Gravitational wave modelling and dipolar tidal effects in scalar-tensor theories

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Journée LISA à l'Observatoire

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LUTH l'Observatoire de Paris | PSL Université de Paris CNRS INSU

Going beyond GR

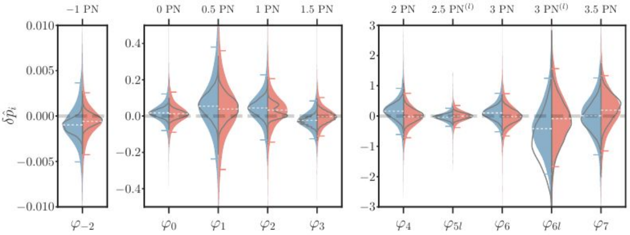
- Why ?
  - dark sectors
  - quantum gravity
  - for the beauty

Going beyond GR

- Why ?
  - dark sectors
  - quantum gravity
  - for the beauty
- How ?
  - higher dimensions
  - new fields
  - etc.

Testing gravity

- Parametrized vs specific theories tests



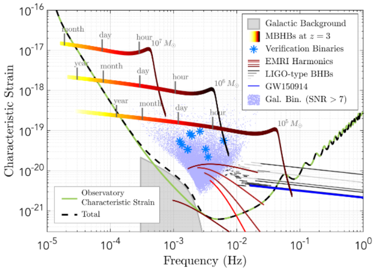
LIGO-Virgo collaboration, 2019

Testing gravity

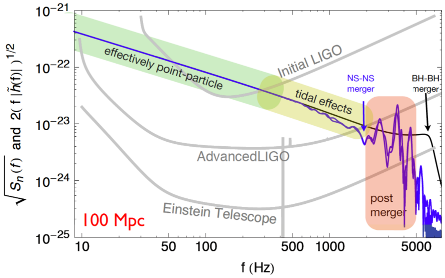
- Parametrized vs specific theories tests
- Challenges for modelisation of strong-field effects beyond GR, specially for analytical models
  - *tidal effects, scalarisation, boson clouds, etc.*
  - *what method : EFT, amplitudes, classical PN ?*
- Degeneracies with other effects, ex : *tidal vs eos for NSs*
  - *I-Love test : theory agnostic and EoS insensitive*

Testing gravity

- Do we really have a chance to be surprised ?
  - *with LIGO-Virgo, LISA, 3rd generation detectors ?*
  - *using multimessenger astronomy (EHT, NICER) ?*

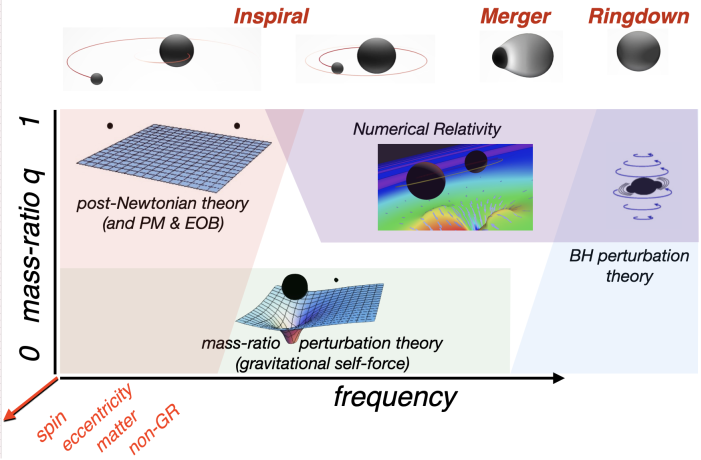


LISA Astro2020 white paper



ET science case

Gravitational wave modelling

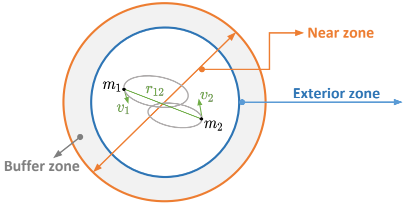


Credits : H. Pfeiffer

Post-Newtonian formalism

Post-Newtonian source

Isolated, compact, slowly moving and weakly stressed source



$$\epsilon \equiv \frac{v_{12}^2}{c^2} \sim \frac{Gm}{r_{12}c^2} \ll 1$$

Near zone post-Newtonian expansion, $n\mathrm{PN} = \mathcal{O}\left(\frac{1}{c^{2n}}\right)$

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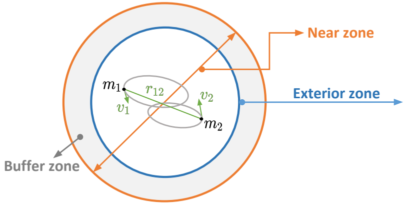

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Exterior zone multipolar expansion in power of $\frac{r_{12}}{R}$

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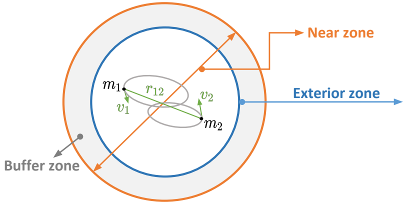

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Exterior zone multipolar expansion in power of $\frac{r_{12}}{R}$

Matching radiative moments $\underset{\text{exp. in } 1/R}{\longleftarrow}$ source moments $\underset{\text{matching}}{\longrightarrow}$ source

Scalar-tensor theories

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[\phi\, R - \frac{\omega(\phi)}{\phi} g^{\alpha\beta}\partial_\alpha\partial_\beta\phi\right] + S_m\left(\mathfrak{m}, g_{\alpha\beta}\right)$$

- well-posed, passes solar system tests
- no hair theorem
- neutron stars : scalarization

Coupling to matter

Violation of the Strong Equivalence Principle

- Incorporate the internal structure of compact, self-gravitating bodies
- Eardley's approach : masses depend on the scalar field $m_A(\phi)$

$$S_{\mathrm{m}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

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▷ Sensitivities : $s_A = \left. \frac{\mathrm{d} \ln m_A(\phi)}{\mathrm{d} \ln \phi} \right|_0$

- Neutron stars : $s_A \sim 0.2$ (depends on the equation of states)
- Black holes : $s_A = 0.5$ (compacity M/R)

Equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} \qquad \underbrace{+\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}}$$

[LB, 2018, 2019]

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Differences w.r.t. GR

- Dissipative effects start at 1.5PN

[LB, 2018, 2019]

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$$+ \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons, nonloc.}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN
- A conservative scalar tail term at 3PN : $\mathbf{A}^{\text{tail}}_{3\text{PN}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, I_i^{(4)}(t')$

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- A conservative scalar tail term at 3PN : $\mathbf{A}^{\text{tail}}_{3\text{PN}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, I_i^{(4)}(t')$
- Tidal effects start at 3PN

[LB, 2018, 2019]

Scalar and gravitational flux

$$\mathcal{F} = \frac{32c^5\nu^2x^5}{5G_{\text{eff}}}\left[1 + \frac{\mathcal{F}^{\mathsf{grav}}_{\text{1PN}}}{c^2} + \frac{\mathcal{F}^{\mathsf{grav}}_{\text{1.5PN}}}{c^3}\right]$$

[LB, Blanchet, Trestini, 2112.xxxxx]

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$$+ \frac{4c^5\nu^2x^5}{3G_{\text{eff}}}\zeta S_-^2\left[x^{-1} \qquad\qquad\qquad\qquad\qquad\qquad \right]$$

Differences w.r.t. GR

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Differences w.r.t. GR

- Scalar flux starts at -1PN
- A scalar tail term at 0.5PN $\delta U_{ij} \propto \int \frac{\mathrm{d}t'}{|t-t'|} I_s{}^{(2)}(t') I_s{}_j^{(2)}(t')$
- A scalar memory term at 1.5PN $\delta U_i^s \propto M \int \mathrm{d}\tau \, \ln\left(\frac{\tau}{\tau_0}\right) \, I_s{}_i^{(3)}(t-\tau)$

[LB, Blanchet, Trestini, 2112.xxxxx]

Scalar and gravitational flux

$$\mathcal{F} = \frac{32c^5\nu^2x^5}{5G_{\text{eff}}}\left[1 + \frac{\mathcal{F}^{\text{grav}}_{1\text{PN}}}{c^2} + \frac{\mathcal{F}^{\text{grav}}_{1.5\text{PN}}}{c^3}\right] + \frac{4c^5\nu^2x^4}{3G_{\text{eff}}}\zeta S_-^2 \cdot \frac{\mathcal{F}^{\text{scal, tidal}}_{2\text{PN}}}{c^4}$$

$$+ \frac{4c^5\nu^2x^5}{3G_{\text{eff}}}\zeta S_-^2\left[x^{-1} + \frac{\mathcal{F}^{\text{scal}}_{0\text{PN}}}{c^0} + \frac{\mathcal{F}^{\text{scal}}_{0.5\text{PN}}}{c^1} + \frac{\mathcal{F}^{\text{scal}}_{1\text{PN}}}{c^2} + \frac{\mathcal{F}^{\text{scal}}_{1.5\text{PN}}}{c^3}\right]$$

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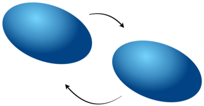
- A scalar tidal contribution at 2PN

[LB, Blanchet, Trestini, 2112.xxxxx]

Tidal effects - Newtonian theory



$$\mathcal{E}_{ij} \sim \partial_i \partial_j U$$

Response to an external tidal field $\mathcal{E}_{ij}$

$$U = \frac{M}{R} - \frac{1}{2}\mathcal{E}_{ij}x^i x^j + \frac{3}{2}\frac{\mathcal{Q}_{ij}x^i x^j}{r^5}$$

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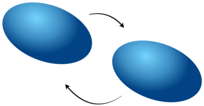

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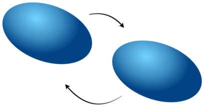

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$$U \; = \frac{M}{R} - \frac{1}{2}\left(1 + 2k_2 \frac{R^5}{r^5}\right) \mathcal{E}_{ij} x^i x^j$$

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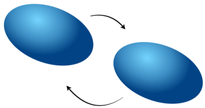

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- Adiabatical approximation : $\mathcal{Q}_{ij} = -\frac{2}{3} k_2 \, R^5 \, \mathcal{E}_{ij}$
- We can generalize to multipolar field $\mathcal{E}_L$ and tidal Love number k_L

Tidal effects - General relativity

- Electric-type multipole moments : $\mathcal{E}_L \propto \nabla_{L-2} C_{0a_1 0a_2}$
- Magnetic-type multipole moments : $\mathcal{B}_L \propto \epsilon_{a_1 bc} \nabla_{L-2} C_{a_2 0bc}$

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In the matter action

$$S_m = S_{\rm pp} + \sum_A \int \mathrm{d}\tau_A \left[\mu_A \mathcal{E}^A_{\mu\nu} \mathcal{E}^{\mu\nu}_A + \sigma_A \mathcal{B}^A_{\mu\nu} \mathcal{B}^{\mu\nu}_A + \cdots \right]$$

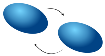
▷ In the phase : $\Delta\psi_{\rm fs} \propto \psi_{\rm N} \cdot k_2 \cdot \dfrac{R^5}{r^5}$

▷ Formally a 5PN effect but $\propto R^5$

Tidal effects - Scalar-tensor theory

- Scalar-type multipole moments : $\mathcal{E}_L^{(s)} \sim \nabla_L \varphi$

Tidal effects - Scalar-tensor theory



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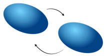

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- Addiabatic approximation : $\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)} \mathcal{E}_\mu^{(s)}$
- Scalar-type Love number : $k_s \propto \lambda_s R^3$

From point particles to extended body

$$m_A(\phi) \longrightarrow m_A[\phi] = m_A(\phi) + N_A(\phi)\nabla_\mu\phi\nabla^\mu\phi$$

In the action

$$S_{\mathrm{m}} = S_{\mathrm{pp}} - \frac{1}{2}\sum_A \lambda_A^{(s)} \int \mathrm{d}\tau_A \, \left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)_A \, + \text{high. orders}$$

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Consequence on the dynamics

$$\Delta\mathbf{a}_{(fs)} \propto \; \mathbf{a}_{\rm (N)} \cdot \left[\tfrac{m_2}{m_1}\,\bar{\delta}_1\,\lambda_1^{(s)} + \tfrac{m_1}{m_2}\,\bar{\delta}_2\,\lambda_2^{(s)}\right]\frac{1}{r^3}$$

- ▷ formally 3PN order correction with small ST parameters

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- ▷ formally 3PN order correction with small ST parameters
- ▷ but scales as $\left(\frac{R}{M}\right)^3$

Effect on the gravitational signal

Phase evolution : $\frac{\mathrm{d}\psi}{\mathrm{d}x} = -\frac{(c^2 x)^{3/2}}{\tilde{G}\alpha m}\frac{\mathrm{d}E/\mathrm{d}x}{\mathcal{F}}$

$$\frac{\mathcal{F}_{\rm dip}}{\mathcal{F}_{\rm quad}} \propto \frac{(s_1 - s_2)^2}{x}$$

Effect on the gravitational signal

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Quadrupolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{32\zeta\eta x^{5/2}}\, k_s\, \frac{R^3}{r^3} \Longrightarrow \text{ non detectable}$$

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Dipolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{(s_1 - s_2)^2\eta\, x^{7/2}}\, k_s\, \frac{R^3}{r^3}$$

- formally 2PN effect in the phase (beyong GR)
- but similar to the ST 1PN contribution
- may contribute $\mathcal{O}(1)$ cycles $\Longrightarrow$ detectable by LISA or 3G

[LB, 2019]

Conclusion

State-of-the-art in ST theories

- Equations of motion at 3PN order
- Total flux and scalar modes at 1.5PN order
- Scalar tidal effect at 3PN order

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To be done

- Total flux and scalar modes at 2.5PN order
- Generalise to higher multipolar scalar tides
- Spin effects $\Longrightarrow$ other theories ?