The updated
International System of Units (SI)

Richard Davis

rdavis@bipm.org

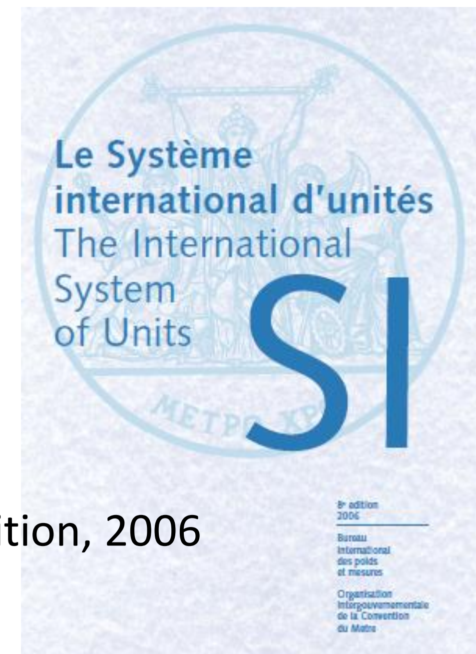
BIPM Consultant

(Former director of BIPM Mass Department)

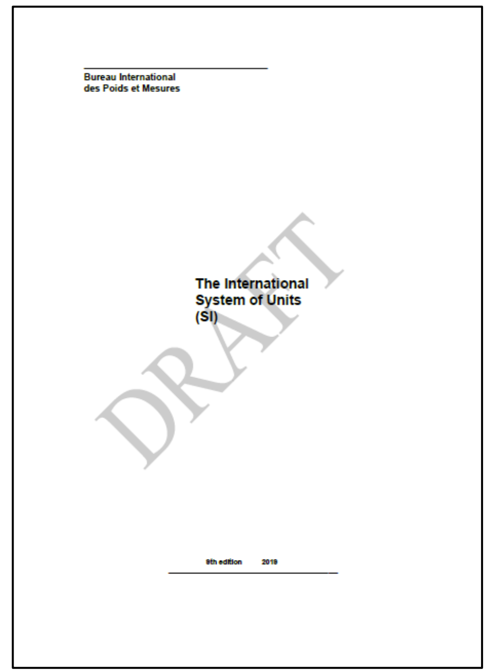
Bureau
International des
Poids et
Mesures

The BIPM and the SI

- BIPM: the intergovernmental organization through which Member States act together on matters related to measurement science and measurement standards



8th edition, 2006



https://www.bipm.org/en/publications/si-brochure/

9th edition, 2019

The present SI is based on 7 reference values

- the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 hertz, (1967) s^{-1}
- the speed of light in vacuum c is 299 792 458 metres per second, (1983) $m\ s^{-1}$
- the mass of the international prototype of the kilogram $m_{\mathcal{K}}$ is 1 kilogram, (1889) kg
- the permeability of vacuum μ_0 is $4\pi \times 10^{-7}$ newton per ampere squared, (1948, 1954) $kg\ m\ s^{-2}\ A^{-2}$
- the thermodynamic temperature of the triple point of water T_{TPW} is 273.16 kelvin, (1954) K
- the molar mass of carbon 12, $M(^{12}C)$, is 0.012 kilogram per mole, (1971) $kg\ mol^{-1}$
- the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, is 683 lumens per watt. (1979) $cd\ kg^{-1}\ m^{-2}\ s^{-3}$

Draft Resolution A (26th CGPM, November 2018)

The International System of Units, the SI, is the system of units in which

- **the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 Hz,** s^{-1}
- **the speed of light in vacuum c is 299 792 458 m/s,** $m\ s^{-1}$
- **the Planck constant h is 6.626 070 15 × 10^{-34} J s,** $kg\ m^2\ s^{-1}$
- **the elementary charge e is 1.602 176 634 × 10^{-19} C,** A s
- **the Boltzmann constant k is 1.380 649 × 10^{-23} J/K,** $kg\ m^2\ s^{-2}\ K^{-1}$
- **the Avogadro constant N_A is 6.022 140 76 × 10^{23} mol^{-1},**
- **the luminous efficacy of monochromatic radiation of frequency 540 × 10^{12} hertz K_{cd} is 683 lm/W.**

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to Hz = s^{-1}, J = $m^2\ kg\ s^{-2}$, C = A s, lm = cd $m^2\ m^{-2}$ = cd sr, and W = $m^2\ kg\ s^{-3}$

https://www.bipm.org/utils/en/pdf/CGPM/Draft-Resolution-A-EN.pdf
https://www.bipm.org/utils/fr/pdf/CGPM/Draft-Resolution-A-FR.pdf

A momentous change in 2018/2019

BIPM photo

1 kg = $m_{\mathcal{K}}$ (at present)

1 kg = 1.475… $\times 10^{40}\ h\,\Delta\nu_{Cs}/c^2$

- Definition of **kilogram** (1889) replaced by fixed value for the Planck constant, h
- Definition of **ampere** (1948, 1954) replaced by fixed value for the elementary charge, e
- Definition of **kelvin** (1954) replaced by fixed value for the Boltzmann constant, k
- Definition of **mole** (1971) replaced by fixed value for the Avogadro constant, N_A

No change to definitions of the **second** ($\Delta\nu_{Cs}$, 1967) and the **metre** (c, 1983) [or the **candela** (1978)]

All definitions will be in 'explicit constant' form--Example

Definition of the metre since 1983:

"The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."

In symbols, 1 m = c·(s/299 792 458)

Definition expected in 2019 :

"The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum *c* to be 299 792 458 when expressed in the unit m s^{-1} , where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$."

In symbols, c = 299 792 458 m s^{-1} $\rightarrow$ 1 m = (c/299 792 458) s

or 1 m = 30.663 318…$c/\Delta\nu_{Cs}$

Before the SI, electrical units were problematic

- 1941 Table of Physical Constants (*Rev. Sci. Instrum.*)

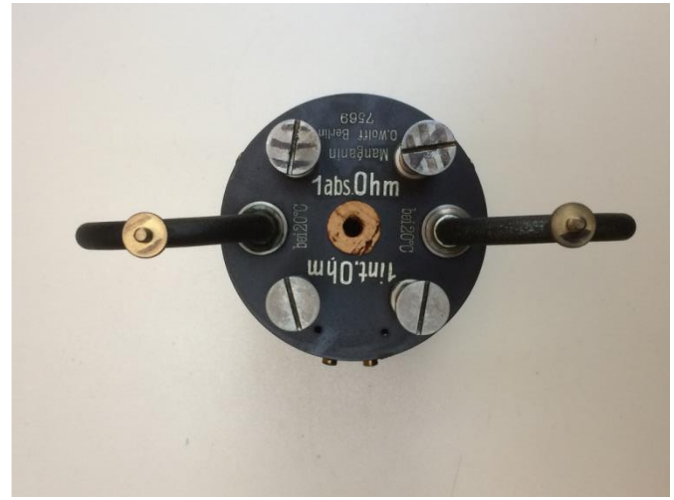
234 RAYMOND T. BIRGE

Table a. Principal Constants and Ratios.*

Section		
A	Velocity of light	$c=(2.99776\pm0.00004)\times10^{10}$ cm·sec^{-1}
B	Gravitation constant	$G=(6.670\pm0.005)\times10^{-8}$ dyne·cm^2·g^{-2}
C	Liter (=1000 ml)	$l=1000.028\pm0.002$ cm^3
D	Volume of ideal gas (0°C, A_0)	$V_0=(22.4146\pm0.0006)\times10^3$ cm^3·atmos·mole^{-1} $V_0'=22.4140\pm0.0006$ liter·atmos·mole^{-1}
D	Volume of ideal gas (0°C, A_{45})	$V_{45}=(22.4157\pm0.0006)\times10^3$ cm^3·atmos·mole^{-1} $V_{45}'=22.4151\pm0.0006$ liter·atmos·mole^{-1}
E	International ohm (= p abs-ohm)	$p=1.00048\pm0.00002$
E	International ampere (= q abs-amp)	$q=0.99986\pm0.00002$
F	Atomic weights (see Table a′)	
G	Standard atmosphere	$A_0=(1.013246\pm0.000004)\times10^6$ dyne·cm^{-2}·atmos^{-1}
G	45° atmosphere	$A_{45}=(1.013195\pm0.000004)\times10^6$ dyne·cm^{-2}·atmos^{-1}
H	Ice-point (absolute scale)	$T_0=273.16\pm0.01$°K
I	Joule equivalent	$J_{15}=4.1855\pm0.0004$ abs-joule·cal$_{15}^{-1}$
I	Joule equivalent (electrical)	$J_{15}'=4.1847\pm0.0003$ int-joule·cal$_{15}^{-1}$
J	Faraday constant	
	(1) Chemical scale	$F=96501._2\pm10$ int-coul·g-equiv^{-1} $=96487._7\pm10$ abs-coul·g-equiv^{-1} $=9648.7_7\pm1.0$ abs. e.m.u·g-equiv^{-1} $F'=Fc=(2.89247\pm0.00030)\times10^{14}$ abs. e.s.u·g-equiv^{-1}
	(2) Physical scale	$F=96514._0\pm10$ abs-coul·g-equiv^{-1} $=9651.4_0\pm1.0$ abs. e.m.u·g-equiv^{-1} $F'=Fc=(2.89326\pm0.00030)\times10^{14}$ abs. e.s.u·g-equiv^{-1}
K	Avogadro number (chemical scale)	$N_0=(6.0228_3\pm0.0011)\times10^{23}$ mole^{-1}
K	Electronic charge	$e=F/N_0=1.60203_3\pm0.00034)\times10^{-20}$ abs. e.m.u. $e'=ec=(4.8025_1\pm0.0010)\times10^{-10}$ abs. e.s.u.
L	Specific electronic charge	$e/m=(1.7592\pm0.0005)\times10^7$ abs. e.m.u·g^{-1} $e'/m=ec/m=(5.2736_6\pm0.0015)\times10^{17}$ abs. e.s.u·g^{-1}
M	Planck constant	h (see Table c).

International *versus* Absolute electrical units

BIPM photo



BIPM photo

International electrical units used by engineers
Absolute electrical units used by physicists

"It surely would be a great misfortune to the whole scientific world if in taking up a standard ohm coil…20 years hence, it should be necessary to ask whether it was standardized for physicists or for electrotechnicians."
-- Arthur Kennelly, 1935

Impact on science due to earlier changes to the SI - 2

- Cohen and Taylor 1973 *J. Phys. Chem. Ref. Data*

TABLE 32.1. Our best set of WQED constants based on adjustment No. 21 of table 29.3. $\chi^2 = 8.75$ for $18 - 5 =$ degrees of freedom; $R_B = 0.82$.[a]

Quantity	Symbol	Value	Uncertainty (ppm)	Units	
				SI	cgs[b]
Speed of light in vacuum	c	299792458(1.2)	0.004	$m \cdot s^{-1}$	$10^{2}\ cm \cdot s^{-1}$
Fine-structure constant, $[\mu_0 c^2/4\pi](e^2/\hbar c)$	α	7.2973461(81)	1.1	10^{-3}	10^{-3}
	α^{-1}	137.03612(15)	1.1		
Elementary charge	e	1.6021876(50)	3.1	10^{-19} C	10^{-20} emu
		4.803238(15)	3.1		10^{-10} esu
Planck constant	h	6.626167(38)	5.7	$10^{-34}\ J \cdot s$	$10^{-27}\ erg \cdot s$
	$\hbar = h/2\pi$	1.0545872(60)	5.7	$10^{-34}\ J \cdot s$	$10^{-27}\ erg \cdot s$
Avogadro constant	N_A	6.022046(31)	5.2	$10^{23}\ mol^{-1}$	$10^{23}\ mol^{-1}$
Electron rest mass	m_e	9.109533(47)	5.1	10^{-31} kg	10^{-28} g
		5.4858026(21)	0.38	10^{-4} u	10^{-4} u
Proton rest mass	m_p	1.6726483(86)	5.2	10^{-27} kg	10^{-24} g
		1.007276470(11)	0.011	u	u
Ratio of proton mass to electron mass	m_p/m_e	1836.15152(70)	0.38		
Neutron rest mass	m_n	1.6749541(86)	5.2	10^{-27} kg	10^{-24} g
		1.008665012(37)	0.037	u	u
Josephson frequency-voltage ratio	$2e/h$	4.835941(13)	2.7	$10^{14}\ Hz \cdot V^{-1}$	

Impact on science due to earlier changes to the SI - 3

- Mohr *et al.*, **2014 values**, *Rev. Mod. Phys.* (2016)

TABLE XXXIII. The CODATA recommended values of the fundamental constants of physics and chemistry based on the 2014 adjustment.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
		UNIVERSAL		
Speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	Exact ←
Magnetic constant	μ_0	$4\pi \times 10^{-7}$	N A^{-2}	
		$= 12.566\,370\,614\ldots \times 10^{-7}$	N A^{-2}	Exact X
Electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817\ldots \times 10^{-12}$	F m^{-1}	Exact X
Characteristic impedance of vacuum $\mu_0 c$	Z_0	376.730 313 461...	Ω	Exact X
Newtonian constant of gravitation	G	$6.674\,08(31) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	4.7×10^{-5}
	$G/\hbar c$	$6.708\,61(31) \times 10^{-39}$	(GeV/c^2)$^{-2}$	4.7×10^{-5}
Planck constant	h	$6.626\,070\,040(81) \times 10^{-34}$	J s	1.2×10^{-8} ←
		$4.135\,667\,662(25) \times 10^{-15}$	eV s	6.1×10^{-9} ←
$h/2\pi$	$\hbar$	$1.054\,571\,800(13) \times 10^{-34}$	J s	1.2×10^{-8} ←
		$6.582\,119\,514(40) \times 10^{-16}$	eV s	6.1×10^{-9} ←
	$\hbar c$	197.326 9788(12)	MeV fm	6.1×10^{-9} ←
Planck mass $(\hbar c/G)^{1/2}$	m_P	$2.176\,470(51) \times 10^{-8}$	kg	2.3×10^{-5}
energy equivalent	$m_P c^2$	$1.220\,910(29) \times 10^{19}$	GeV	2.3×10^{-5}
Planck temperature $(\hbar c^5/G)^{1/2}/k$	T_P	$1.416\,808(33) \times 10^{32}$	K	2.3×10^{-5}
Planck length $\hbar/m_P c = (\hbar G/c^3)^{1/2}$	l_P	$1.616\,229(38) \times 10^{-35}$	m	2.3×10^{-5}
Planck time $l_P/c = (\hbar G/c^5)^{1/2}$	t_P	$5.391\,16(13) \times 10^{-44}$	s	2.3×10^{-5}
		ELECTROMAGNETIC		
Elementary charge	e	$1.602\,176\,6208(98) \times 10^{-19}$	C	6.1×10^{-9} ←
	e/h	$2.417\,989\,262(15) \times 10^{14}$	A J^{-1}	6.1×10^{-9} ←
Magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,831(13) \times 10^{-15}$	Wb	6.1×10^{-9} ←
Conductance quantum $2e^2/h$	G_0	$7.748\,091\,7310(18) \times 10^{-5}$	S	2.3×10^{-10} ←
inverse of conductance quantum	G_0^{-1}	12 906.403 7278(29)	Ω	2.3×10^{-10} ←
Josephson constant[a] $2e/h$	K_J	$483\,597.8525(30) \times 10^9$	Hz V^{-1}	6.1×10^{-9} ←
von Klitzing constant[b] $h/e^2 = \mu_0 c/2\alpha$	R_K	25 812.807 4555(59)	Ω	2.3×10^{-10} ←

(Table continued)

quantum transducer:
potential difference → frequency

- **Josephson effect (c)**

According to theory,

Nobel Prize 1973

Brian Josephson

$$\frac{V_n}{f} = n\left(\frac{h}{2e}\right) = n / K_J$$

V_n **: "voltage" of the n^{th} step**

f **: frequency of e-m radiation shining on the J-junction**

Niemeyer and Grimm, PTB
Metrologia, 1988

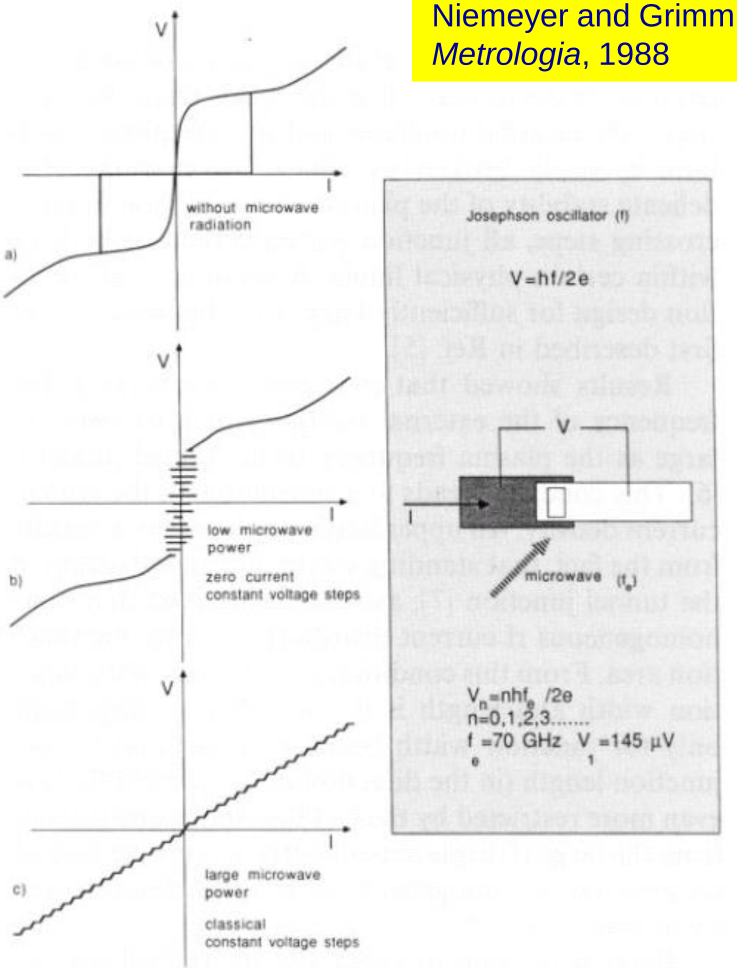


Fig. 1 a–c. *I-V* characteristic of a highly hysteretic Josephson tunnel junction. **a** without microwave radiation; **b** with low-power microwave radiation; **c** with high-power microwave radiation

h/e^2, a quantum resistor of ~ 26 kΩ

- **Resistance via the quantum-Hall effect**

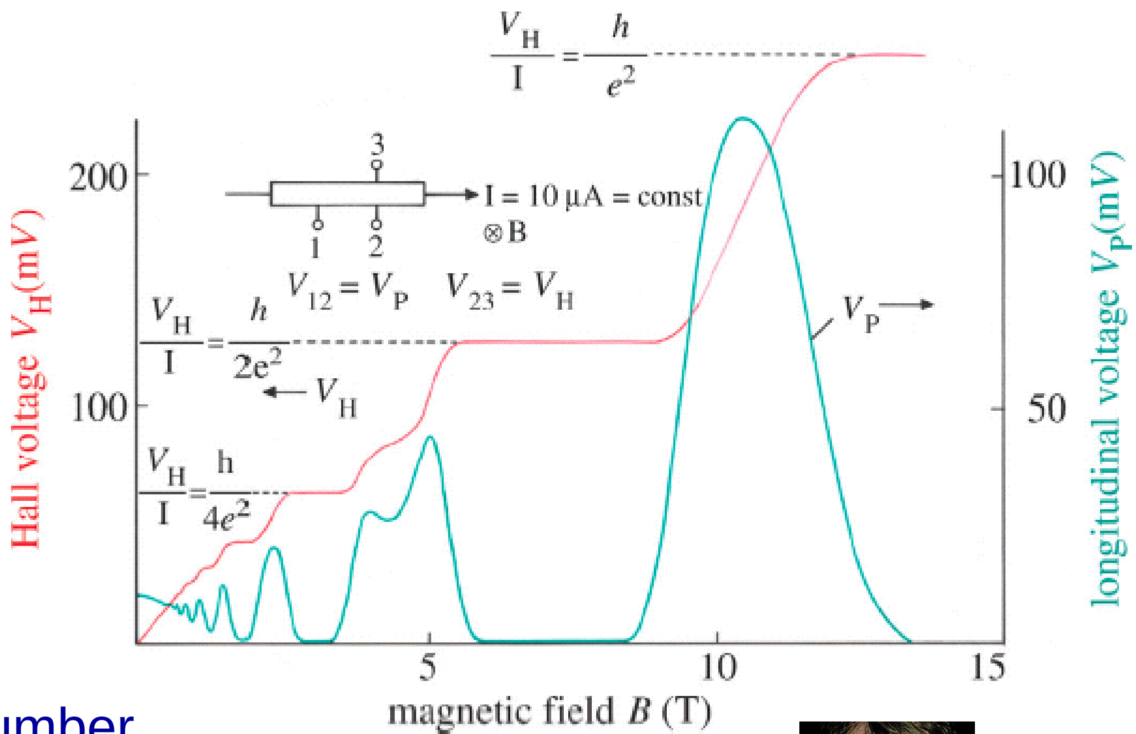
From von Klitzing, *Phil. Trans. R. Soc. A* (2005)

According to theory,

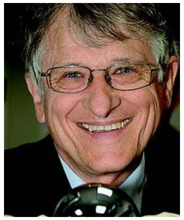
$$R_H(i) = \frac{1}{i}\left(\frac{h}{e^2}\right) = \frac{R_K}{i}$$

$$\approx \frac{26\ \text{k}\Omega}{i}$$

where i is the plateau number



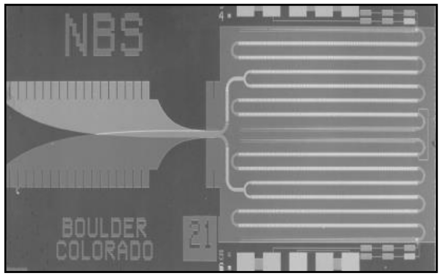
Nobel Prize 1985



Klaus V. Klitzing

Realization of quantum electrical devices

Josephson effect



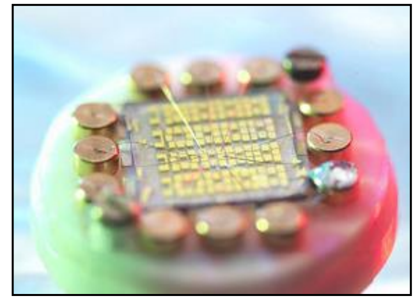
NIST / Wikimedia Commons

$$V_{\mathrm{n}} = n\frac{f}{K_{\mathrm{J}}}, \quad K_{\mathrm{J}} = \frac{2e}{h}$$

(universality tests at 10^{-16})

K_{J} = 483 597.8525(30) GHz/V (CODATA 2014)

quantum-Hall effect



Graphene(!) qu-Hall device
NPL
BIPM
Chalmers Univ.
Linköping Univ.
Lancaster Univ.

$$R_{\mathrm{H}}(i) = \frac{R_{\mathrm{K}}}{i}, \quad R_{\mathrm{K}} = \frac{h}{e^2}$$

(universality: Si-MOSFET; GaAs; graphene)

R_{K} = 25 812.807 4555(59) Ω (CODATA 2014)

Deriving the definitions of the base units:
s, m, kg, A, K, mol, cd

Bureau International des Poids et Mesures

The SI units of the 7 defining constants

Any SI unit can be written as the product of powers of the 7 base units:

$$\mathrm{s}^{\alpha}\ \mathrm{m}^{\beta}\ \mathrm{kg}^{\gamma}\ \mathrm{A}^{\delta}\ \mathrm{K}^{\varepsilon}\ \mathrm{mol}^{\zeta}\ \mathrm{cd}^{\eta}$$

	s	m	kg	A	K	mol	cd
$\Delta\nu_{Cs}$	-1	0	0	0	0	0	0
c	-1	1	0	0	0	0	0
h	-1	2	1	0	0	0	0
e	1	0	0	1	0	0	0
k	-2	2	1	0	-1	0	0
N_A	0	0	0	0	0	-1	0
K_{cd}	3	-2	-1	0	0	0	1

e.g. unit of *h* is J s = kg m^2 s^{-1} = s^{-1} m^2 kg

Invert the matrix of exponents, transpose the labels…

…to see the product of powers of the defining constants, whose unit is an SI base unit:

$$\Delta\nu_{\mathrm{Cs}}^{\alpha'} c^{\beta'} h^{\gamma'} e^{\delta'} k^{\varepsilon'} N_{\mathrm{A}}^{\zeta'} K_{\mathrm{cd}}^{\eta'}$$

	$\Delta\nu_{\mathrm{Cs}}$	c	h	e	k	N_{A}	K_{cd}
s	-1						
m	-1	1					
kg	1	-2	1				
A	1			1			
K	1		1		-1		
mol						-1	
cd	2		1				1

e.g., the kilogram row (kg):

$$\Delta\nu_{\mathrm{Cs}}^{1} c^{-2} h^{1} e^{0} k^{0} N_{\mathrm{A}}^{0} K_{\mathrm{cd}}^{0} = \Delta\nu_{\mathrm{Cs}} c^{-2} h = \frac{h\Delta\nu_{\mathrm{Cs}}}{c^{2}}$$

From the Draft Resolution A (2018)...

$$\frac{h\Delta\nu_{\text{Cs}}}{c^2} = \frac{\left(6.626\,070\,15\times10^{-34}\right)\left(9\,192\,631\,770\right)}{\left(299\,792\,458\right)^2}\ \text{kg}$$

$$1\ \text{kg} = \frac{\left(299\,792\,458\right)^2}{\left(6.626\,070\,15\times10^{-34}\right)\left(9\,192\,631\,770\right)}\frac{h\Delta\nu_{\text{Cs}}}{c^2} = 1.475\,521\,3997\ldots\ \times10^{40}\ \frac{h\Delta\nu_{\text{Cs}}}{c^2}$$

See: Section 2.3.1 in Draft 9th Edition, SI Brochure (slide 2)
and Davis, Rapport BIPM-2018/02

We can do the same analysis for the present SI

- the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 hertz, (1967) s^{-1}
- the speed of light in vacuum c is 299 792 458 metres per second, (1983) $m\ s^{-1}$
- the mass of the international prototype of the kilogram $m_{\mathcal{K}}$ is 1 kilogram, (1889) kg
- the permeability of vacuum μ_0 is $4\pi \times 10^{-7}$ newton per ampere squared, (1948, 1954) $kg\ m\ s^{-2}\ A^{-2}$
- the thermodynamic temperature of the triple point of water T_{TPW} is 273.16 kelvin, (1954) K
- the molar mass of carbon 12, $M(^{12}C)$, is 0.012 kilogram per mole, (1971) $kg\ mol^{-1}$
- the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, is 683 lumens per watt. (1979) $cd\ kg^{-1}\ m^{-2}\ s^{-3}$

Comparison of updated SI with present SI

updated (2019)

	$\Delta\nu_{Cs}$	c	h	e	k	N_A	K_{cd}
s	-1						
m	-1	1					
kg	1	-2	1				
A	1			1			
K	1		1		-1		
mol						-1	
cd	2		1				1

at present

	$\Delta\nu_{Cs}$	c	m_K	μ_0	T_{TPW}	$M(^{12}C)$	K_{cd}
s	-1						
m	-1	1					
kg			1				
A	1/2	1/2	1/2	-1/2			
K					1		
mol			1			-1	
cd	1	2	1				1

Continuity condition (similar for all redefined units)

1 kg = $m_{\mathcal{K}}$ (at present)

1 kg = 1.475 521 3997…× 10^{40} $h\,\Delta\nu_{Cs}/c^2$

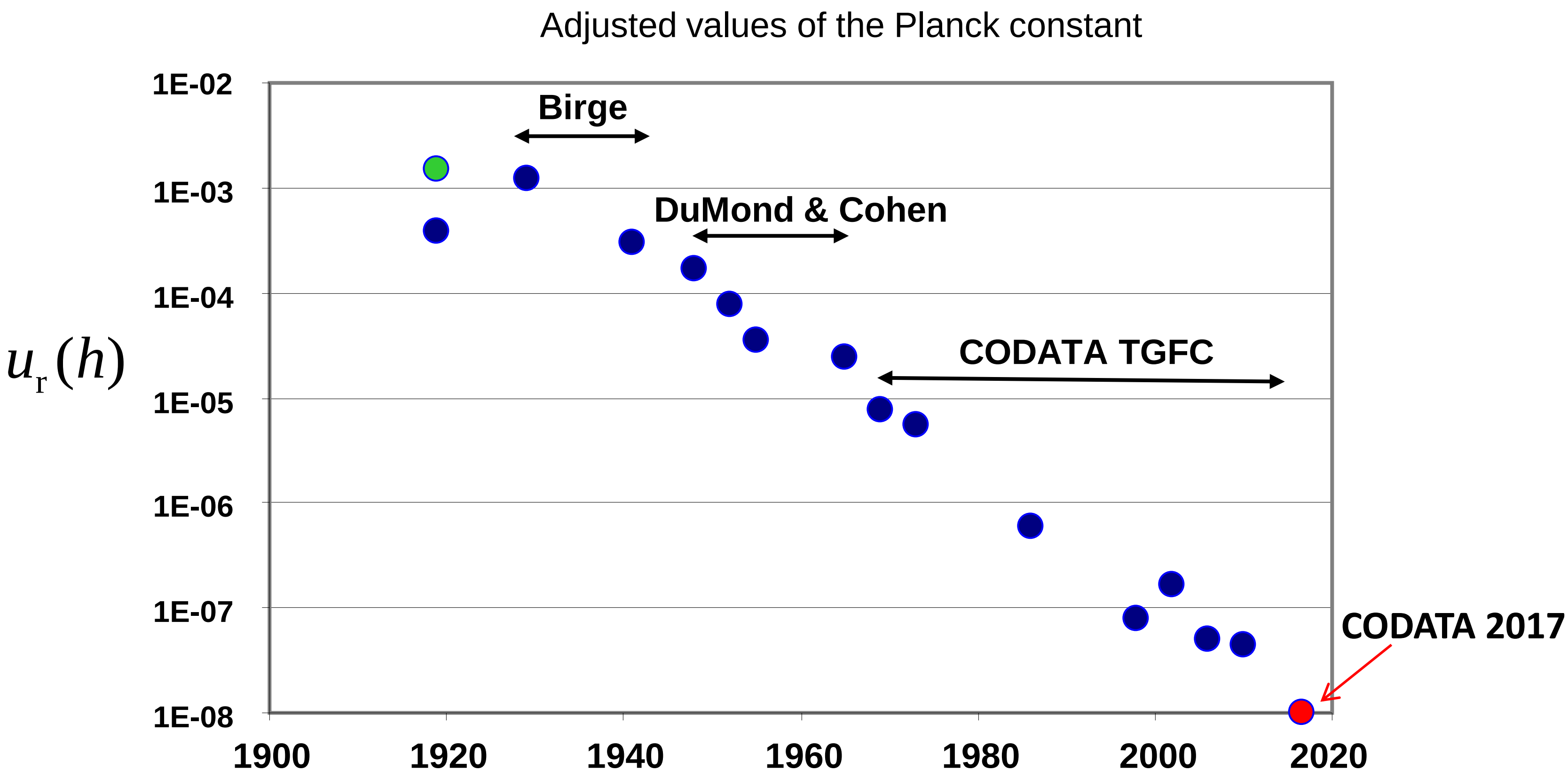
SI value of *h* fixed to be consistent with:

$$m_{\mathcal{K}} \doteq 1.475\,521\,3997\ldots \times 10^{40}\,\frac{h\Delta\nu_{\mathrm{Cs}}}{c^2}$$

in practice, uncertainty will be imperceptible

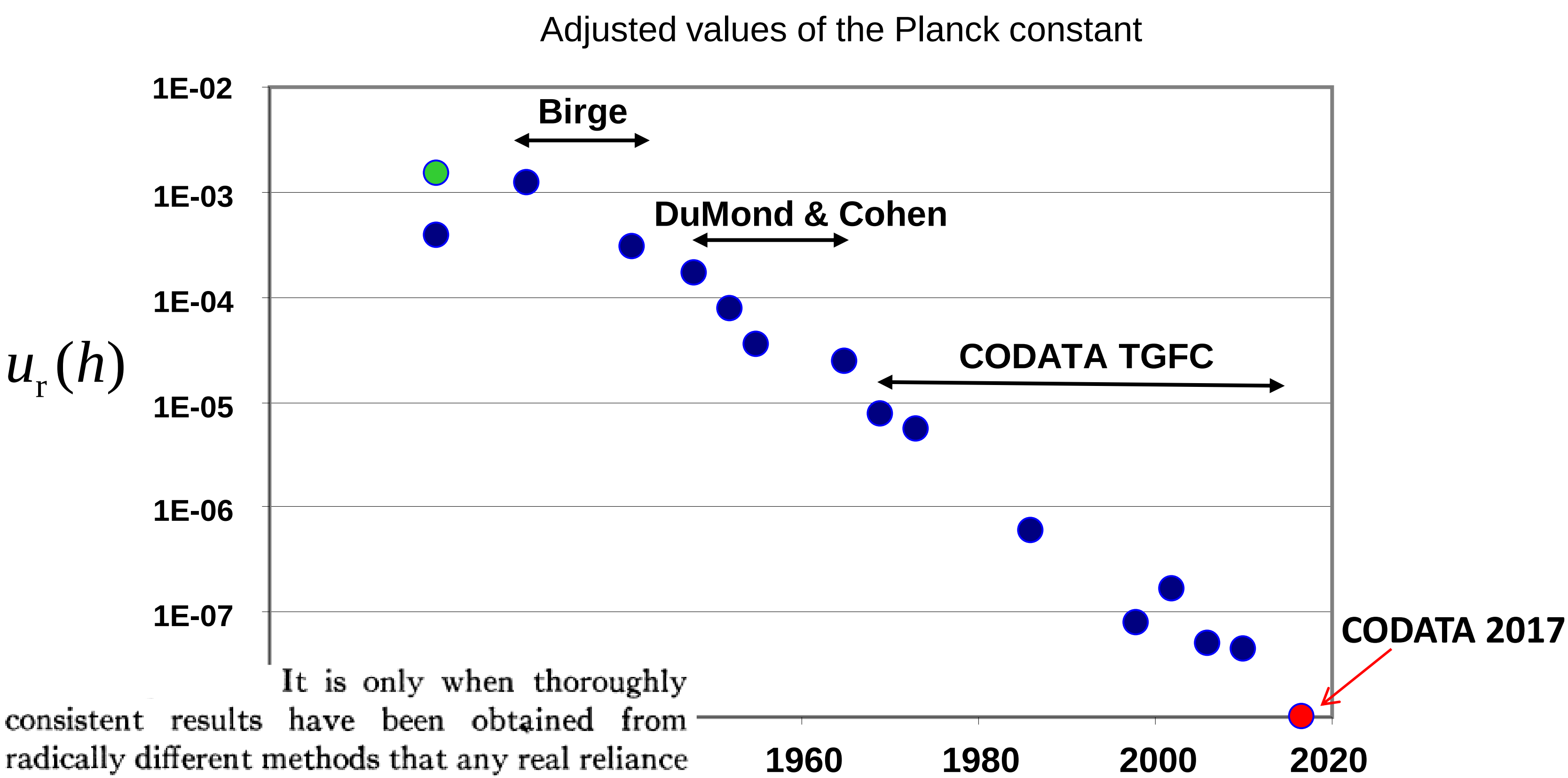
Newell *et al.* (2018) ; Mohr *et al.* (2018)

What took so long?



After a graph originally created in 2012 by Estefanía de Mirandés, BIPM

What took so long?



It is only when thoroughly consistent results have been obtained from radically different methods that any real reliance can be placed on the final weighted average value.

--R.T. Birge, 1941

After a graph originally created in 2012 by Estefanía de Mirandés, BIPM

Two radically different methods to consider

- **Kibble balance (watt balance)**

An electromagnetic balance weighs a mass $m \sim 1$ kg to determine $h/m = Q_{\mathrm{wb}}$; quantum electrical devices are used.

Q_{wb} combines **auxiliary measurements of two frequencies**, **one velocity**, the **local gravitational acceleration**, and dimensionless scaling factors. $\boldsymbol{m}$ **is traceable to** $\boldsymbol{m_{\mathcal{K}}}$.

- X-ray crystal density (**XRCD**), also known as '**Avogadro**' **method**

Determines the atomic mass of $^{28}\mathrm{Si}$, $m_{\mathrm{a}}(^{28}\mathrm{Si})$, by determining the number *N* of atoms in a perfect crystal of mass $m \sim 1$ kg. $\boldsymbol{m}$ **is traceable to** $\boldsymbol{m_{\mathcal{K}}}$; $m_{\mathrm{a}}(^{28}\mathrm{Si}) = m/N$.

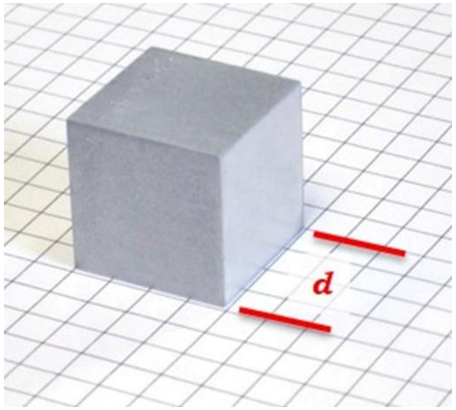
$hN/m = Q_{\mathrm{e}} \cdot (m_{\mathrm{e}}/m_{\mathrm{a}}(^{28}\mathrm{Si}))$, already known to 0.45 parts in 10^9.

atomic mass of $^{27}\mathrm{Al}$, $m_\mathrm{a}(^{27}\mathrm{Al})$ traceable to the mass of $\mathcal{K}$ by the X-ray crystal density method (**XRCD**).

Make a 20-g cube of high-purity aluminium, with side *d* and mass *m*.

measured: *d* = 19.54 mm; *m* = 20.05 g.

If there are N atoms in the cube,

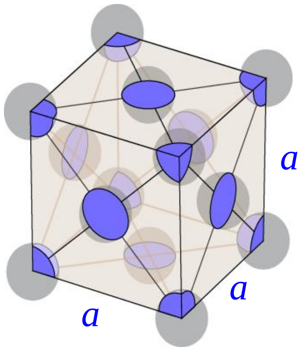


$$m = N \cdot m_\mathrm{a}(\mathrm{Al})$$

$$N = 4\frac{d^3}{a^3} = 4\left(\frac{d}{a}\right)^3$$

$$m_\mathrm{a}(\mathrm{Al}) = \frac{m}{N} = \frac{m}{4}\left(\frac{a}{d}\right)^3$$

$$\frac{m}{d^3} = \frac{4 \cdot m_\mathrm{a}(\mathrm{Al})}{a^3}$$

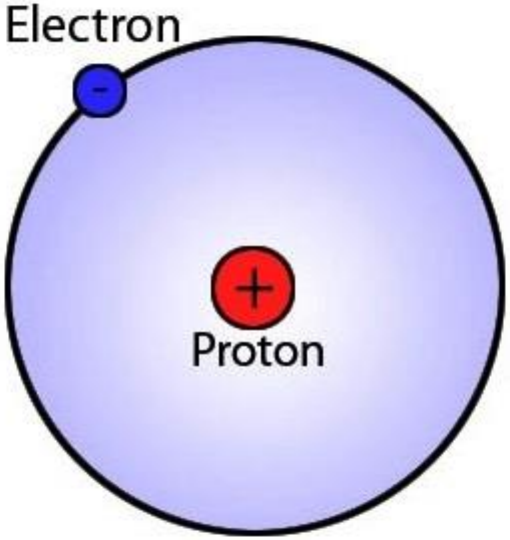


Al unit cell has
4 atoms.
a = 405 x 10^{-12} m

The Planck constant h from XRCD : one possibility

The Bohr model of the hydrogen atom relates the Planck constant, h, to the the electron mass, m_e. As written today:

$$h \cdot (cR_\infty) = \frac{1}{2} m_\mathrm{e} \cdot (\alpha c)^2$$



In this equation, proton

is a point with infinite mass.

R_∞ includes corrections for infinite mass, Special Relativity, and the multitude other deficiencies of the Bohr model.

The Planck constant h from XRCD

The Bohr result as now written:

$$h = \left[\frac{\alpha^2 c}{2R_\infty}\right] m_\mathrm{e} \longrightarrow \boxed{h = Q_\mathrm{e} m_\mathrm{e}}$$

α	: fine structure constant (a pure number)
c	: speed of light in vacuum (in m/s)
R_∞	: Rydberg constant (in 1/m)

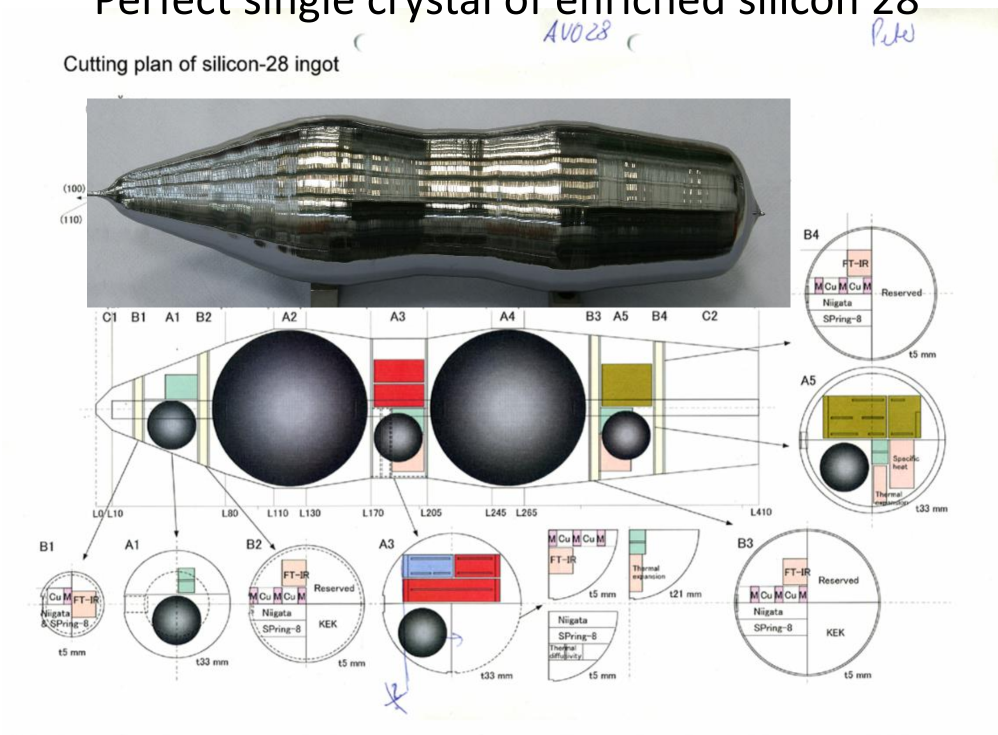
relative uncertainty of Q_e is 0.45 parts in 10^9 (CODATA 2014)

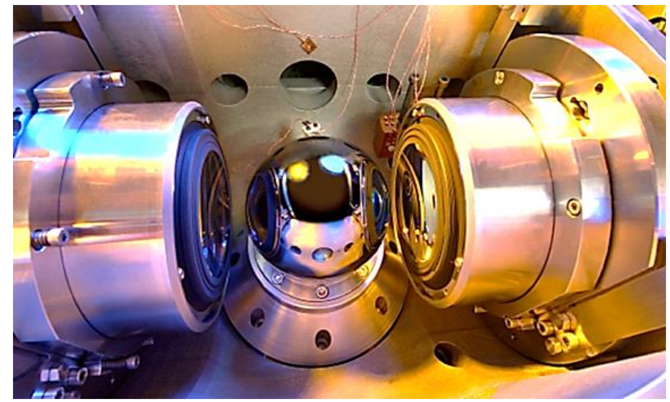
$$h = \left[Q_\mathrm{e}\left(\frac{m_\mathrm{e}}{m_\mathrm{a}(\mathrm{Al})}\right)\frac{a^3}{4d^3}\right] m$$

$$h = \left[Q_\mathrm{e}\left(\frac{m_\mathrm{e}}{m_\mathrm{a}\left({}^{28}\mathrm{Si}\right)}\right)\frac{a_\mathrm{Si}^3}{8\left(\pi/6\right)d^3}\right]\cdot m = Q_\mathrm{SiXRCD}\cdot m$$

Some key features of the silicon XRCD experiment

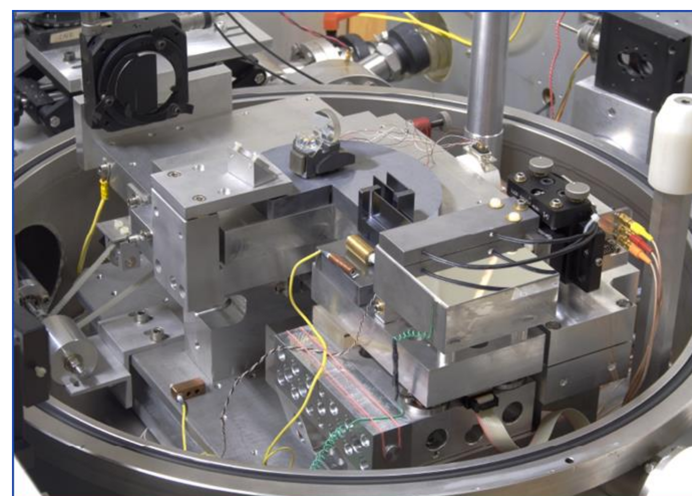
'Perfect single crystal of enriched silicon 28'





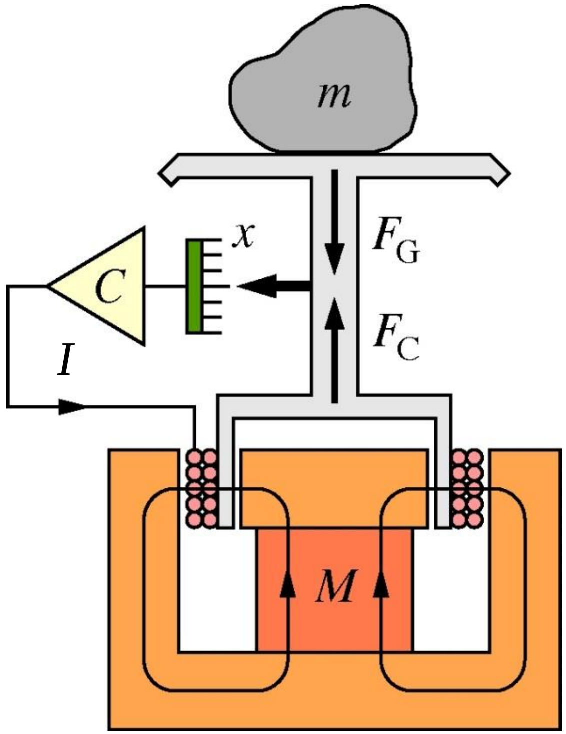
optical interferometer
to measure $V_{sphere} = (\pi/6)d^3$

X-ray interferometer
to determine a_{Si}



Bureau International des Poids et Mesures

The Planck constant h from a Kibble balance – Step 1



from Mettler-Toledo documentation describing servocontrol of an analytical balance.

Gravitational force

$F_G = mg$

Electromagnetic force

$F_C = I(BL)$

$$mg = I(BL)$$

An analytical balance is calibrated by an internal or external mass standard ultimately traceable to $m_{\mathcal{K}}$

Bureau International des Poids et Mesures

watt balance – Step 2

- Move the coil of wire (length L) vertically through the magnetic field (B) at a controlled velocity (v).

- A potential difference V appears across the ends of the wire:

$$V = v(BL)$$

$$mg = \frac{V'}{R}(BL)$$

(from previous slide)

- Combining equations, **mechanical power = electrical power**

$$mgv = V\frac{V'}{R}$$

Quantum electrical effects revisited

$$h = \frac{4}{K_{\mathrm{J}}^2 R_{\mathrm{K}}}$$

The **Josephson effect** relates a d.c. potential difference, V, to a microwave frequency, f, through the Josephson constant K_{J} ;

$$V = X_V \frac{f}{K_{\mathrm{J}}} = X_V \left(\frac{h}{2e} \right) f$$

The **quantum Hall effect** relates a resistance, R, to the von Klitzing constant R_{K} ;

$$R = X_R R_{\mathrm{K}} = X_R \frac{h}{e^2}$$

X_V and X_R are dimensionless experimental scaling factors .

Kibble balance – the link to the Planck constant

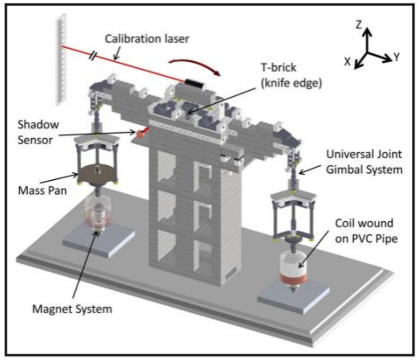
- Measure V, V' and R with **quantum electrical devices**, first discovered in the last century and now widely used:

$$mgv = V\frac{V'}{R} \rightarrow h = \left[4b\frac{gv}{f \cdot f'}\right] \cdot m = Q_{wb} \cdot m$$

f and f' are **microwave frequencies** associated with the voltage measurements;

b is a product of integers and dimensionless scaling factors;

$$Q_{wb} = \left[4b\frac{gv}{f' \cdot f}\right]$$



Kibble balance made of Lego bricks
https://doi.org/10.1119/1.4929898

From remarks of M. André Danjon, President, Comité International des Poids et Mesures

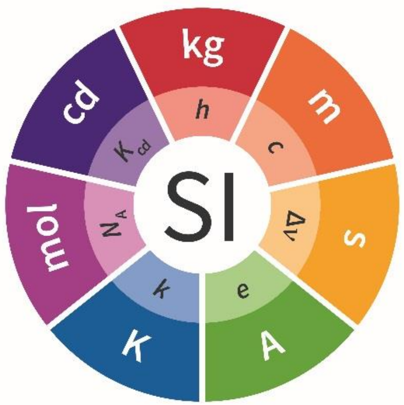
…Seule des trois grandeurs fondamentales de la mécanique, la masse conservera un étalon artificiel, le Kilogramme en platine iridié du Pavillon de Breteuil…

[I]l faut bien avouer que son invariabilité tient un peu du miracle. En pratique, on ne l'utilise que rarement, de peur de l'altérer. Il y a là une faiblesse du Système Métrique à laquelle les métrologistes devront tôt ou tard porter remède.

Mais ils ont d'autres tâches plus urgentes…

*Compte rendu de la onzième réunion de la CGPM (**1960**), page 23*

	$\Delta\nu_{Cs}$	c	h	e	k	N_A	K_{cd}
s	-1						
m	-1	1					
kg	1	-2	1				
A	1			1			
K	1		1		-1		
mol						-1	
cd	2		1				1



Bureau
International des
Poids et
Mesures

www.bipm.org