A geometrical insight into the self-force for modelling Extreme Mass-Ratio Inspirals

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LUTH (long-term visitor)

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Outline

1. EMRIs and self-force

2. Wave propagation and self-force

3. Conclusions

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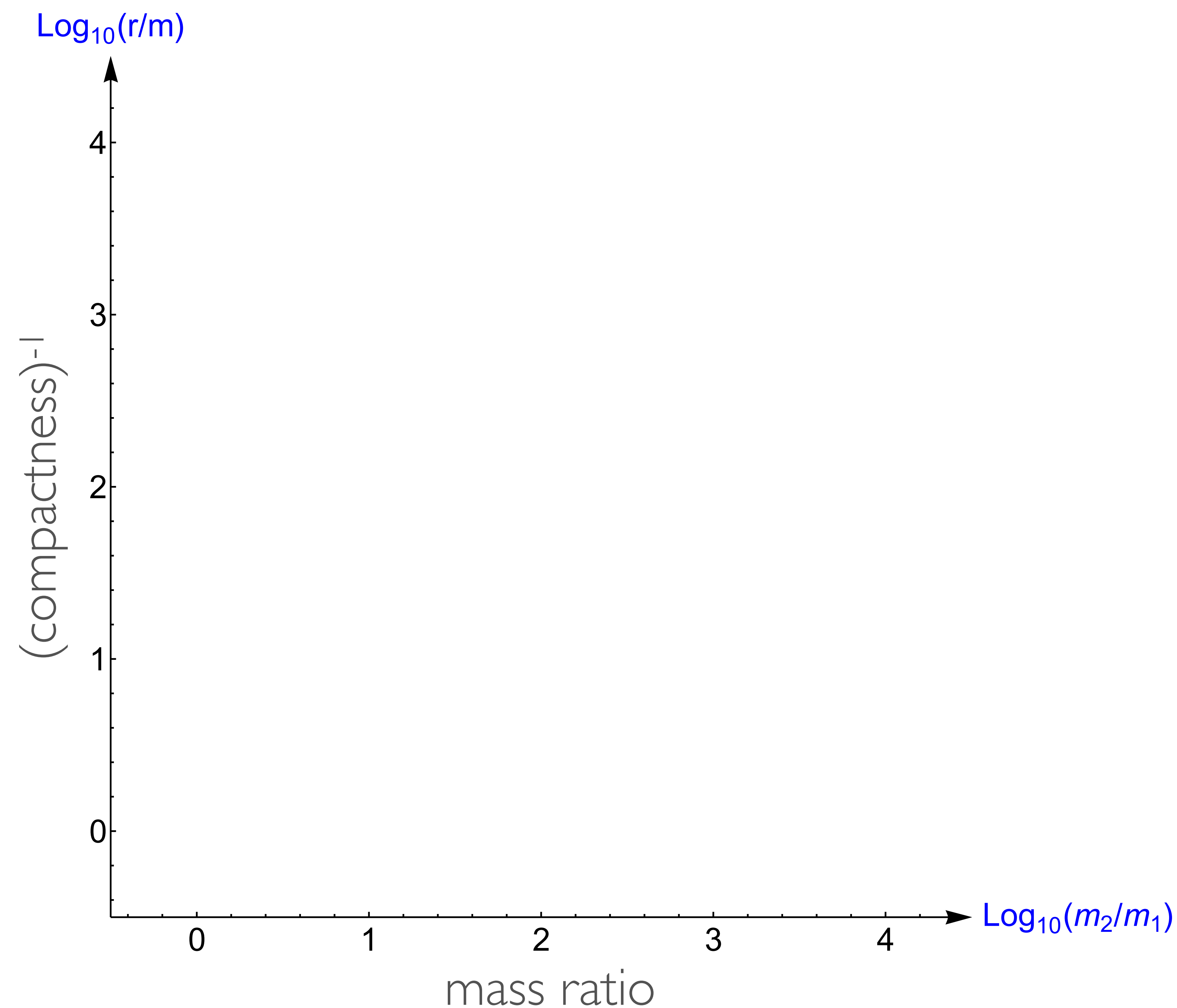
Methods for solving Einstein eqs. for binary inspirals:



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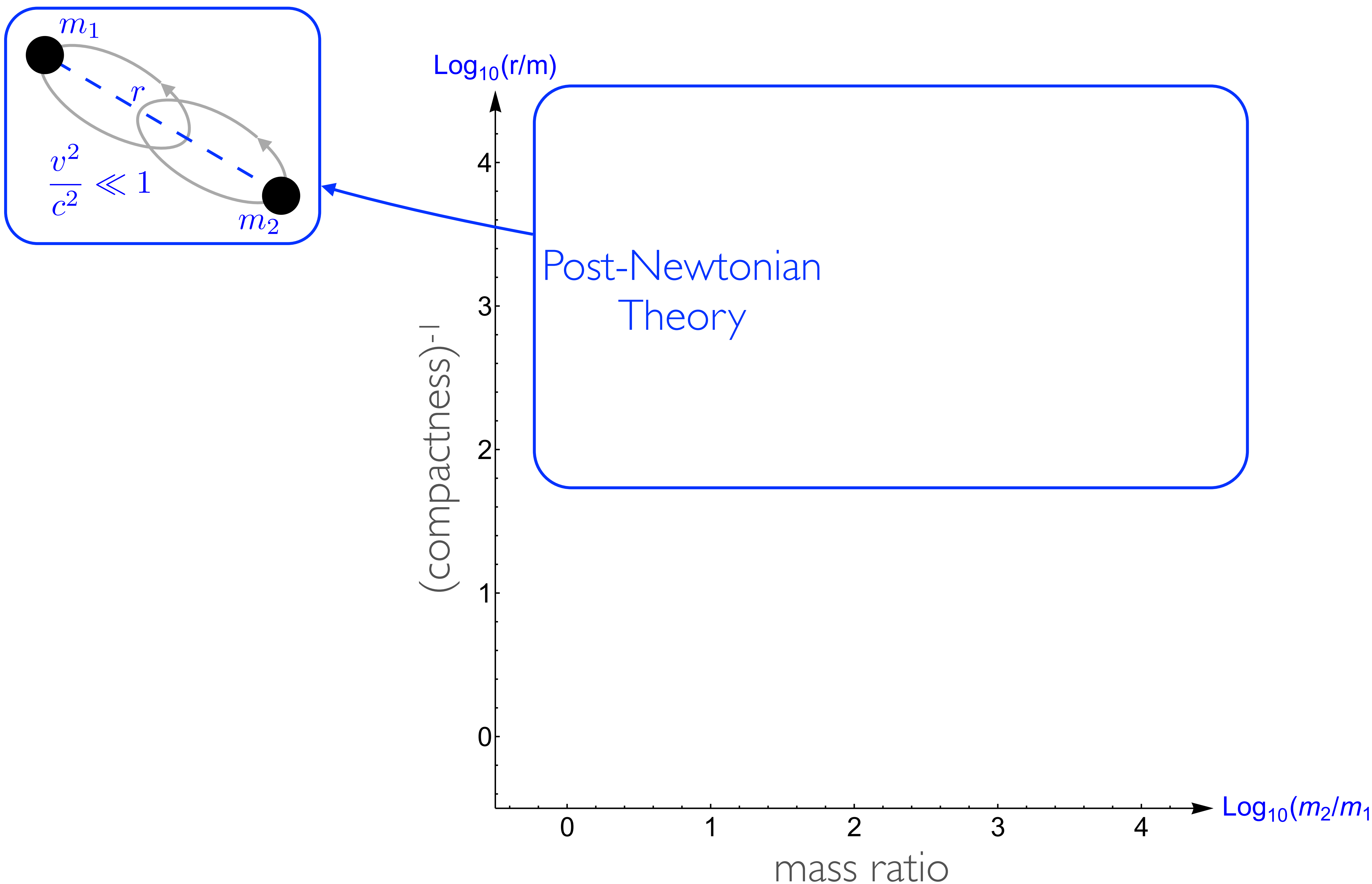

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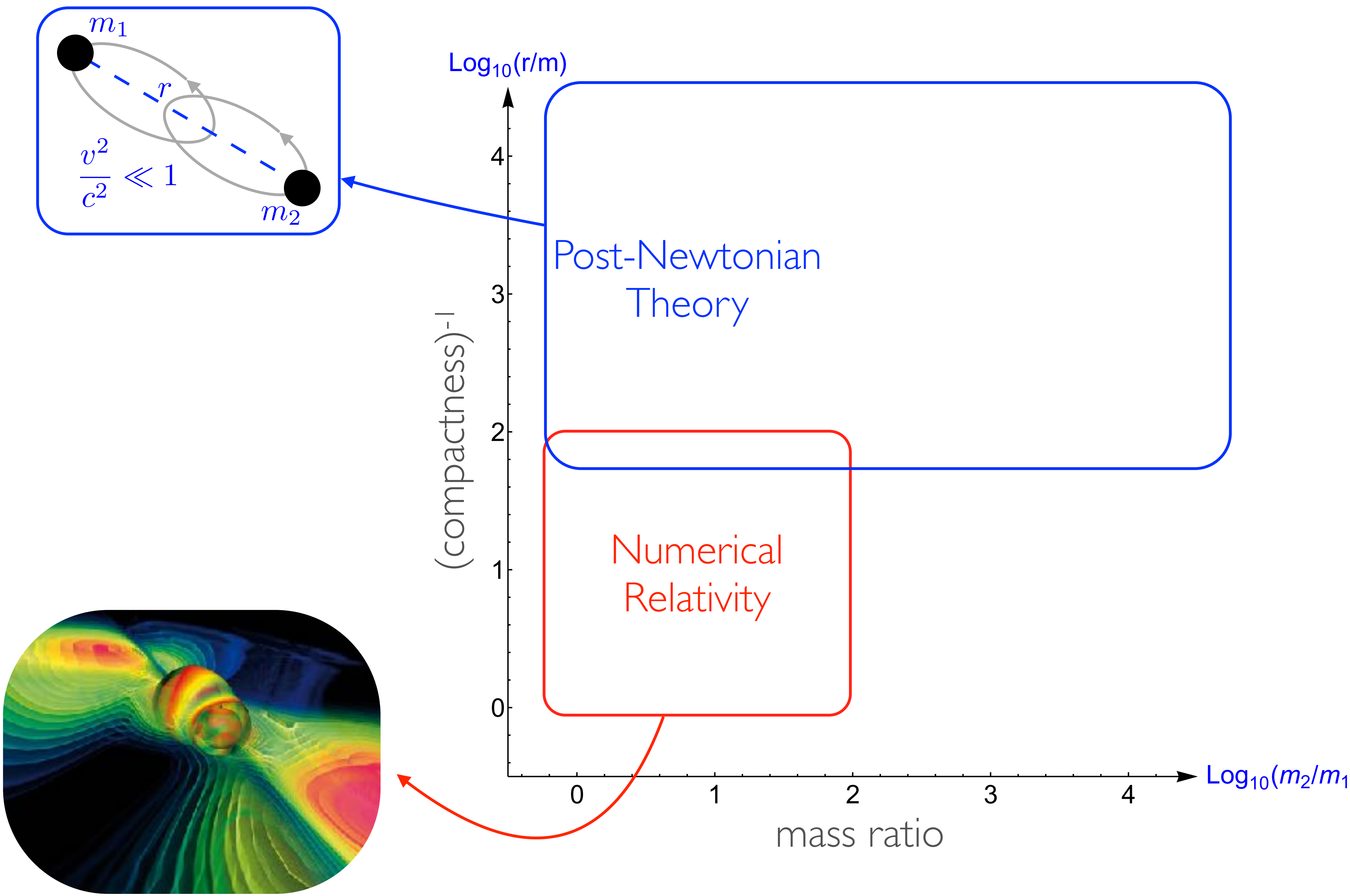

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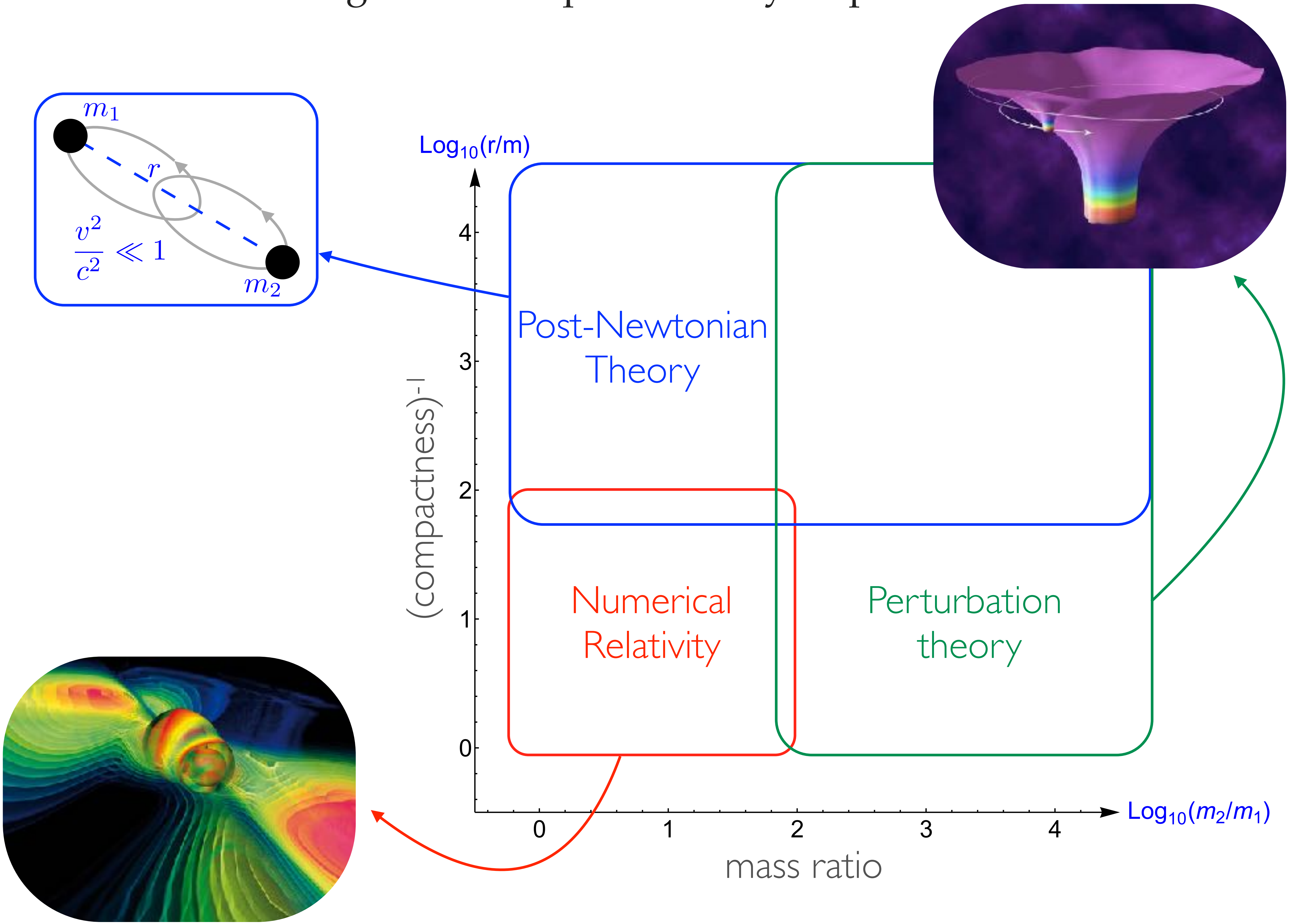

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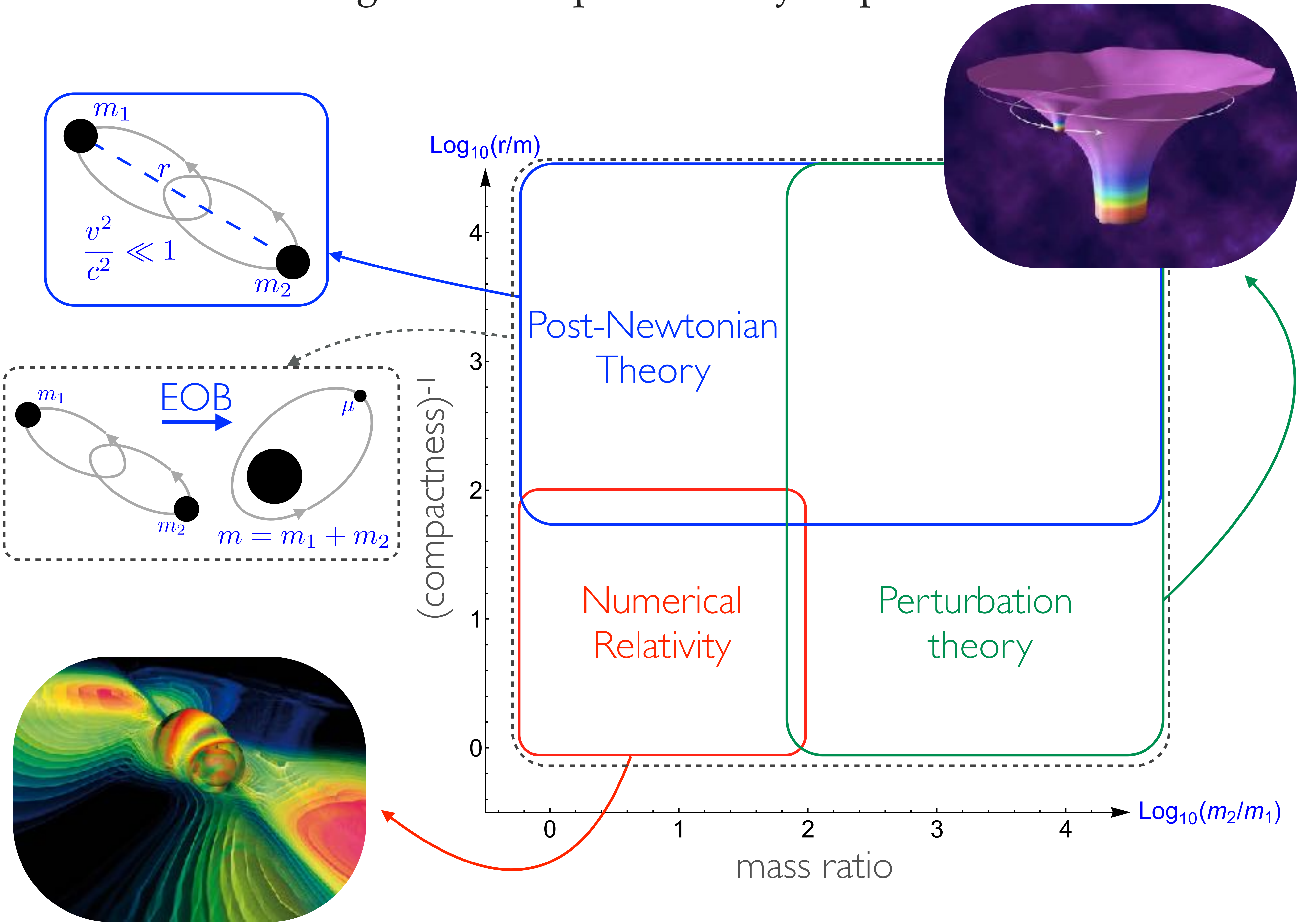

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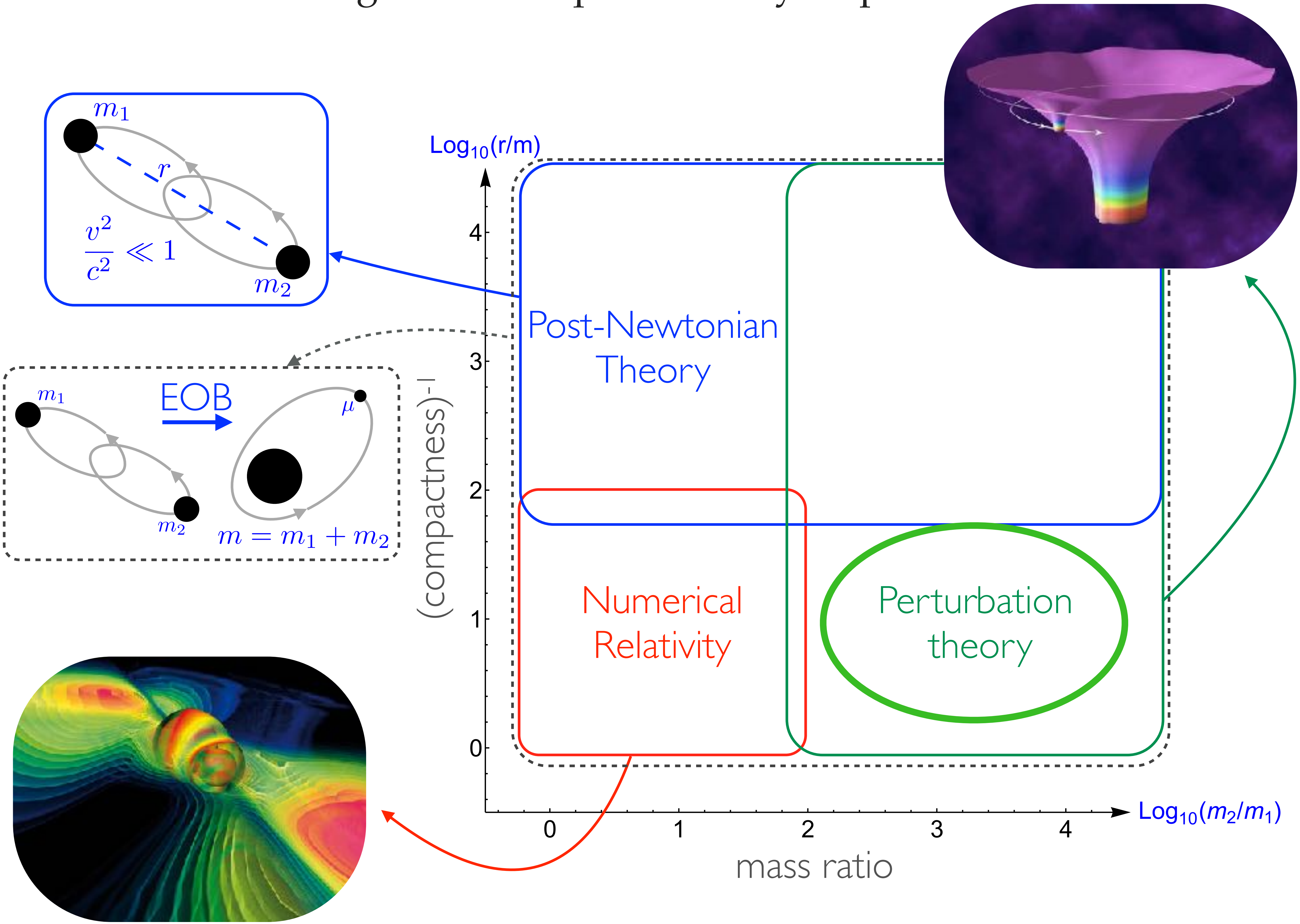
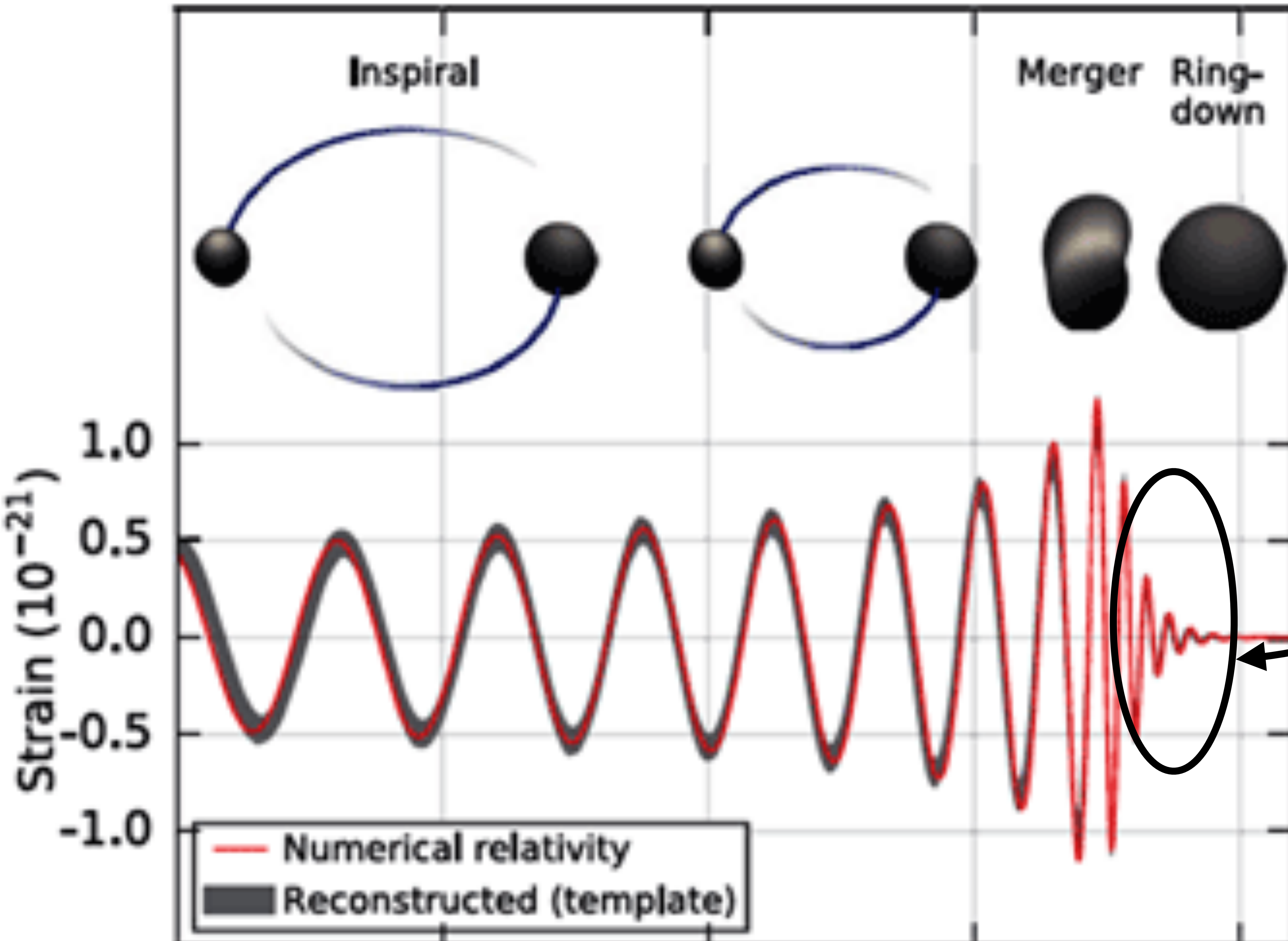


Ringdown

Perturbation theory also serves to model the late (ringdown) stage of a Gravitational Waveform for any mass ratio:



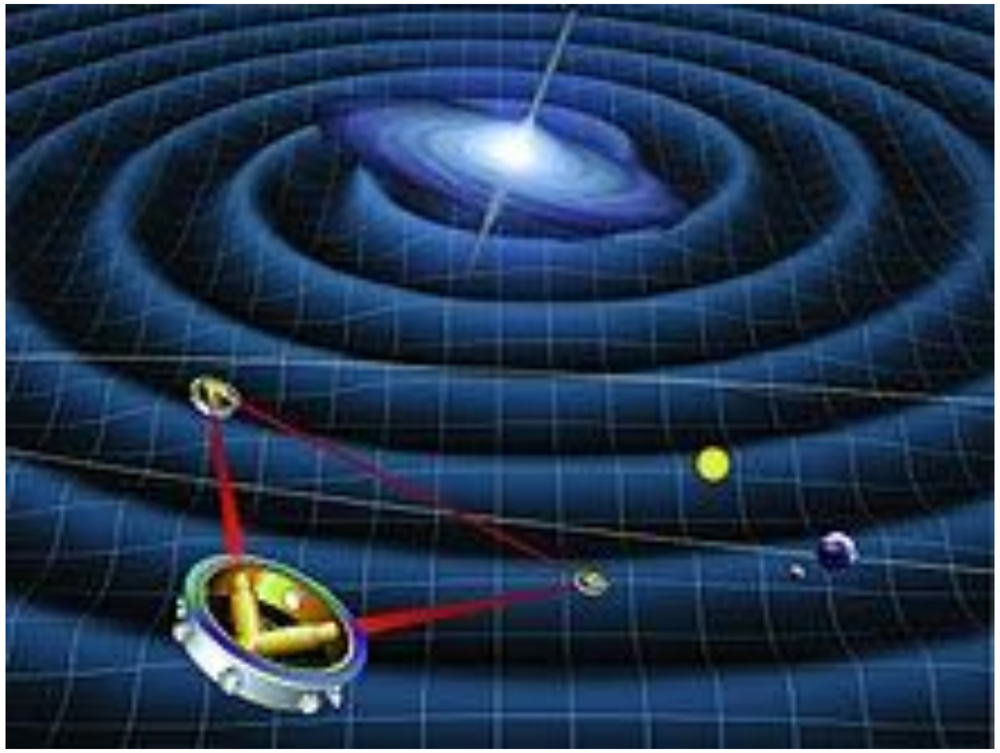
Ringdown: final BH shedding off 'hair' (QuasiNormal Modes)

Credit: LIGO & Virgo

Extreme Mass Ratio Inspirals

- Extreme Mass-Ratio Inspirals (EMRIs) $\frac{M}{m} \sim 10^4 - 10^8$

  are expected to be one of the main sources of GWs for LISA



LISA is expected to see 10-1000 EMRIs/yr (Gair et al'04)

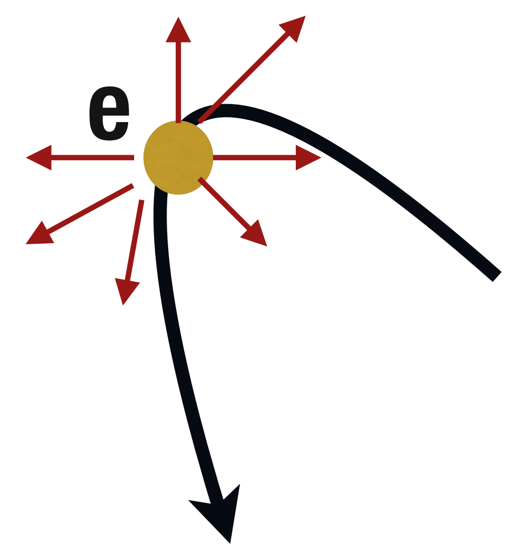
- Numerical Relativity cannot model EMRIs but Perturbation th./ self-force can

Abraham-Lorenz Dirac Self-force

EMRIs can be modeled with the gravitational equivalent of the Abraham-Lorenz-Dirac (1938) force on an accelerated *electric charge* in *flat* space-time:

perpendicular projector to velocity

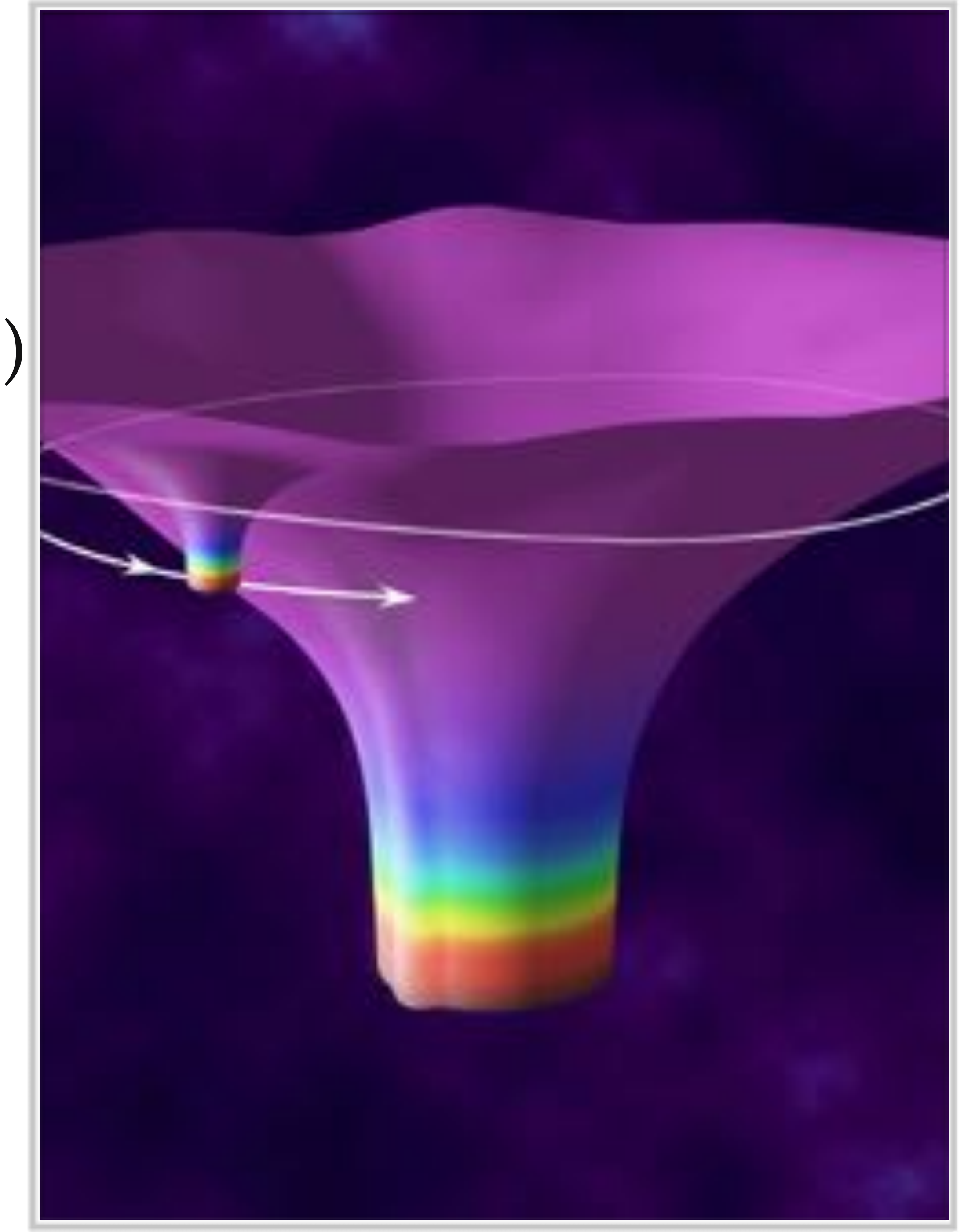
$$ma^\mu = f^\mu_{ext} + \underbrace{\frac{2e^2}{3m} P^\mu{}_\nu \frac{df^\nu_{ext}}{d\tau}}_{\text{emag SF in flat s-t}}$$



It's all local: all quantities are evaluated at the *current* time

Self-force for EMRIs

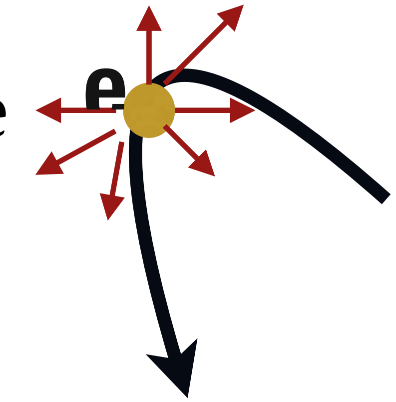
EMRI: inspiral of small mass ($\sim 10 M_{\odot}$) around supermassive BH ($\sim 10^5 - 10^9 M_{\odot}$)

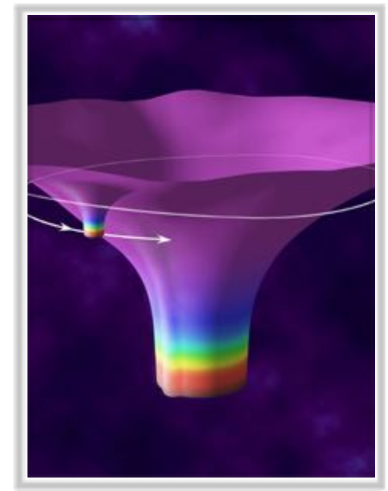


Credit: NASA

Small mass deviates from geodesic of the space-time of the supermassive BH due to the action of its own (regularized) field: gravitational self-force

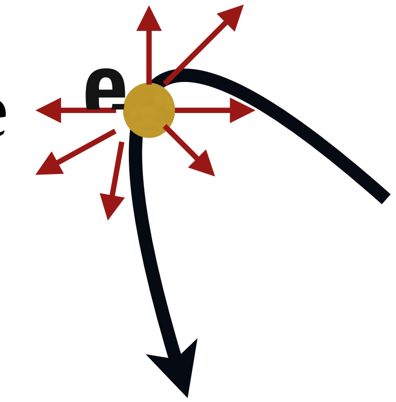
(Note: the small mass is modelled as a *point* particle and the field evaluated at that point diverges -> regularization is needed)

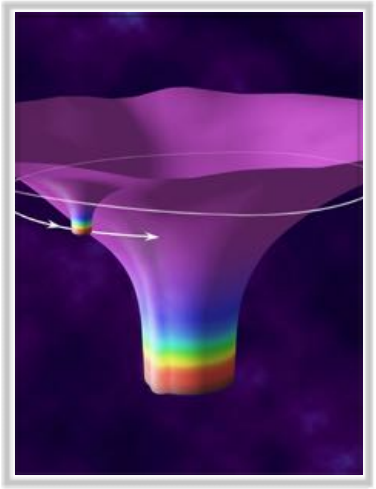
Differences between the SF in the Abraham-Lorenz-Dirac case 

and the SF in the EMRI case  :

(1) in the A-L-D case, the SF is due to emag field, in the EMRI case it's due to grav field

(2) in the A-L-D case, the SF is on particle moving on flat s-t, in the EMRI case it's on curved s-t

Differences between the SF in the Abraham-Lorenz-Dirac case 

and the SF in the EMRI case  :

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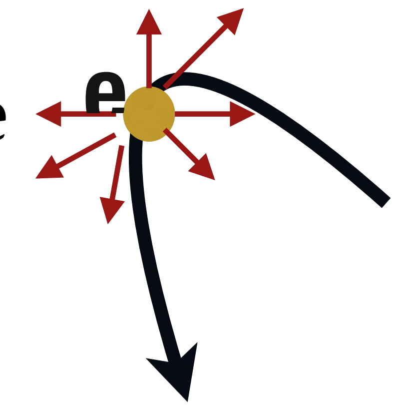
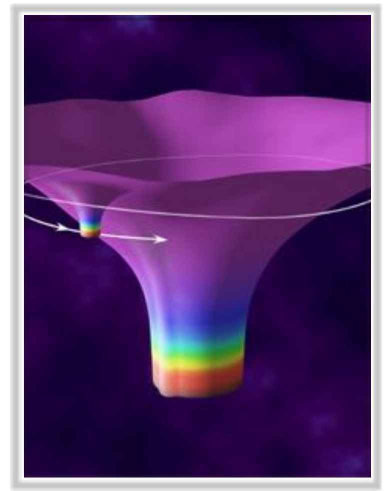
In this talk, we will focus on difference (2)

Differences between the SF in the Abraham-Lorenz-Dirac case 

and the SF in the EMRI case  :

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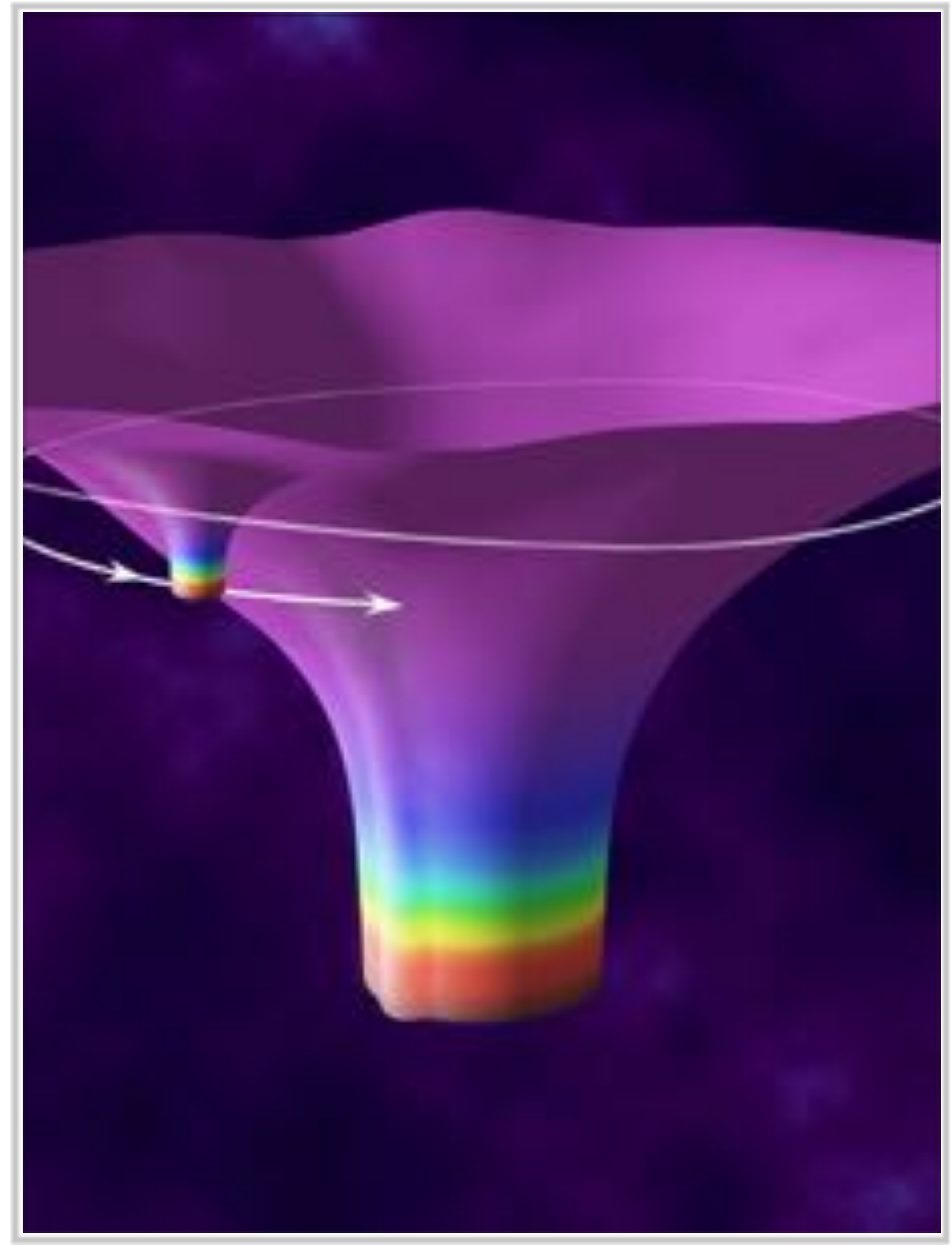
How to calculate the scalar/emag/grav SF on a point scalar charge/electrical charge/mass moving on a *curved* s-t?

Linearized Einstein Eqs.

Smaller BH (m) moving on the background metric $g_{\alpha\beta}$ of massive BH (M) causes perturbation metric $h_{\alpha\beta}$

Linearize Einstein eqs.:

$$\text{Total metric } = \overbrace{g_{\mu\nu}}^{\text{due to M}} + \underbrace{h_{\mu\nu}}_{\text{perturbation due to m}} + O\left(\frac{m}{M}\right)^2$$



Credit: NASA

Background BH spacetime

Background metric $g_{\alpha\beta}$ should in principle be that of a *rotating* (Kerr) BH

Some times, for simplicity, the metric of a *non*-rotating (Schwarzschild) BH is used instead

Wave equation for the perturbation

The linear gravitational perturbation satisfies a wave eq.:

$$\text{“}\Box h_{\mu\nu}\text{”} = T_{\mu\nu}$$

stress-energy tensor of the small BH

$$\Box \equiv g_{\mu\nu}\nabla^{\mu}\nabla^{\nu}$$

background metric due to large BH

Other linear field perturbations of a BH satisfy a similar wave eq.

Eg, scalar case:

$$\Box\phi(x) = T(x)$$

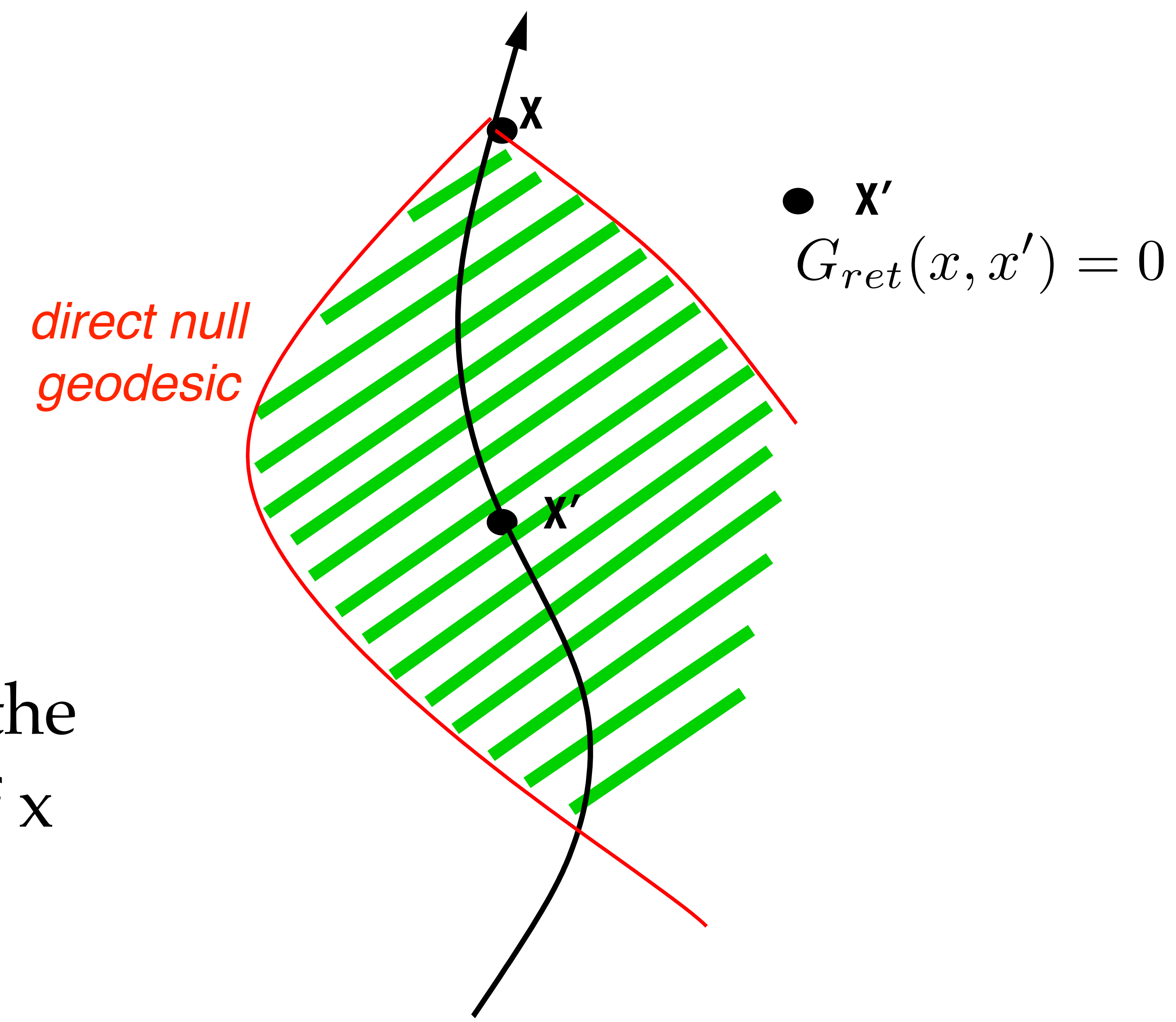
Retarded Green Function

A crucial object is the retarded Green function

$$\Box G_{ret}(x, x') = \delta_4(x, x')$$

with causal b.c.:

$$G_{ret}(x, x') = 0$$ if x′ is not in the causal past of x



The GF is the value of the field at x resulting from an ‘impulse’ at x′

MiSaTaQuWa eq.: SF can be calculated by integrating the GF over the past worldline $z(\tau)$ of the particle

In the case a scalar charge q, the *non-local* part of the SF is:

$$f^\alpha(\tau) = q^2 \nabla^\alpha \int_{-\infty}^{\tau^-} G_{ret}(z(\tau), z(\tau'))d\tau'$$

Remember that the Abraham-Lorenz-Dirac force did not contain a non-local part!

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Where does the contribution to this non-local integral come from?

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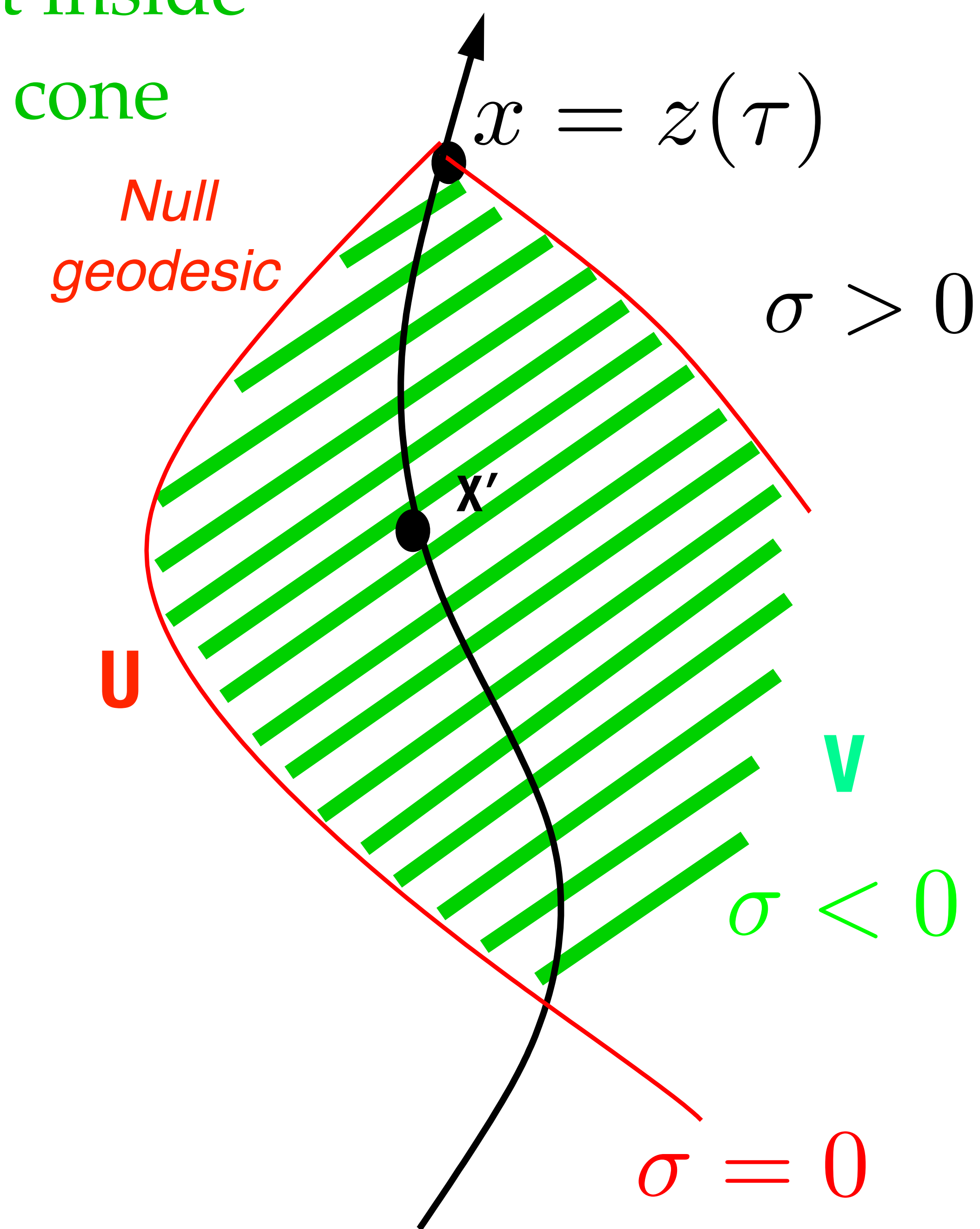
Contribution 1: Backscattering of waves

Locally-valid (ie, for points x and x′ near) Hadamard form for the Green function:

$$G_{ret}\left(x,x'\right)=\underbrace{\theta\left(\Delta t\right)}_{\text{support in past of point x}}\left\{\underbrace{U\ \delta\left(\sigma\right)}_{\text{support on light cone}}-\underbrace{V\ \theta\left(-\sigma\right)}_{\text{support inside light cone}}\right\}$$

U & V: regular

σ : square of geodesic distance between x & x′



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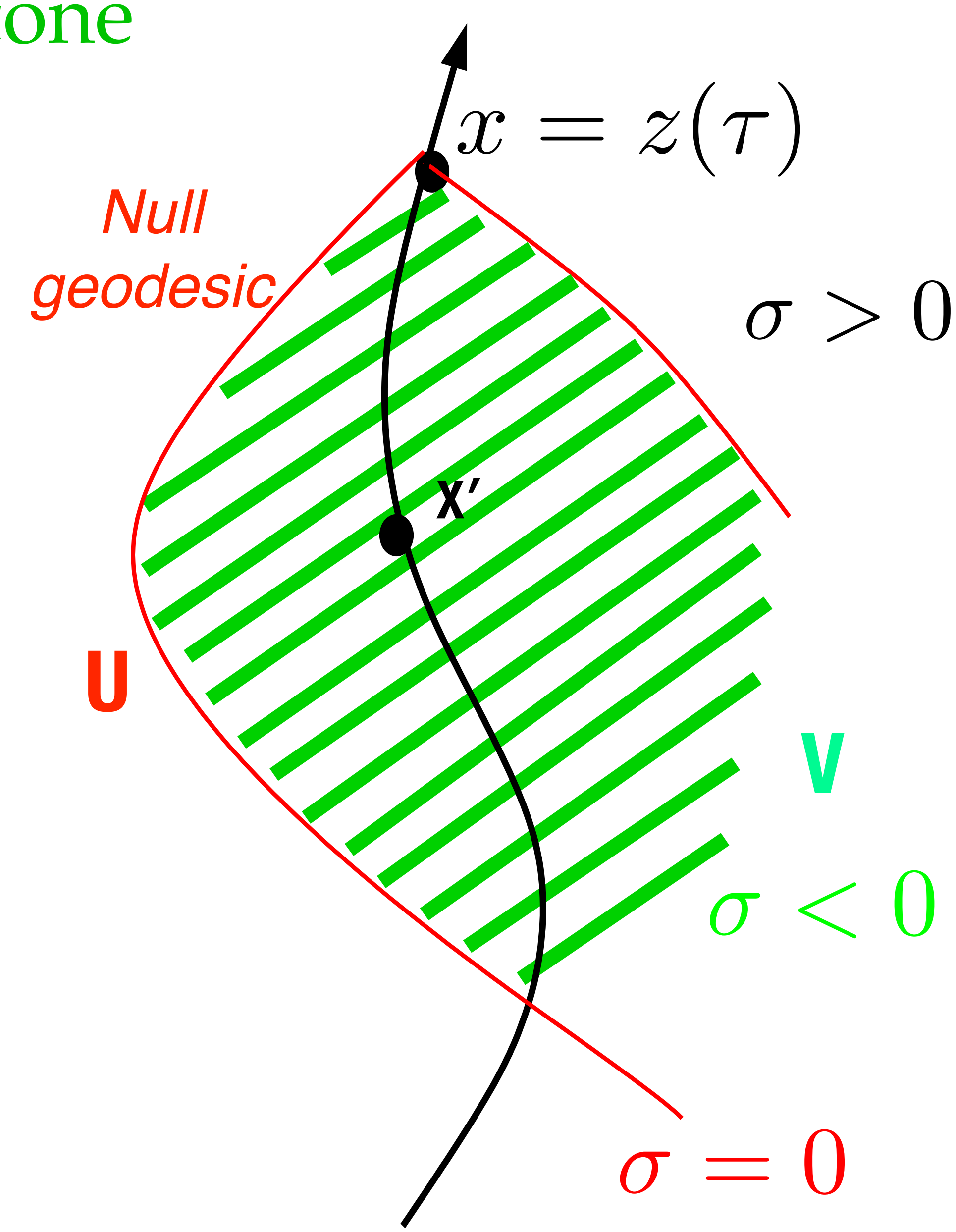
In flat s-t, V=0

In curved s-t, generally, $V \neq 0$

and so scalar/emag/grav waves propagate at all speeds $\leq c$!

This is a contribution to the SF from *timelike* paths ("backscattering" of waves; Huygens principle not held)



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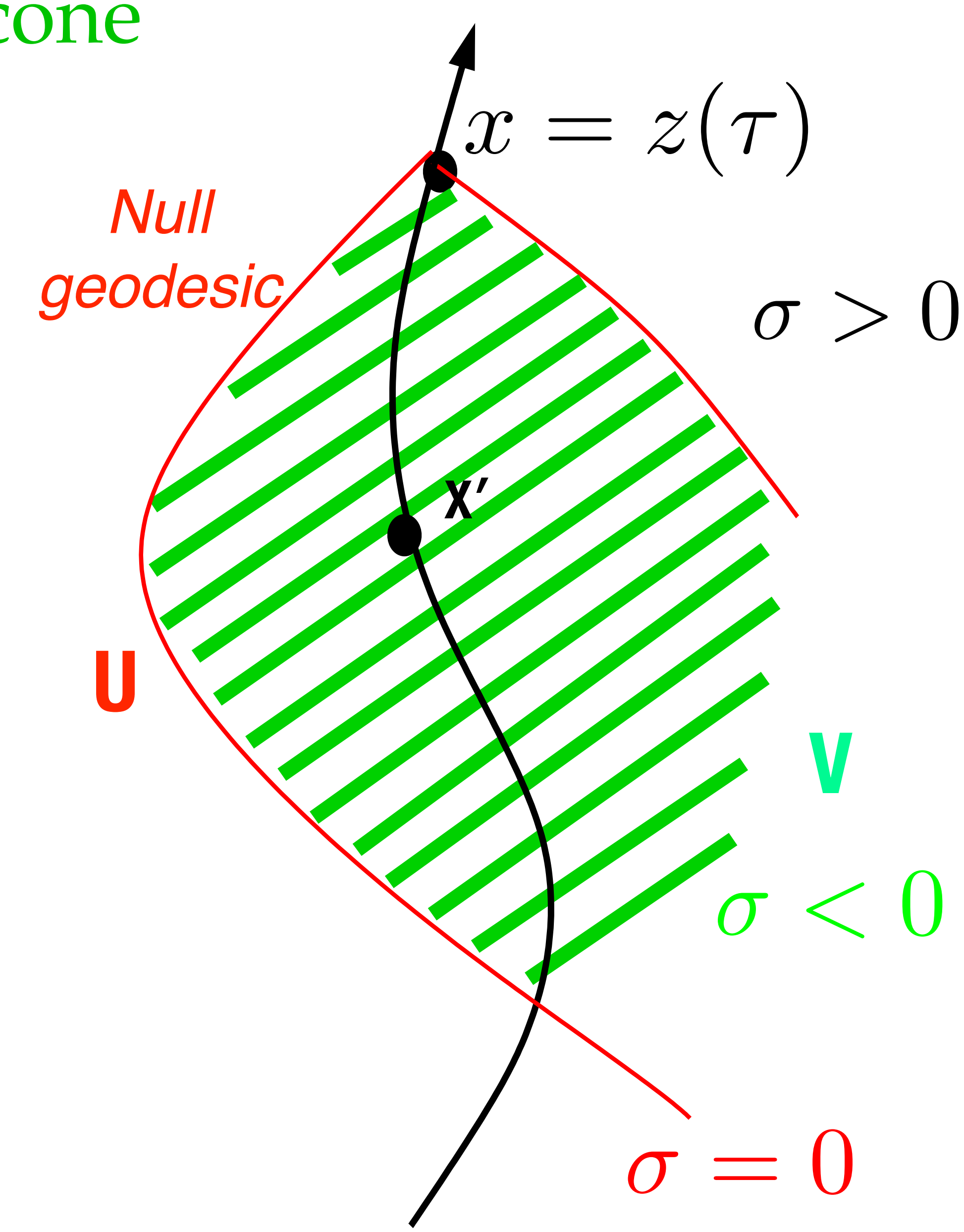

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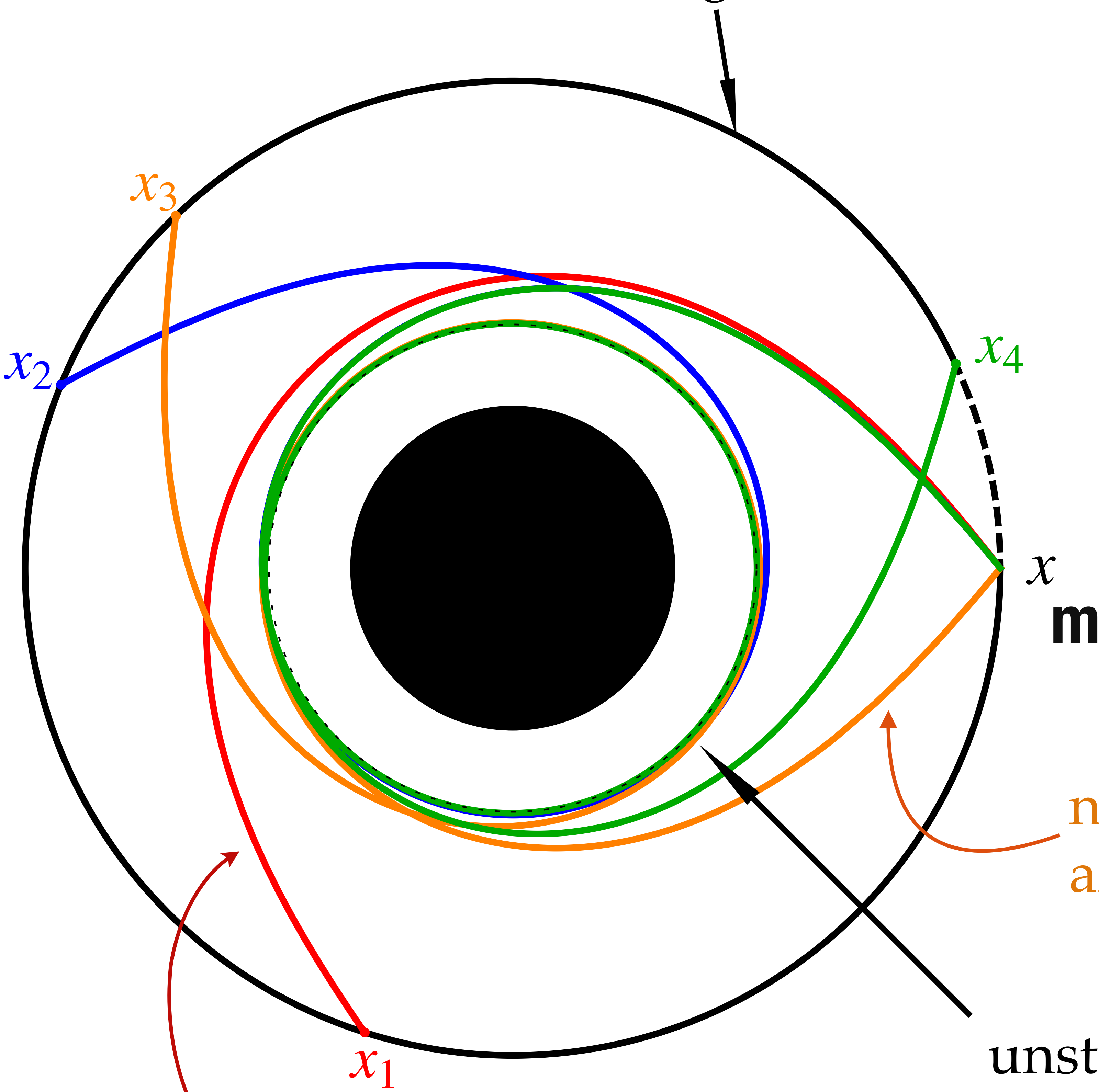
But Hadamard form is only valid for x and x' near - what other contributions are there for points far apart?

Contribution 2: orbiting *null* geodesics

Point mass m on a circular geodesic at r=6M in Schwarzschild

There's an infinite num. of null geods. from x that go around the BH an arbitrary num. of times (remember *BH shadow*!)

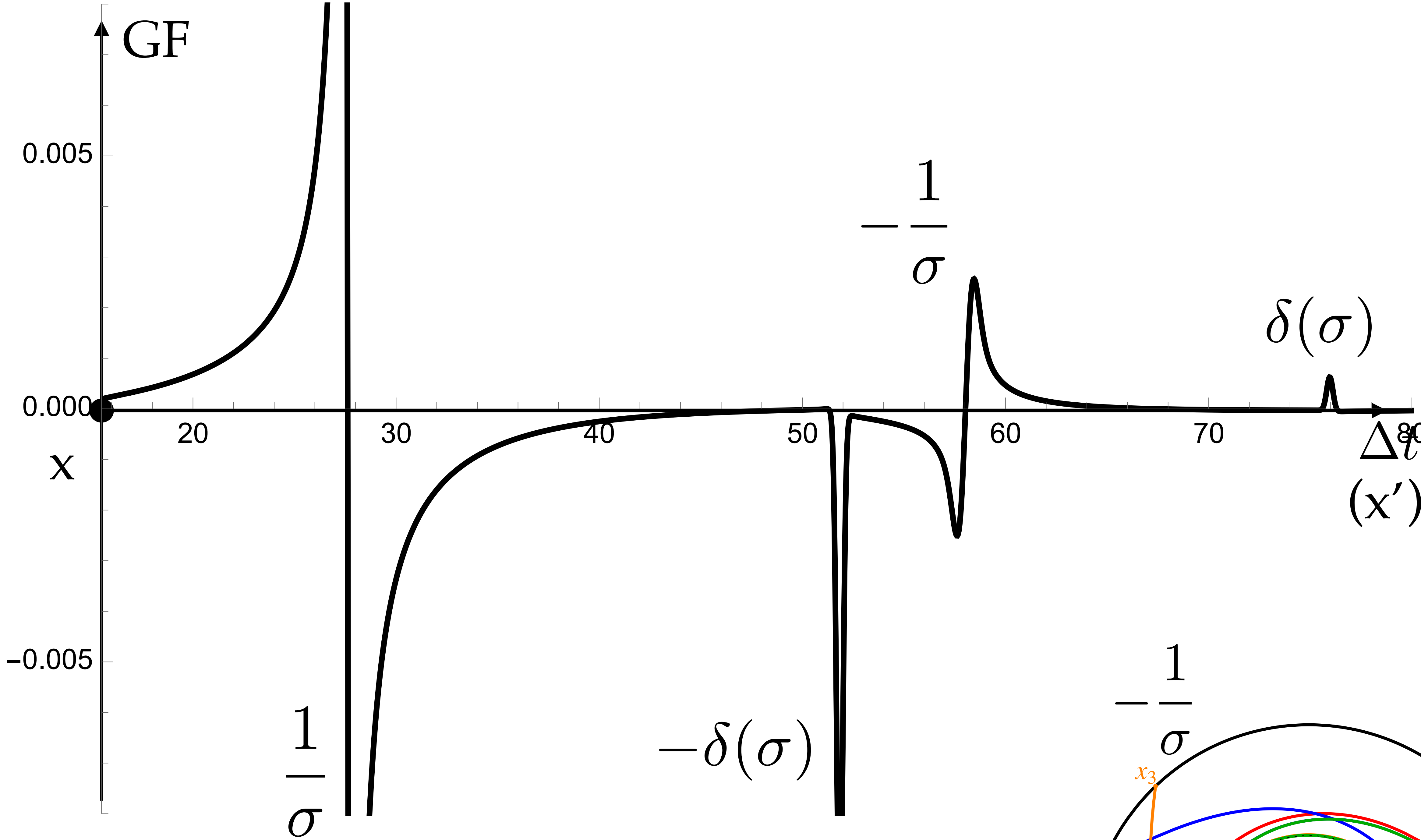


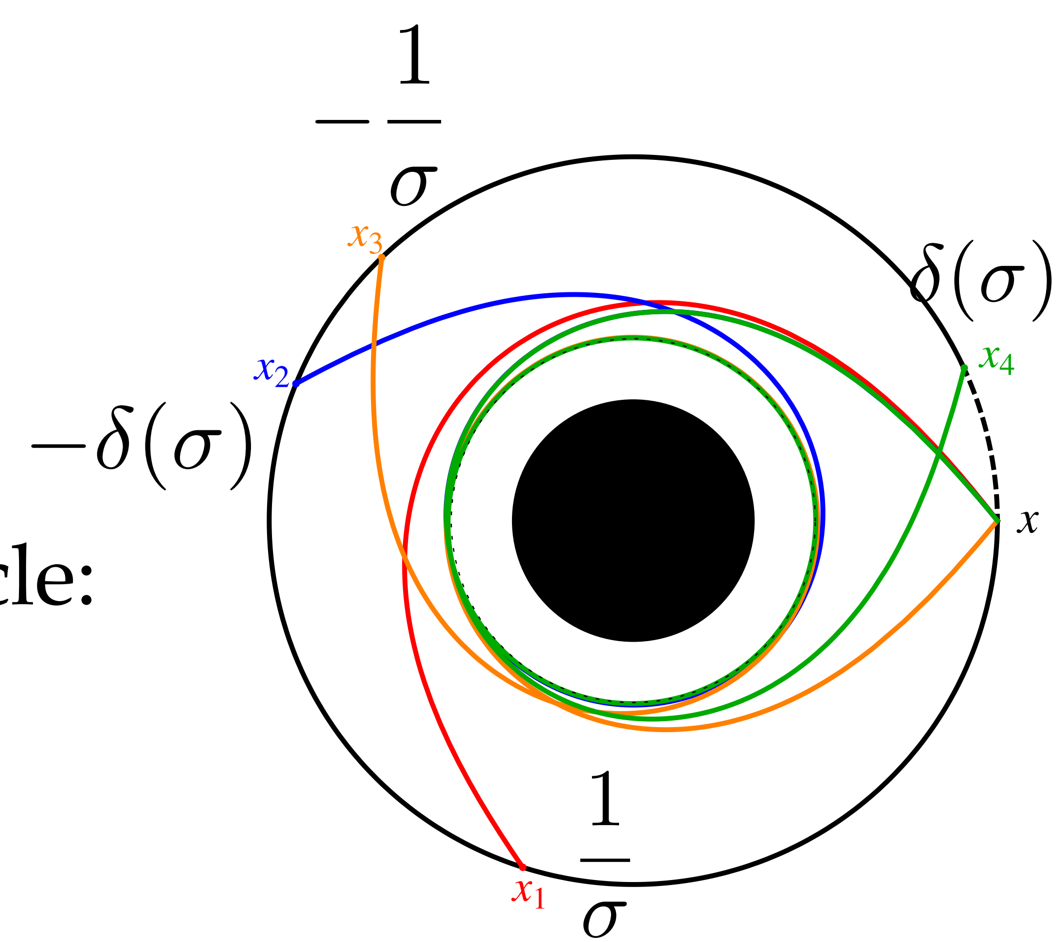
null geod. going around the BH *twice*

unstable *photon orbit*

$r = 3M$

null geod. going around the BH *once*

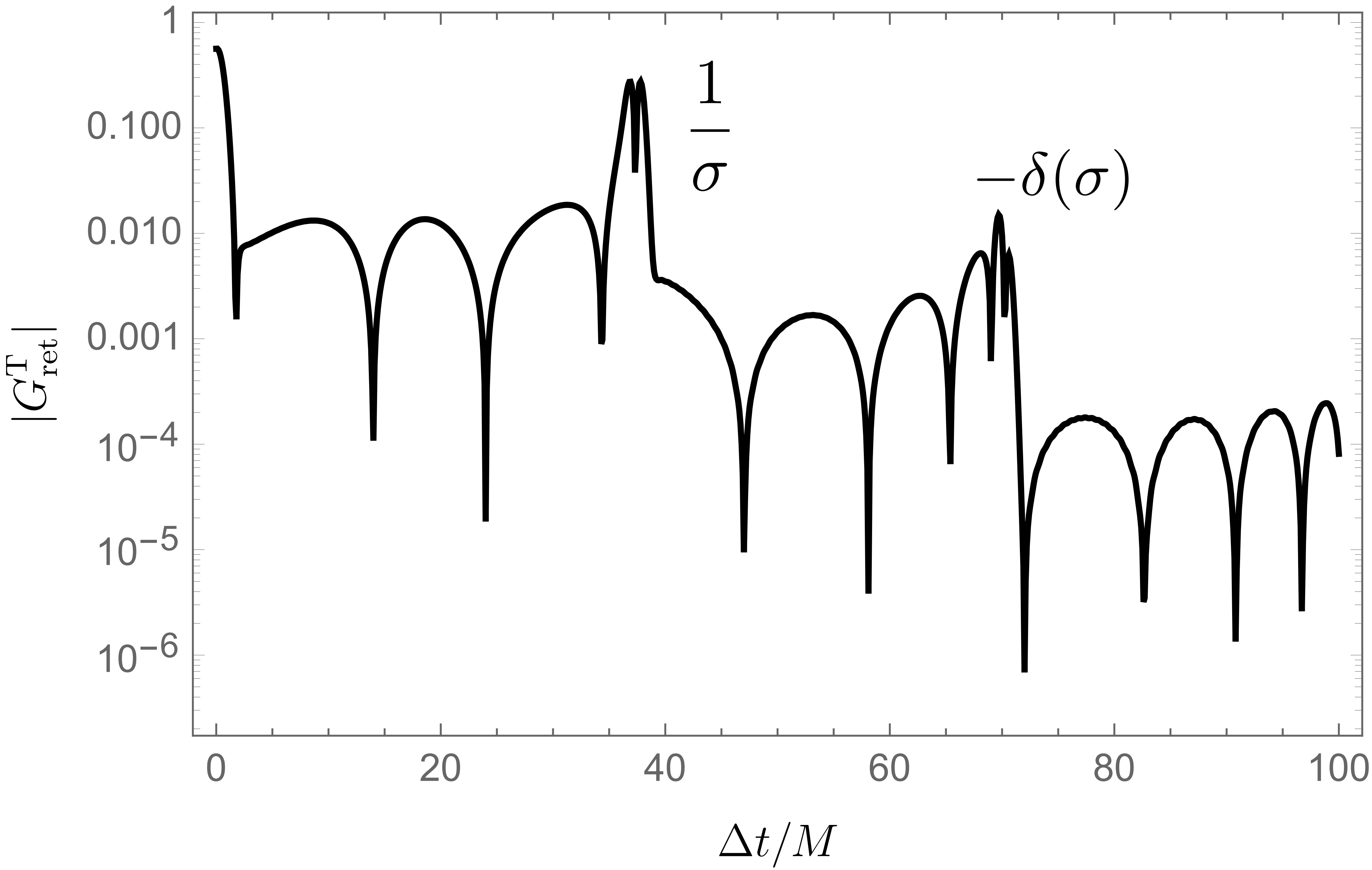




GF diverges at light-crossings in 4-fold cycle:

$$G_{ret} \sim \delta(\sigma) \to \frac{1}{\sigma} \to -\delta(\sigma) \to -\frac{1}{\sigma}$$

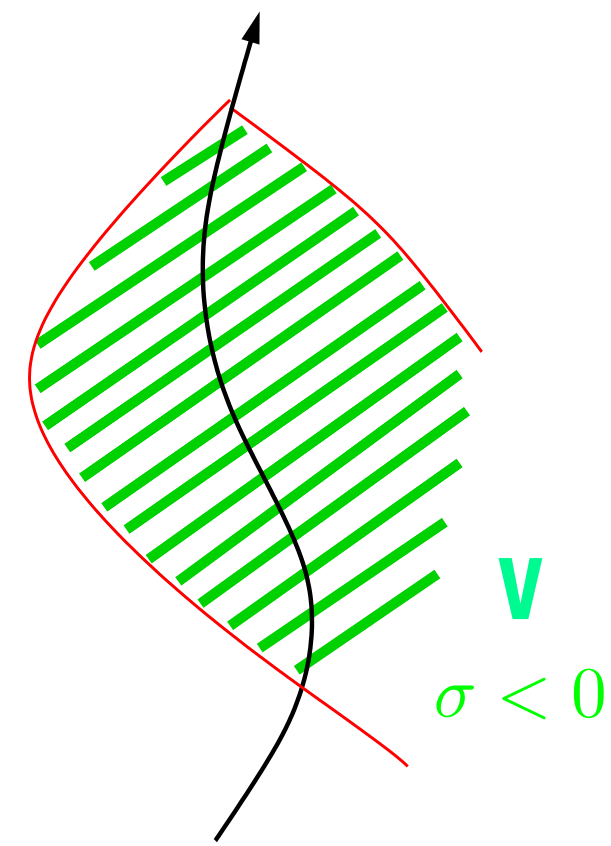
Gravitational (Teukolsky) GF



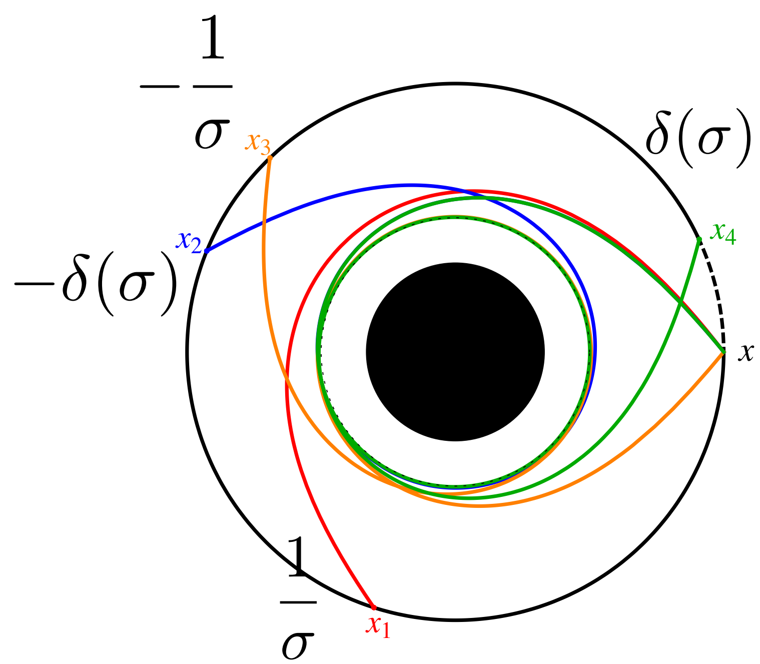
The contribution to the non-local integral in the SF comes from:

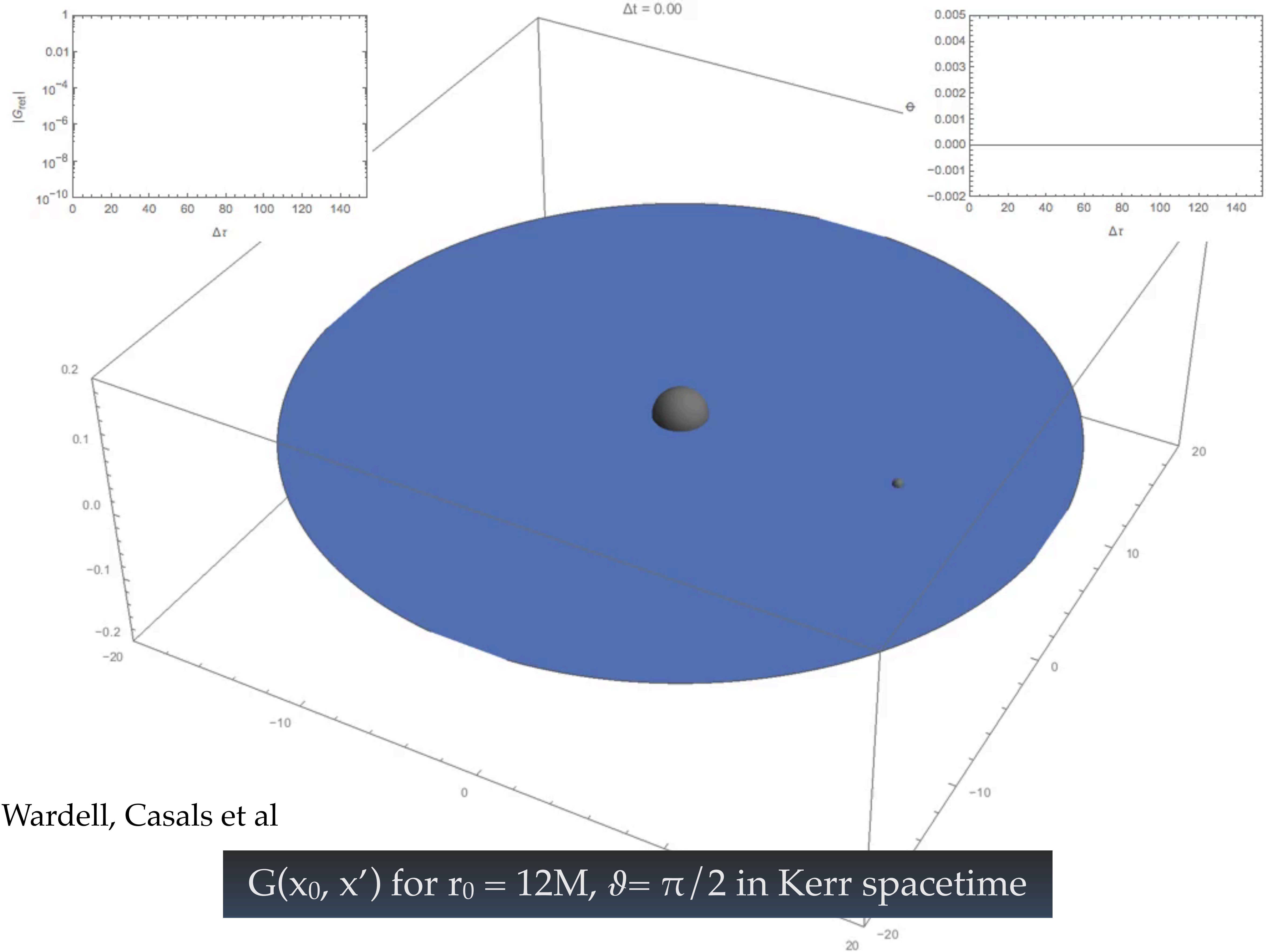
- backscattering of waves (Hadamard $V \neq 0$)

  (timelike paths)



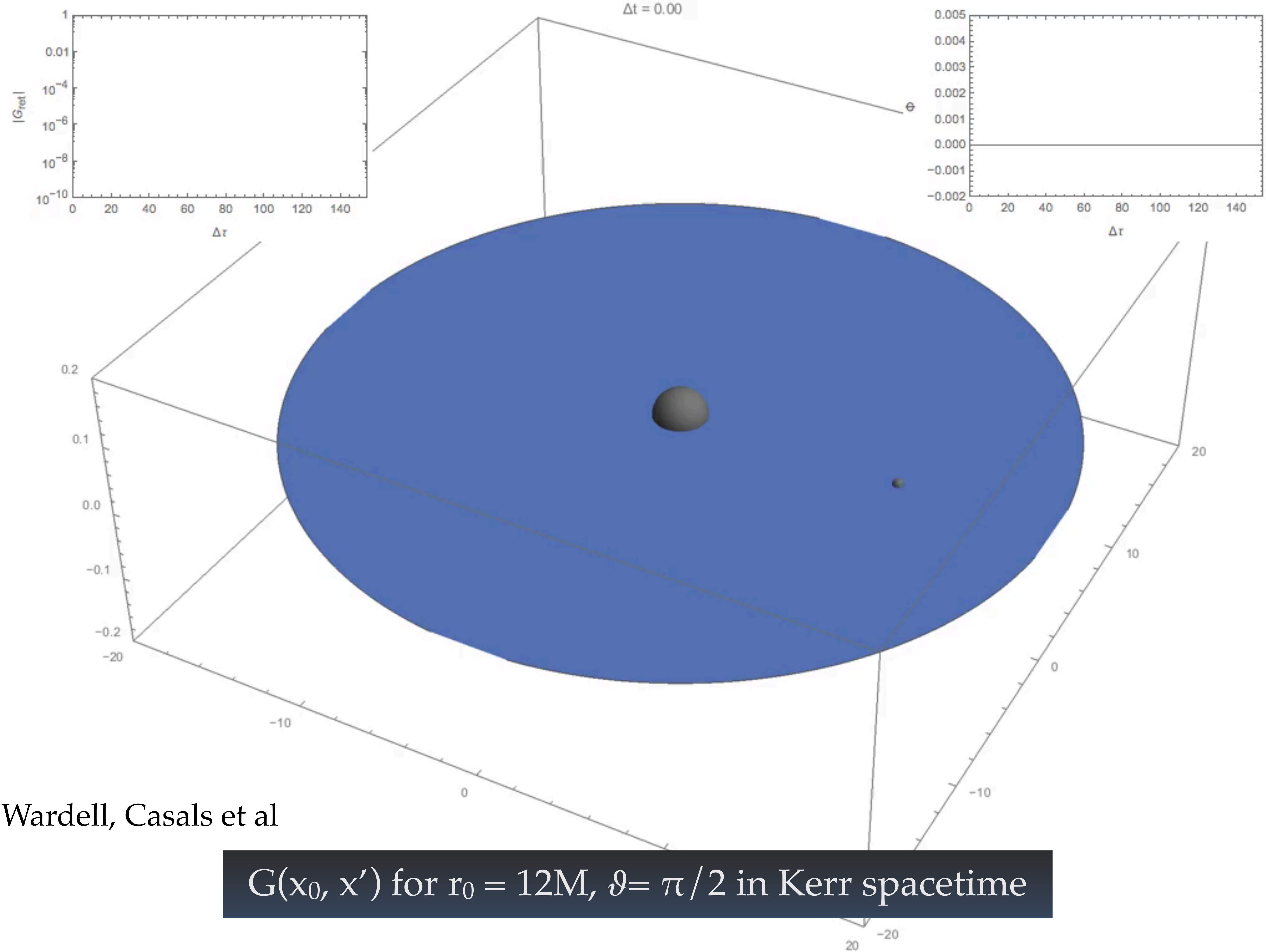
- orbiting null geodesics





Wardell, Casals et al

G(x_0, x′) for r_0 = 12M, ϑ= $\pi/2$ in Kerr spacetime

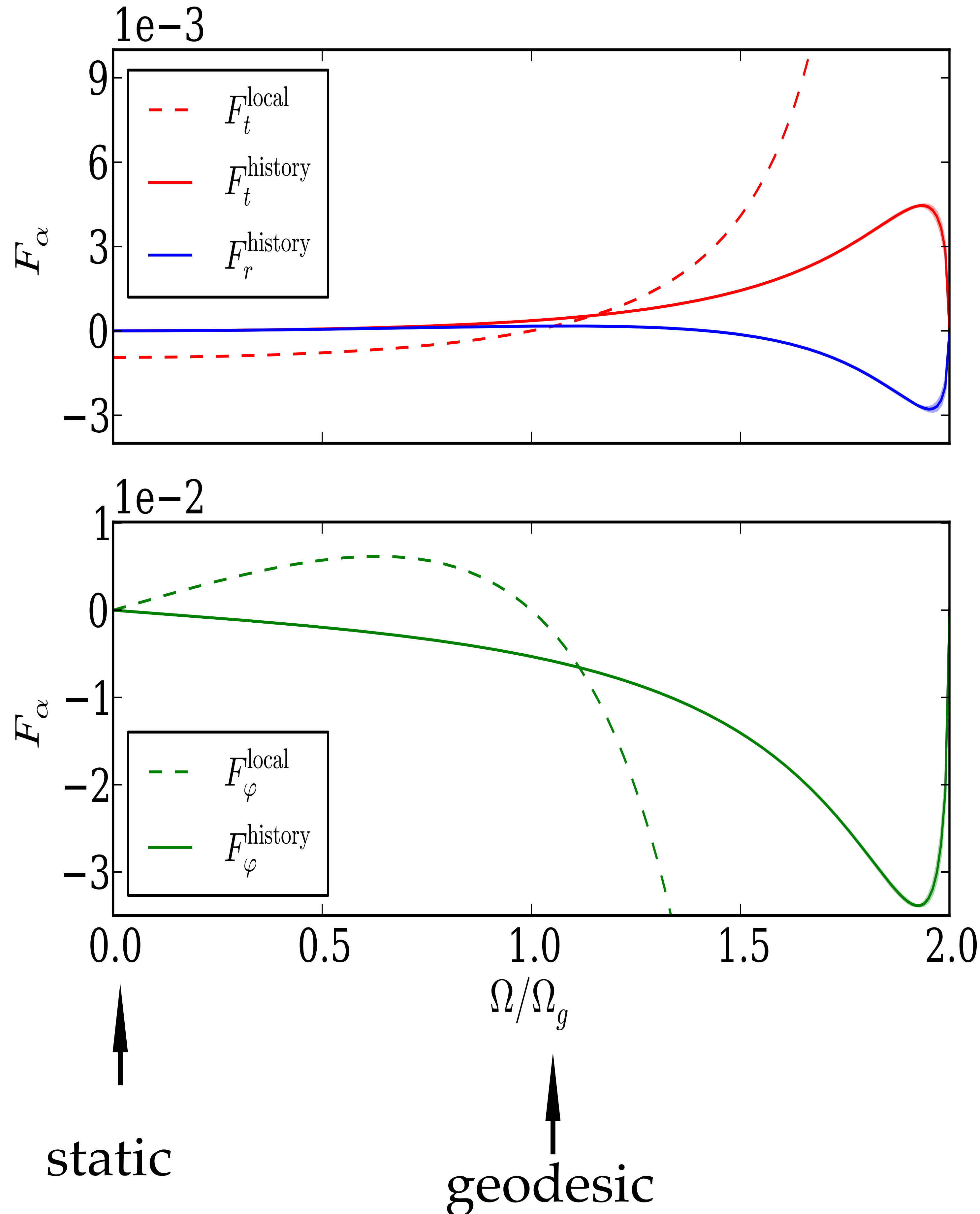


Wardell, Casals et al

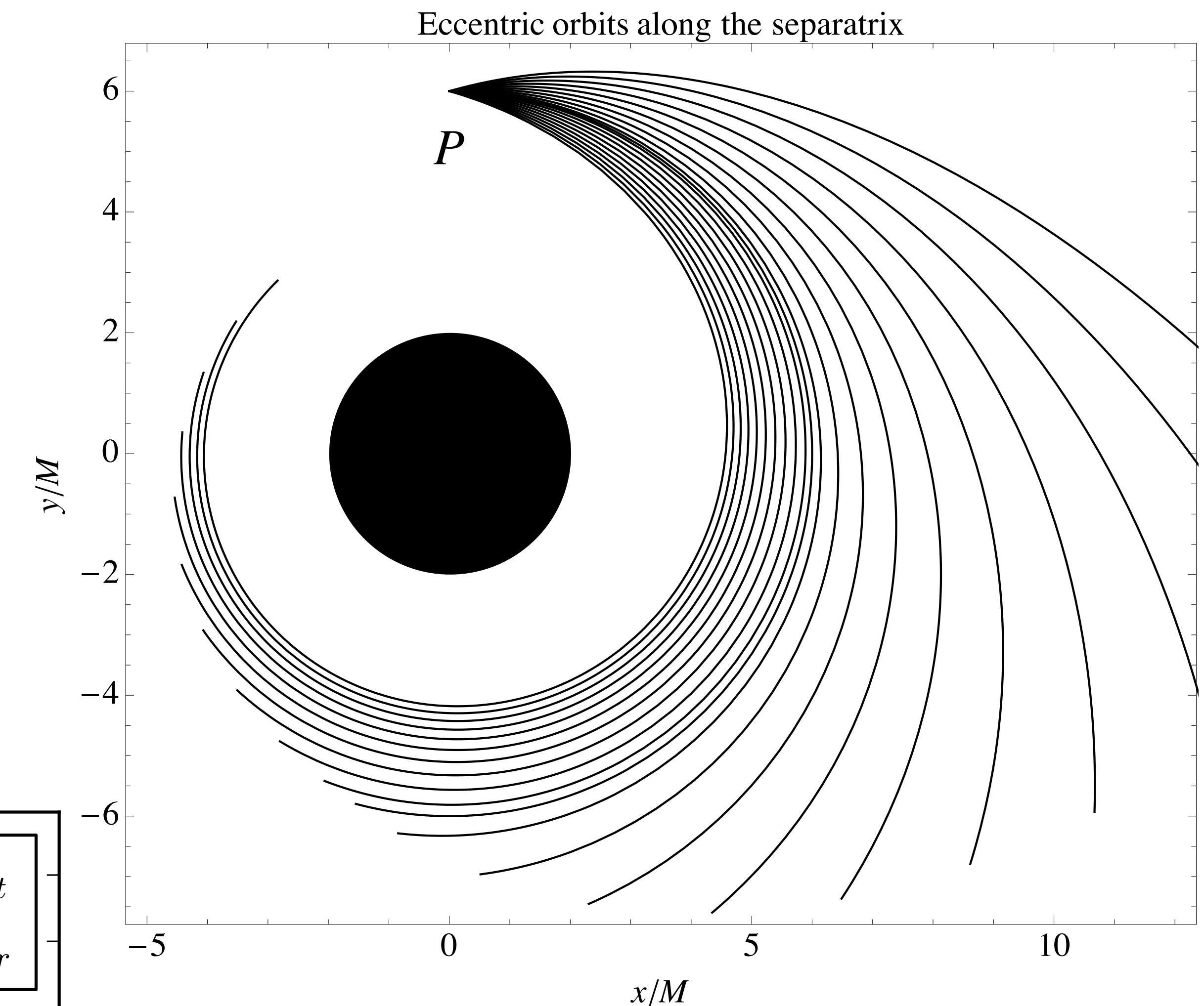
G(x_0, x′) for r_0 = 12M, $\vartheta = \pi/2$ in Kerr spacetime

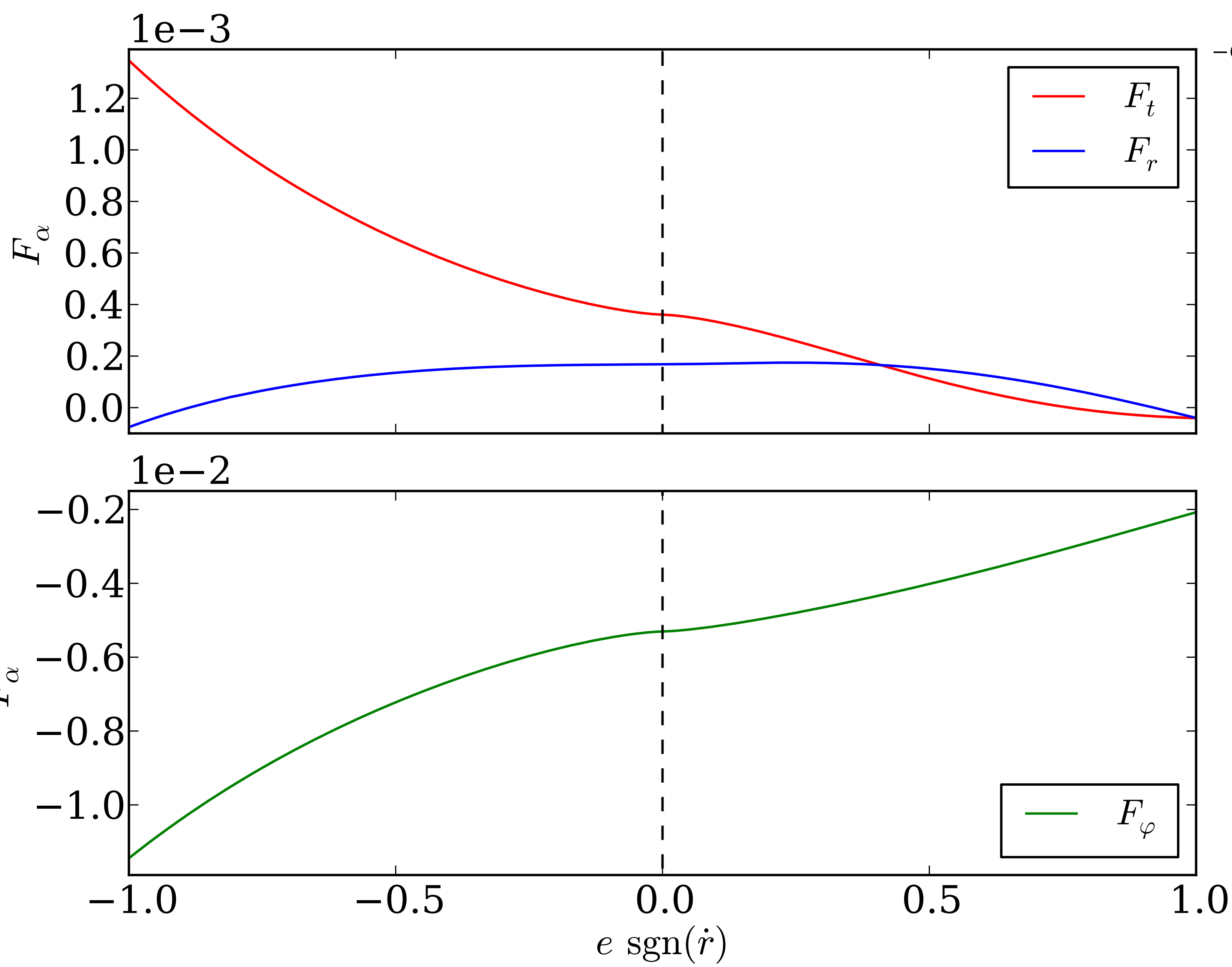
SF results via GF

Scalar SF on a charge in a circular orbit (r=6M) around a Schwarzschild BH (Wardell, Galley, Zenginoglu, Casals et al'14)



Scalar SF on a charge r=10M (P) on *eccentric* geodesics in Schwarzschild





(Wardell, Galley, Zenginoglu, Casals et al'14)

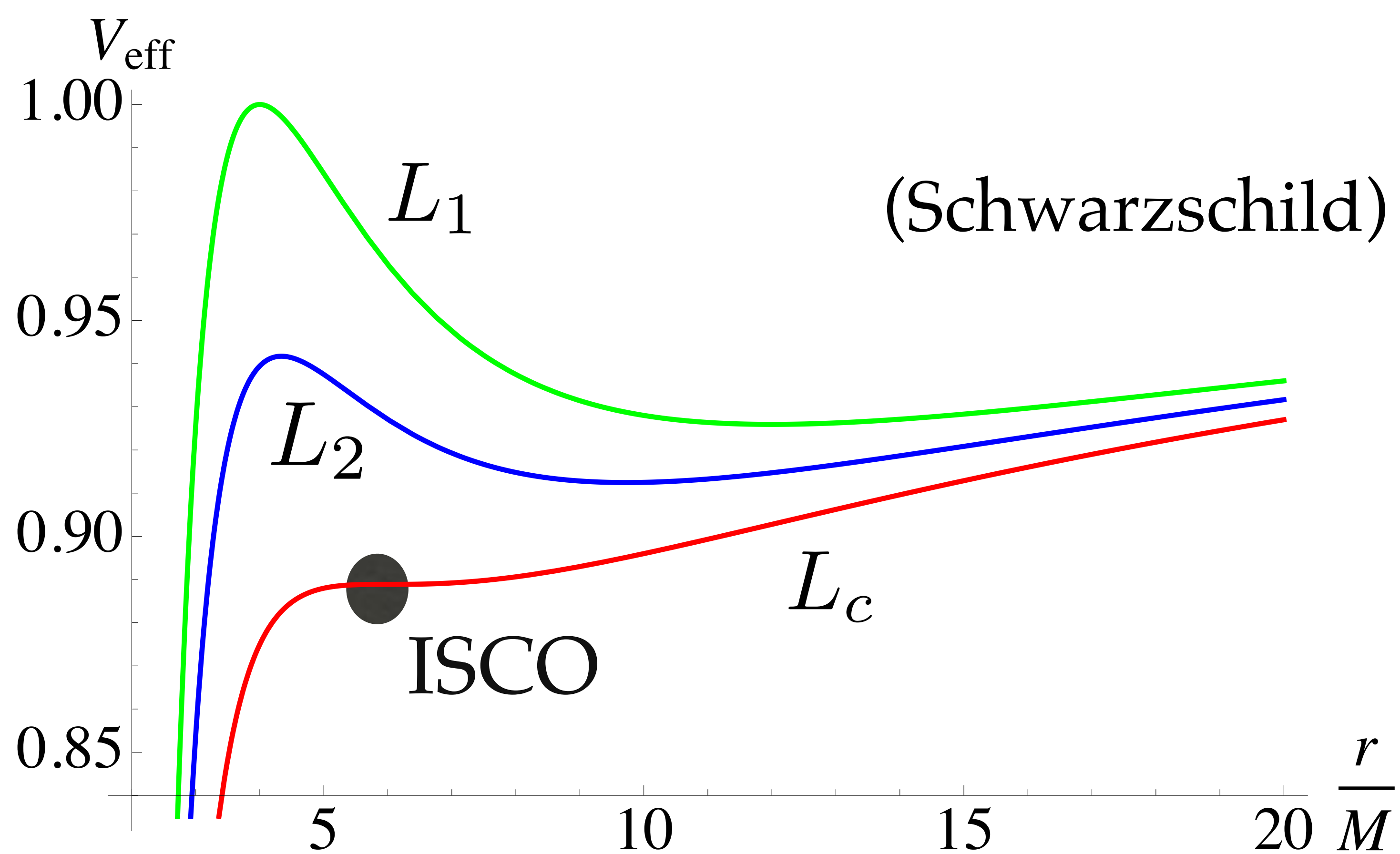
Gauge-Invariants

- It's also useful to compute coordinate-invariant quantities since:

  (1) they're observables in GW astronomy

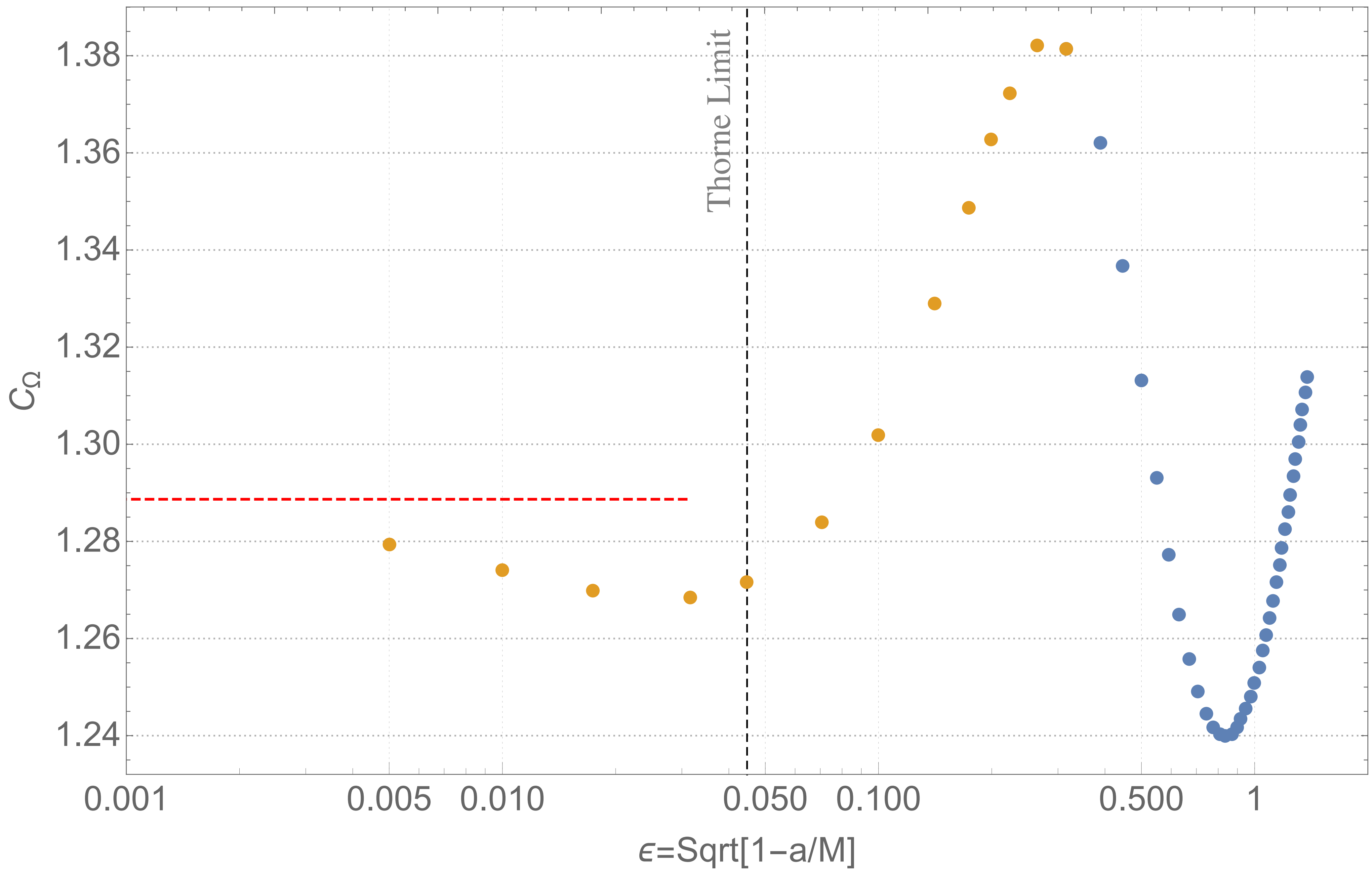
  (2) they allow for comparison with Numerical Relativity and Post-Newtonian

- An interesting one is the frequency of the innermost stable circular orbit (ISCO):



Correction to orbital frequency at ISCO of Kerr due to small mass:

$$(M+m)\Omega_{ISCO} = \underbrace{M\Omega^{(0)}_{ISCO}}_{\text{test particle}} \left\{ 1 + \underbrace{C_\Omega}_{\text{correction}} \frac{m}{M} + O\left(\frac{m}{M}\right)^2 \right\}$$



(Warburton, Casals et al; Cf. Meent'16)

• Calculation of the SF via GF yields physical insight from wave propagation and may be practical for orbit evolution

• But GF method is not the standard one for calculating the SF. Other methods have given impressive results:

- Gravitational SF in Kerr (Meent'18) and first results in 2nd order SF (see Le Tiec's talk)

- Correction to various gauge-invariants (rate of periastron advance, spin precession, redshift, etc) (Le Tiec, Dolan, etc)

- Orbit evolution: self-consistent (solve for SF eq. and EOM simultaneously) in scalar case (Diener et al'11) and 'geodesic' SF (SF calculated for instantaneously tangent geodesic) in gravitational case (Warburton et al'12)

etc

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Conclusions

Geometrical insight into SF - it arises from wave propagation in a curved s-t via:

- *wave scattering* (timelike paths)
- *orbiting null geodesics*

GF method is not the current mainstream method for calculating SF but may be suitable for evolution including SF

Objective for LISA sources: evolution of orbits in Kerr including SF (…to 2nd order!)

Merci bien!