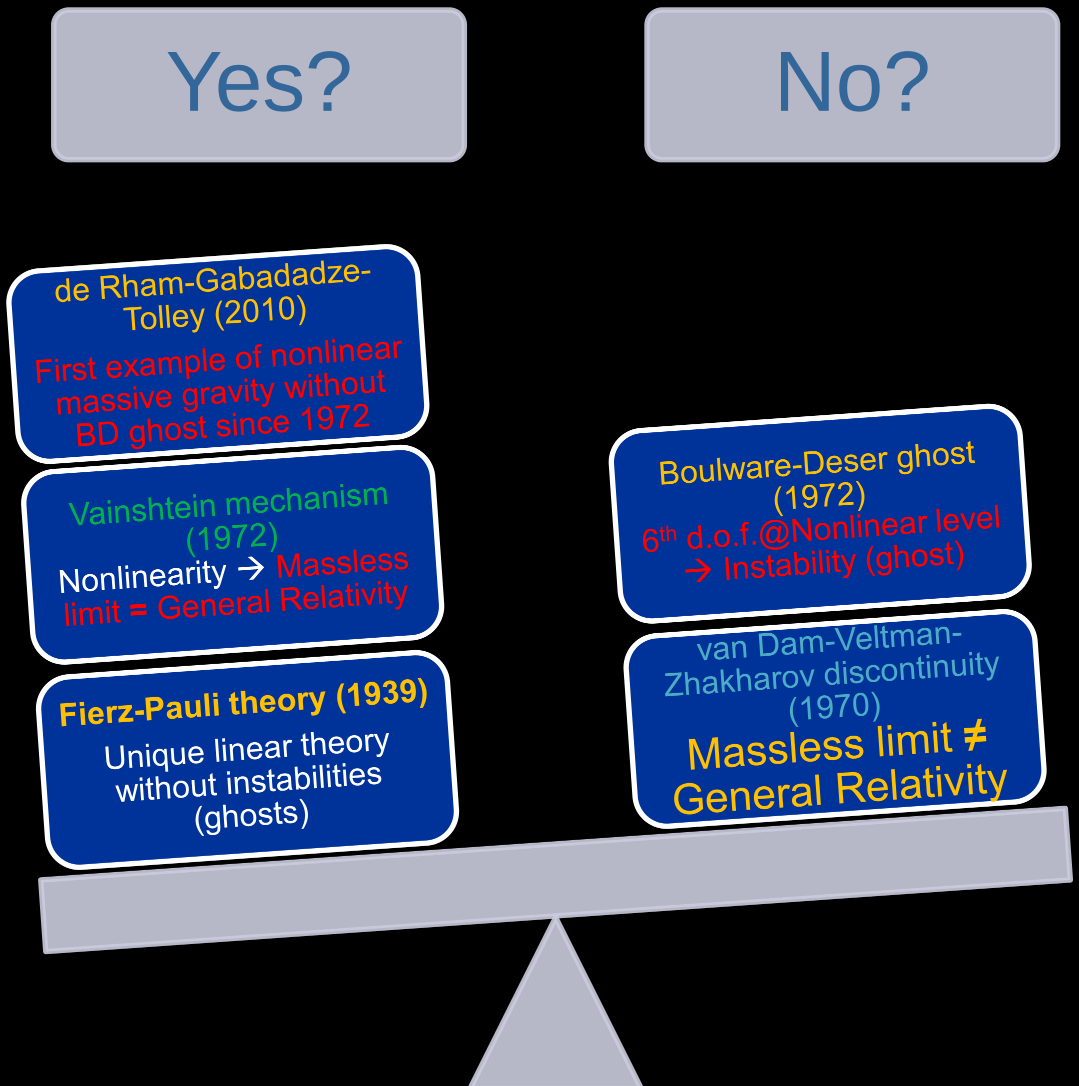
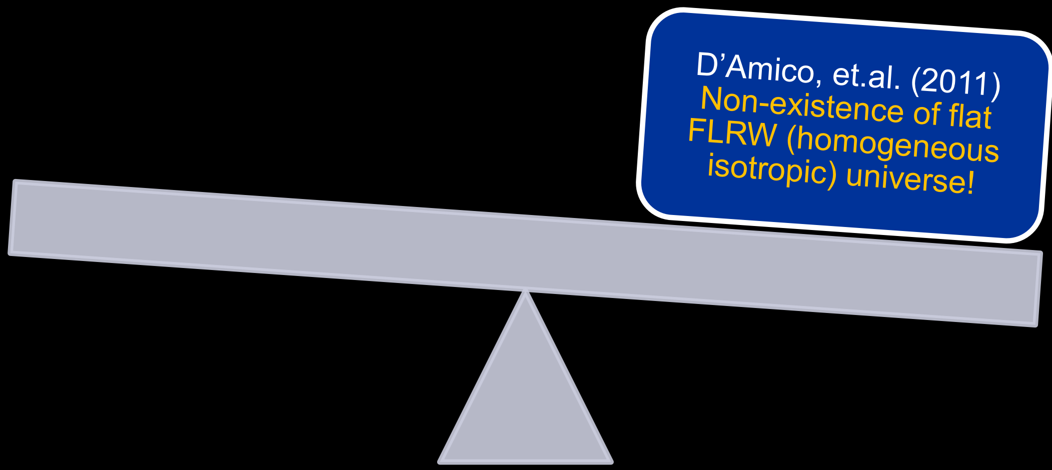


Cosmological solutions in nonlinear massive gravity

Good?

Bad?



Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Open universes with self-acceleration
GLM (2011a)

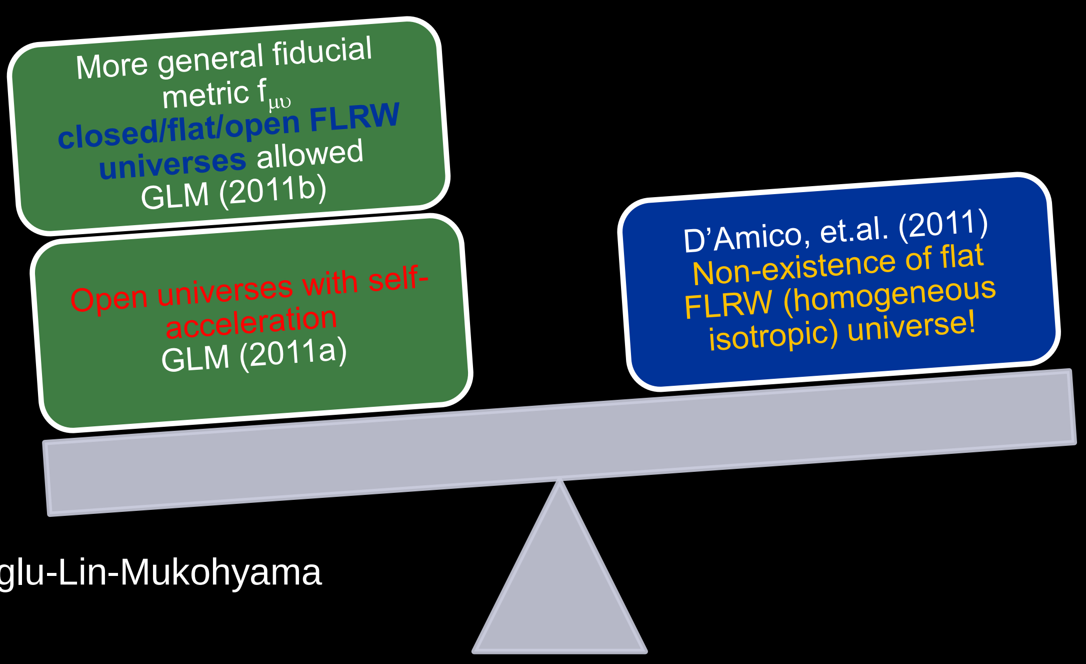
D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

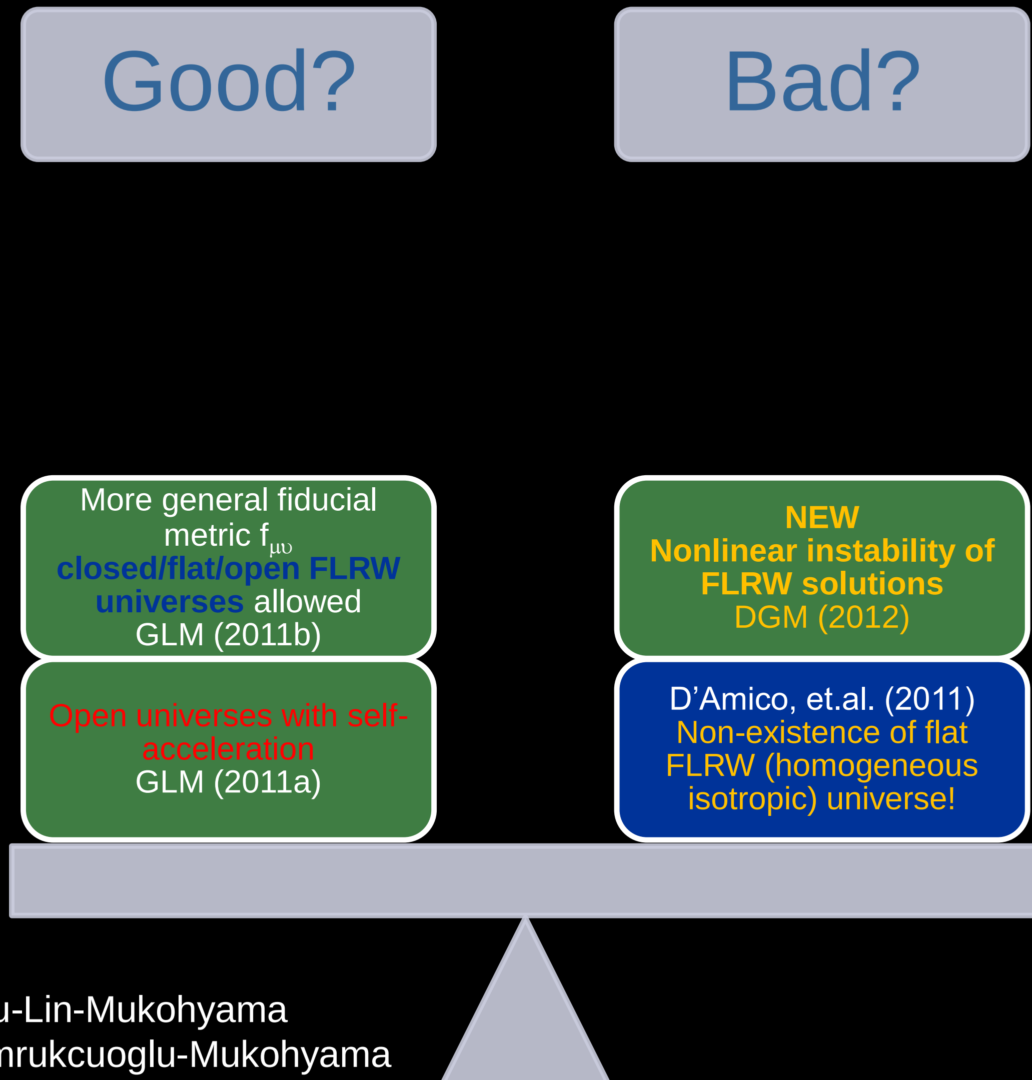
Good?

Bad?



GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Minimal Theory of Massive Gravity
DeFelice&Mukohyama (2015)

More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama

Minimal theory of massive gravity (MTMG)

De Felice & Mukohyama, PLB752 (2016) 302; JCAP1604 (2016) 028

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- positivity bound does not apply

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Lorentz-violation due to graviton loops is suppressed by m^2/M_{Pl}^2 and thus consistent with all constraints for $m = O(H_0)$

Cosmology of MTMG I

- Constraint $\mathcal{C}_0 \approx 0$ $X \doteq \tilde{a}/a$

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0$$

- Self-accelerating branch

$$X = X_\pm \doteq \frac{-c_2 \pm \sqrt{c_2^2 - c_1 c_3}}{c_1} \qquad \lambda = 0$$

$$3M_{\rm P}^2 H^2 = \frac{m^2 M_{\rm P}^2}{2}\left(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3\right) + \rho$$

$\Lambda_{\rm eff}$ from graviton mass term (even with c_4=0)

Scalar/vector parts are the same as ΛCDM

Time-dependent mass for gravity waves

Cosmology of MTMG II

- Constraint $\mathcal{C}_0 \approx 0$ $\qquad X \doteq \tilde{a}/a$

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0$$

- "Normal" branch

$$H = XH_f \qquad \lambda = \frac{4(H_f X - H)\,N}{m^2\,(c_1 X^2 + 2c_2\,X + c_3)\,M}$$

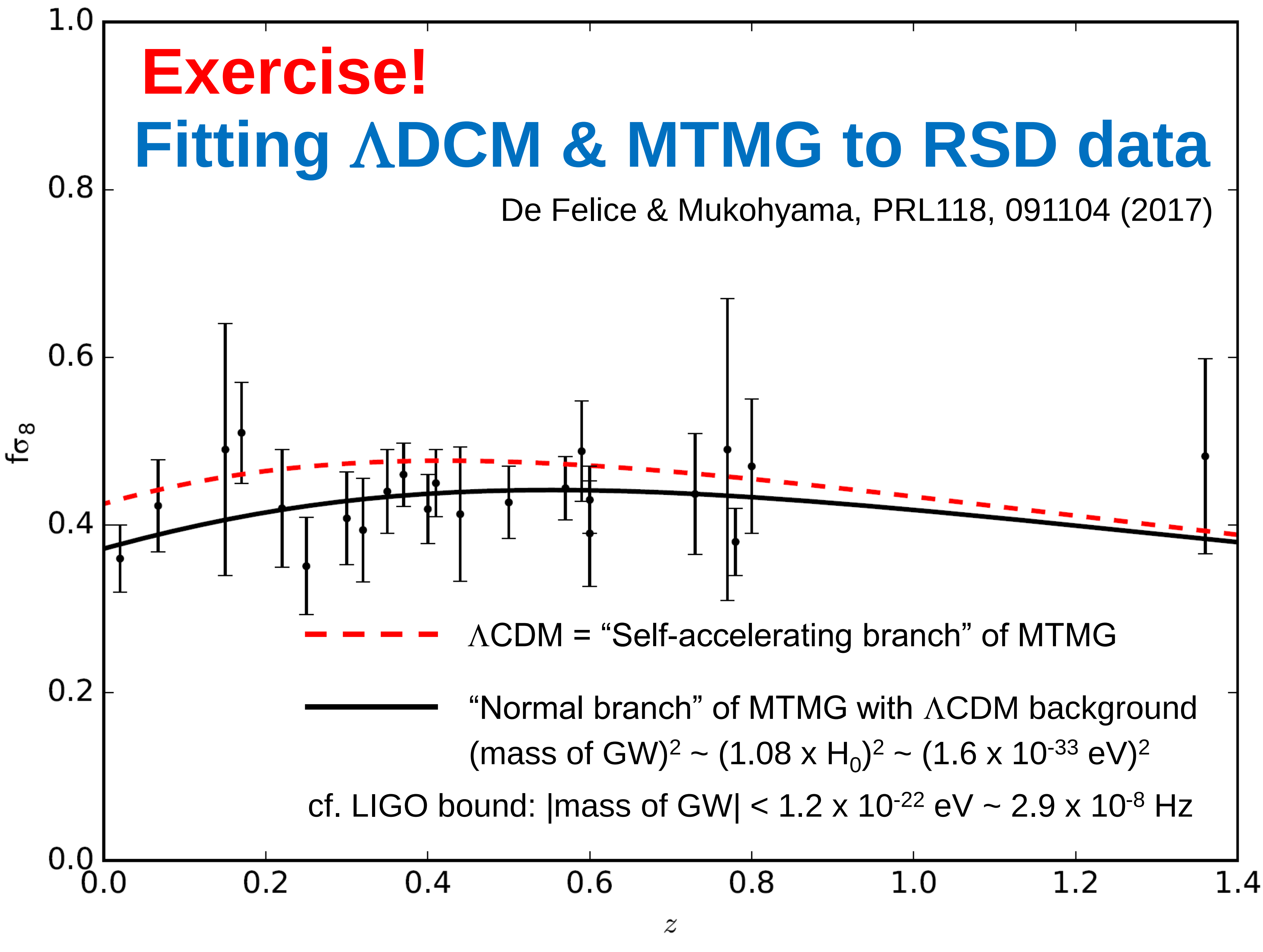
$$3M_{\mathrm{P}}^2 H^2 = \frac{m^2 M_{\mathrm{P}}^2}{2}\left(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3\right) + \rho$$

Dark component **without extra dof**

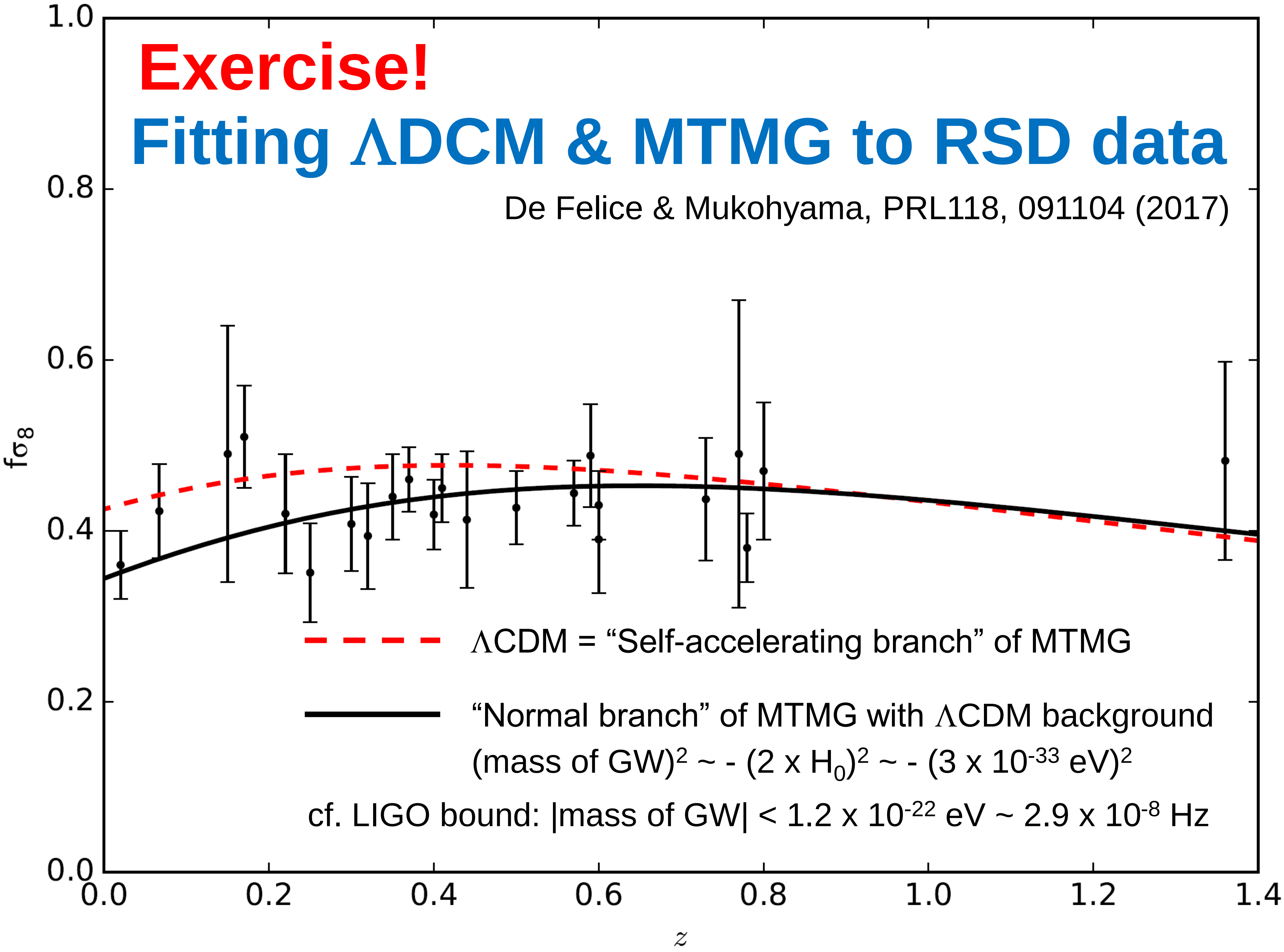
Scalar part recovers GR in UV ($L \ll m^{-1}$) but deviates from GR in IR ($L \gg m^{-1}$)

Vector part is the same as GR

Non-zero mass for gravity waves



Exercise!
Fitting ΛDCM & MTMG to RSD data
De Felice & Mukohyama, PRL118, 091104 (2017)
ΛCDM = "Self-accelerating branch" of MTMG
"Normal branch" of MTMG with ΛCDM background
(mass of GW)^2 ~ (1.08 x H_0)^2 ~ (1.6 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4


Exercise!
Fitting ΛDCM & MTMG to RSD data
De Felice & Mukohyama, PRL118, 091104 (2017)
ΛCDM = "Self-accelerating branch" of MTMG
"Normal branch" of MTMG with ΛCDM background
(mass of GW)^2 ~ - (2 x H_0)^2 ~ - (3 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

BH and Stars in MTMG

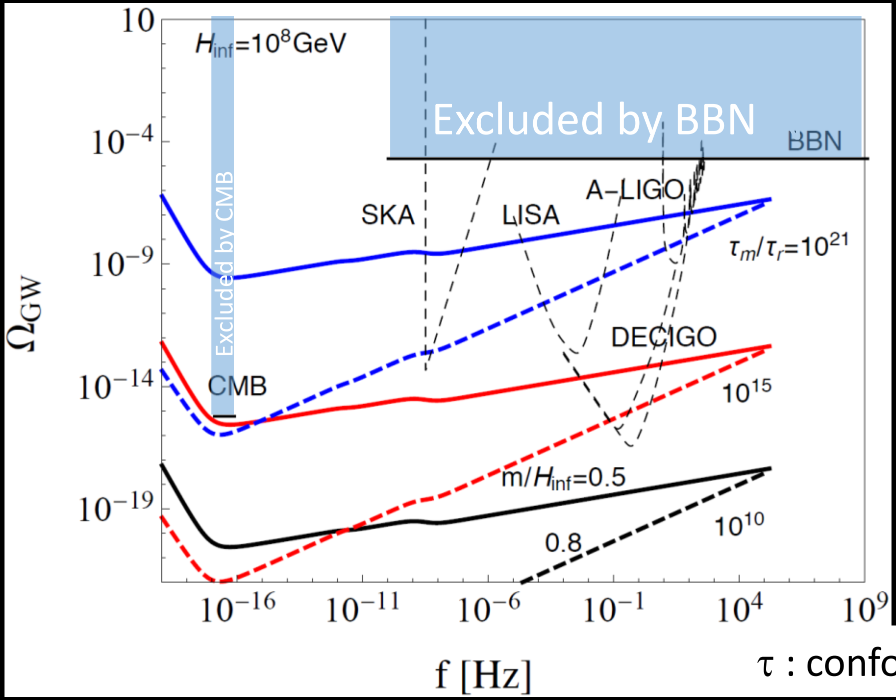
De Felice, Larrouturou, Mukohyama, Oliosi,
PRD98, 104031 (2018)

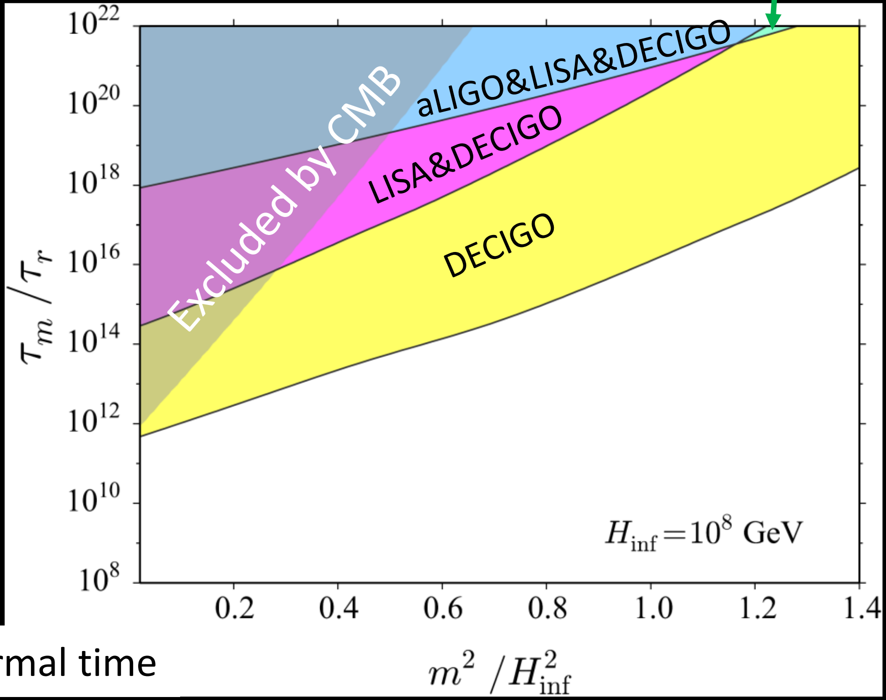
- Any solution of GR that can be rendered spatially flat by a coordinate change is also a solution of the self-accelerating branch of MTMG, with or without matter.
- Schwarzschild sol → **BH, star exterior**
- Spherical GR sol with matter → **gravitational collapse, star interior**
- **No strong coupling**
- **No singularities** except for those in GR

Blue-tilted & amplified primordial GW from MTMG

Fujita, Kuroyanagi, Mizuno, Mukohyama, arxiv: 1808.02381

- Simple extension: $c_i \rightarrow c_i(\phi)$ with $\phi = \phi(t)$
- m large until t_m ($t_{reh} < t_m < t_{BBN}$) but small after t_m
  cf. no Higuchi bound in MTMG
- Suppression of GW in IR due to large m → **blue spectrum**
- $\rho_{GW} \propto a^{-3}$ for $t_{reh} < t < t_m$ → **amplification relative to GR**





τ : conformal time

Summary

- Minimal # of d.o.f. in modified gravity = 2
  can be saturated → minimally modified gravity (MMG)
- Type-I MMG: $\exists$ Einstein frame
  Type-II MMG: no Einstein frame
- Example of type-I MMG
  GR + canonical tr. + gauge-fixing + adding matter
  Rich phenomenology: w_{DE}, G_{eff}, etc.
- Example of type-II MMG
  Minimal theory of massive gravity (MTMG)
  Cosmology: self-accelerating branch & normal branch
  BHs and stars: no strong coupling, no new singularity
  Stochastic GWs: blue-tilted & largely amplified