Low Frequency Gravitational Wave Detection With Ground Based Atom Interferometer Arrays


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• Introduction to Gravitational Wave (GW) detection
  → Why using atom interferometry?
• Principle of atom interferometry
• Low frequency GW detection with an array of Atom Interferometers
• The MIGA instrument.
Introduction to Gravitational Waves and their detection

View from an experimentalist in atomic physics...
Gravitational Waves

- GW : Perturbation of space-time propagating at the speed of light
- Sources of GW : accelerated massive objects (e.g. compact binaries)

- Illustration of the effect of a plane GW on matter (this picture is coordinate system dependent)

Generation of GW

Gravitationnal luminosity (energy radiated in unit of time in the form of GW)

\[ L = \frac{1}{5} \frac{G}{c^5} \langle \dddot{Q}_{ij} \dddot{Q}^j \rangle \]

Comparison with the electromagnetic energy radiated by an oscillating dipole

\[ L_{e.m.} = \frac{\mu_0}{12\pi c} \langle \dddot{D}_i \dddot{D}^i \rangle \]

Order of magnitude for a mass distribution of radius \( R \), mass \( M \) and asymmetry coeff. \( s \)

\[ Q \sim sM R^2 \quad \dddot{Q} \sim s \omega^3 M R^2 \]

\[ L \sim \frac{G}{c^5} s^2 \omega^6 M^2 R^4 \]
**Generation of GW**

\[ L \sim \frac{G}{c^5} s^2 \omega^6 M^2 R^4 \]

\[ \frac{c^5}{G} \sim 10^{52} \text{ W} ! \]

→ GW generated by laboratory masses are extremely small

\[ L \sim 10^{-29} \text{ W} \text{ for a 490 ton cylindric bar of 20 m and rotating at 28 rad/s !!} \]

For astrophysical objects, this formula can be re-written as

\[ L \sim \frac{c^5}{G} s^2 \left( \frac{R_S}{R} \right)^2 \left( \frac{v}{c} \right)^6 \]

with \( R_S = \frac{2GM}{c^2} \) the Schwarzschild radius and \( v = R\omega \) the radial velocity (Weber, 1974)

Orbiting black holes or neutron stars : \( L \sim 10^{52} \text{ W} !! \)

Strain amplitude of the GW :

\[ h \sim 2 \times 10^{-19} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{1 \text{ Mpc}}{r} \right) \left( \frac{1 \text{ kHz}}{f} \right) \left( \frac{1 \text{ ms}}{\tau} \right)^{1/2} \varepsilon^{1/2} \]

Very small on Earth...
GW astronomy

The big picture of gravitational-wave astronomy

Cosmic Microwave Background

- Primordial gravitational waves

Pulsar Timing Arrays

- Supermassive black hole binaries
- Primordial gravitational waves

Space-based Interferometers

- Stellar mass compact binaries
- Supermassive black hole mergers

Ground-based Interferometers

- Neutron star mergers
- Black hole mergers

McLaughlin, GRG 46, 1810 (2014)
Detection of GW with laser interferometers

- Quadrupole nature of GW $\rightarrow$ use a Michelson interferometer
- GW effect: one arm of the interferometer is increased in length while the other arm is decreased
- The induced length change results in a change of the light intensity observed at the interferometer output.

\[
\Delta L(t) = h(t)L \quad \Delta \varphi = 2\pi \frac{2\Delta L}{\lambda} \quad L = 3 \text{ km} \rightarrow \Delta L \sim 3 \times 10^{-19} \text{ m}
\]
Strain sensitivity curve

Measured sensitivity curve
GW detection with AI?

**Motivation:** at low frequencies (<10 Hz), optical GW detectors are limited by *motion noise*

- Residual seismic noise
- Suspension thermal noise
- Mirror thermal noise
- Etc.

More on the noise sources: see, e.g.,

*The VIRGO sensitivity curve* - *VIR-NOT-PER-1390-51 (2004)*

Why not using « perfectly » free falling test masses to measure the laser phase?

→ Atom interferometry !
GW detection with AI: principle

Very brief history of the developments

- Theoretical studies on the effect of GW in laser spectroscopy started in ~ 1980’s (Bordé, Tourrenc et al., …)
- New gain of interest in ~ 2004 because of many advances in the field of AI
- Description of the effect of a GW on the AI phase (2004-2008) ; Chiao et al, Roura et al, Delva et al, Vetrano et al, Bordé, …
- Reading the optical phase using two distant AI and a differential measurement (Tino, Dimopoulos, Delva, …)
Principle of Atom Interferometry

Atomic physics part of the talk
Principle of Atom Interferometry

- Analogy with a Mach-Zehnder optical interferometer
- Use lasers to coherently manipulate a matter-wave
Two photon transitions

Example: AI with stimulated Raman transition

Momentum transfer

\[ k_{\text{eff}} = k_1 + k_2 \]

Laser phase difference imprinted on the atoms

\[ \varphi = \phi_1 - \phi_2 = \vec{k}_{\text{eff}} \cdot \vec{r}'(t) \]
Rabi oscillation - Interferometer building blocks

Rabi oscillation between $|f\rangle$ and $|e\rangle$

Transition Probability $f \rightarrow e$

Pulse duration

1

1/2

“$\pi$” pulse = mirror

$|e, p + \hbar \vec{k}_{\text{eff}}\rangle$

$|f, \vec{p}\rangle$

$|\bar{k}_1, \omega_1\rangle$

“$\pi/2$” pulse = beam splitter

$|e, p + \hbar \vec{k}_{\text{eff}}\rangle$

$|f, \vec{p}\rangle$

$|\bar{k}_1, \omega_1\rangle$
Phase difference

UP: \[ \varphi(0) - \varphi(T) + \varphi(2T) \]

DOWN: \[ 0 + \varphi(T) + 0 \]

Simple picture of the AI: sampling of the atomic trajectory by the lasers at 3 different times.

\[ \Delta \Phi = \vec{k}_{\text{eff}} \cdot \vec{a}T^2 \]
Measurement of the phase difference

\[ \Delta \Phi = \vec{k}_{\text{eff}} \cdot \vec{a}T^2 \]

Interferometer output signal

\[ P = P_0 + A \cos \Delta \phi \]
Why do we need cold atoms?

The contrast of the interferometer is limited by:

- The transverse spread of the cloud in the Raman lasers
- The velocity selectivity of the Raman transition (Doppler effect)

→ need atoms with rms velocities ~ 1 cm/s → temperature ~ 1 µK
Typical experimental sequence

**Typical values:**

- $2T = 150$ ms, $10^6$ atoms @ 1 µK
- Lasers of ~100 mW power, 1 cm beam radius, phase locked @ < mrad level
- Cycle time ~ 0.3 – 1 s.
Cold atom gravimeter (2003 →)
Transportable gravimeter
Orders of magnitude of performances

• Atomic gravimeters (SYRTE & HU Berlin):
  → sensitivity $\sim 5 \times 10^{-9} \, g / \sqrt{Hz}$; long term: $5 \times 10^{-11} \, g$; accuracy $\sim 3 \times 10^{-9} \, g$

• Atomic gyroscope of SYRTE:
  → Sensitivity 1 nrad/s after 10,000 s of integration time

• Measurement of the gravitational constant G with 150 ppm accuracy (group of Tino, Italy)

• ... about 25 research groups worldwide.
Gravitational Wave detection with an array of Atom Interferometers
Effect of the GW on the AI

- Simple view: the GW « moves » the mirror by $hL/2$ and the AI records the position of the mirror at the time of the 3 light pulses.
- Rigorous calculation with the metric to compute the phase of the lasers imprinted on the atom at the light pulse:

$$\Delta \phi_x (X, t) = \epsilon (X, t) + 2nk \times \left[ \left( \frac{\Delta \nu(t)}{\nu} \right) + \frac{h(t)}{2} \right] (L - X) + \Delta x_2(t) - \Delta x(X, t) \otimes s(t)$$

- Detection noise
- Laser frequency noise
- GW signal
- Position noise of the retro-mirror
- Effect of local gravity
- AI sensitivity function
AI gradiometer

- Measurement of the differential phase between 2 physically separated AIs
- Gradiometer signal = \( \phi(X) - \phi(X + L) \)

- Position noise of the retro-reflecting mirror is common \( \rightarrow \) no sensitivity to \( \Delta x_2 \)
AI gradiometer

- Gradiometer signal $= \phi(X) - \phi(X + L)$

\[
\psi(X, t) = 2nk \left[ \frac{L \ddot{h}(t)}{2} + a_x(X + L, t) - a_x(X, t) \right] \otimes s_\alpha(t)
\]

GW signal
Gravity gradient noise

It is impossible to distinguish a fluctuating gravity gradient from the GW signal with 2 test masses.

- « Newtonian Noise » (fundamental limit) ; well known in optical GW detectors
- Limit for observations on ground below few Hz.
Newtonian Noise (gravity gradient noise)

Terrestrial gravitational noise on a gravitational wave antenna

Peter R. Saulson
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 December 1983)

• Mass fluctuations in one region of space:

\[
\frac{F_x}{m} = G \Delta M(\omega) \frac{\cos \theta}{r^2}
\]

Main sources of NN at frequencies $< 10$ Hz

- Mass fluctuations in one region of space:

\[ \frac{F_x}{m} = G \Delta M(\omega) \frac{\cos \theta}{r^2} \]

- Saulson: assume that the space is filled with regions of fluctuating mass and that the size of a coherently fluctuating region is $\lambda/2$ (= noise correlation length)

<table>
<thead>
<tr>
<th>Seismic NN (Density fluctuations due to ground motion)</th>
<th>Infrasound NN (Density fluctuations in the near atmosphere)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground motion</td>
<td>Air density fluctuations caused by turbulence</td>
</tr>
<tr>
<td>Velocity of P-waves</td>
<td>Sound velocity</td>
</tr>
<tr>
<td>Correlation length: 1 km (at 1 Hz)</td>
<td>200 m (at 1 Hz)</td>
</tr>
</tbody>
</table>
Beating Newtonian Noise with AI arrays

**General idea** (W. Chaibi): repeat the gradiometer experiment to obtain several realizations of the NN.

The NN characteristic length (few km at most) is $\ll$ GW wavelength

→ average the NN to zero.

$$H_N(t) = \frac{1}{N} \sum_{i=1}^{N} \psi_i(t),$$
Assume $N$ independent realizations of the gradiometer experiment. Resulting noise:

$$H_N(t) = \frac{1}{N} \sum_{i=1}^{N} \psi_i(t),$$

$$\sigma_{H_N} = \frac{\sqrt{2}\sigma_\eta}{\sqrt{N}},$$

$$\sigma_\eta = \sqrt{\sigma_{\alpha}^2 + \sigma_{\epsilon}^2},$$

Gravity acceleration noise

Detection noise
Implementation with an AI array
We consider an upgrade of the Saulson model where we take into account mass conservation between adjacent cells

→ introduces mass correlations between adjacent cells (Beccaria et al, 1998)

→ Modification of the gravity acceleration correlation function \( \langle a(X_1, t)a(X_2, t + \tau) \rangle \)
Gravity acceleration correlation function

![Graph showing the spatial behavior of the normalized NN correlations between two distant points separated by the relative distance $x = |X_j - X_i|/\mathcal{L}_\rho(\omega)$, where $\mathcal{L}_\rho(\omega)$ is the NN correlation length. The anti-correlation is a consequence of mass conservation between adjacent cells of fluctuating density.](image)

**FIG. 2.** Spatial behavior of the normalized NN correlations between two distant points separated by the relative distance $x = |X_j - X_i|/\mathcal{L}_\rho(\omega)$, where $\mathcal{L}_\rho(\omega)$ is the NN correlation length. The anti-correlation is a consequence of mass conservation between adjacent cells of fluctuating density.
Variance reduction

**Idea 2** (W. Chaibi): in Monte Carlo methods, you can choose your sampling points in a clever way to obtain an error lower than $1/\sqrt{N} \rightarrow \text{"variance reduction"}$ (e.g. $0.3/\sqrt{N}$)

$\rightarrow$ use the correlations/anti-correlations between the sampling points

In the AI array, we tune the separation $\delta$ between the different gradiometers to exploit the gravity acceleration correlation between the sampling points.
Proposition of implementation

N = 80 AIs per sub-harm (320 in total) ; L = 16 km ; δ = 200 m between the gradiometers

Al phase noise = -140 dB rad^2/Hz ; n = 1000 LMT beam splitters

Challenge for cold atom physics to reach such low detection noise !!
Closer look at the rejection efficiency

Maximum rejection obtained when the NN correlation length = distance corresponding to anticorrelation of the gravity acceleration correlation function: 1.1 Hz for INN / 6.5 Hz for SNN.
Many sources are predicted in the 0.3-3 Hz frequency band (see e.g. Harms et al, PRD 88, 122003 (2013))
The MIGA project:
Matter wave laser Interferometric Gravitation Antenna

References
• B. Canuel et al, E3S Web of Conferences 4, 01004 (2014)
The MIGA project

- **Equipex project**: 10 years (2013 – 2023), 9 M€, 13 research institutes, 2 companies
- **Goal**: precision gravity measurements with Atom Interferometry (AI)
- Design and realization of an instrument for 2 applications:
  1. **Monitoring of underground mass distributions**
     - Applications: geophysics, hydrology
  2. **Test setup for applications of AI to gravitational wave (GW) detection**

- At SYRTE: realization of the cold atom source units, AI metrology expertise; modelling of the instrument (this presentation)
Overview of the MIGA project

Implementation site

• Low noise underground laboratory
• Site of (hydro)-geological interest
Orders of magnitude

Interrogation time $2T \approx 0.5 \text{ s}$; Phase sensitivity $= 1/$SNR $\sim 1 \text{ mrad/shot}$

Interferometer phase shift at position $x$:

$$\Delta \phi(x) = 2kT^2 a(x)$$

Acceleration sensitivity $\sim 10^{-10} \text{ m. s}^{-2}/\sqrt{\text{Hz}}$

Gravity gradient sensitivity $\sim 10^{-13} \text{ s}^{-2}/\sqrt{\text{Hz}} \Rightarrow 1 \text{ ton at 100 m}$

Rb87 atoms:

$\lambda = 780 \text{ nm}$

$L = 300 \text{ m cavity}$

Orders of magnitude
MIGA main subsytems

- SYRTE (Paris): cold atom source and detection system, AI expertise
- LP2N (Talence): cavity design, vacuum system
- ARTEMIS (Nice): cavity mirror suspensions, GW detection expertise
- µQuans (Talence): laser systems
- LSBB (Rustrel): tunnels & site management, geophysics expertise

MIGA installation at LSBB : mid 2018
MIGA vacuum system (L2PN)
Cold atom source (SYRTE)

Design: Louis Amand

$10^8$ atoms at 2 µK
launched at 4 m/s
MIGA : status and perspectives

- First cold atom source delivered by SYRTE to LP2N (June 2015)
- 6 m Al gradiometer in the optical cavity under design
- Beginning of the digging of the MIGA galleries at LSBB (Summer 2016)
- MIGA installation at LSBB in 2018
Conclusions

- **Light-pulse atom interferometry**: measure the phase of lasers using atoms → precision inertial measurements
- **GW detection with AI**: use free falling atoms instead of suspended mirrors → potential gain at low frequency (< 10 Hz)
- Previous proposals did not address the fundamental issue of **Newtonian Noise**
- We propose a detection strategy using an **AI array to efficiently reject the NN**
- Many **challenges** in cold atom physics to reach ~ $10^{-23}/\sqrt{Hz}$ strain sensitivity levels around 1 Hz → **AI could nicely complement** optical interferometry in the ~ 0.3 – 3 Hz band
- **MIGA**: proof of concept + applications in geosciences.
The MIGA team (part of it)

**SYRTE**
- D. Holleville
- L. Amand
- A. Landragin
- R. Geiger

**ARTEMIS**: W. Chaibi

**The SYRTE gyroscope team**: I. Dutta, D. Savoie, B. Fang

**LP2N**: I. Riou, J. Gillot
- P. Bouyer
- B. Canuel
- A. Bertoldi

**LSBB**: S. Gaffet

**N. Mielec**
Thank you!

IACI team of SYRTE, Jan. 2015
Implementation with an AI array

We need to obtain random realizations of the noise \( \Rightarrow \) do this by measuring at different positions, and average the measurements.

\[
\{ \psi(X_i, t) \equiv \psi_i(t) \}_{i=1..N} \quad H_N(t) = \frac{1}{N} \sum_{i=1}^{N} \psi_i(t),
\]

Power Spectral Density of the detector output:

\[
S_{H_N}(\omega) = (2nkL)^2 \omega^4 S_h(\omega) |\hat{s}_\alpha(\omega)|^2 + (2nk)^2 S_a(\omega) |\hat{s}_\alpha(\omega)|^2 + \frac{4S_\epsilon(\omega)}{N},
\]

GW signal

Shot noise uncorrelated between distant AIs

NN remaining after averaging
Noise spectra

(a) Acceleration noise (m.s^{-2}.Hz^{-1/2})

(b) Air pressure noise (Pa.Hz^{-1/2})
Sensitivity of future detectors

Adhikari, RMP 2014
Geoscience @ LSBB: motivation

Karstic aquifers: complex multi-scale hydrodynamics

Need for non-invasive measurements in (3+1)D in order to construct and constrain hydrodynamics numerical models.

Courtesy: C. Danquigny, Univ. Avignon
Advantage of AI: non-invasive localisation of masses, e.g. water

- Sensitivity to masses of the order of 1-10 tons
- Spatial resolution depending on the number of AIs in the antenna
- Long term stability (monitoring over years is in principle possible)

→ Typical timescales which can be explored: few hours to months.

AI complements other methods such as:

- GPR = Ground Penetration Radar
- Seismic imaging
- DC Electrical probing.
Title