Particle simulations in special and general relativity

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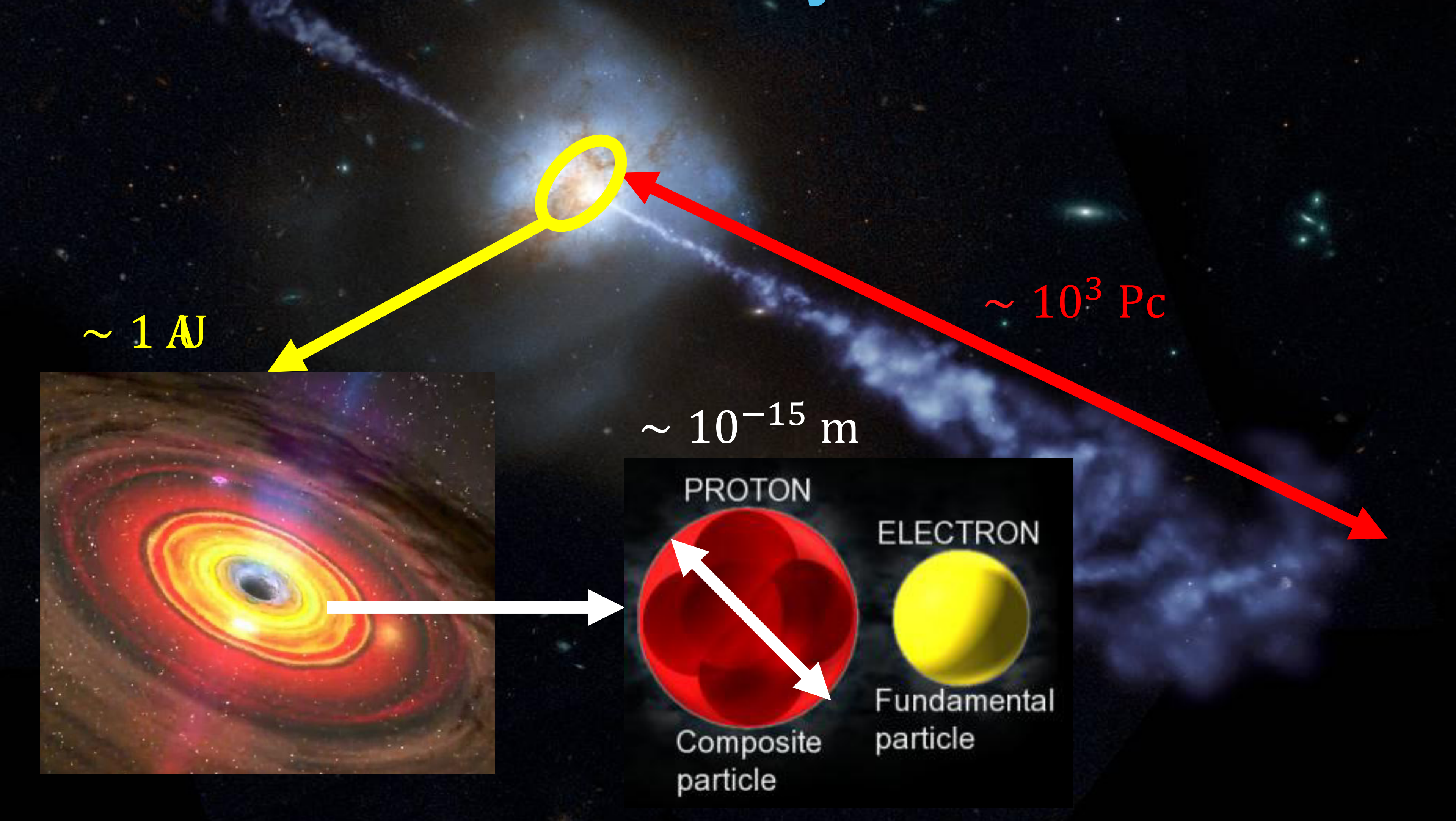
Extreme particle acceleration in the universe

Extreme phenomena in the universe are responsible for the emission of highly energetic particles

The precise mechanisms are not fully understood and involve large separation of scales
→ macroscopic phenomena affect the micro scales and vice versa

Modelling of such systems is computationally very demanding

Separation of scales in stationary GR



Modelling tools

Each scale can be efficiently described by dedicated models:

SCALE

- Jets → (SR) Hydrodynamics
- Accretion disks and jet launching
  → GR (Resistive) Magnetohydrodynamics
- Local plasma phenomena
  → Particle/Kinetic physics in SR/GR

Need for highly efficient simulation codes

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Plasma dynamics around black holes

Typically modelled with GRMHD and radiation + photon geodesics

$$\begin{cases} \nabla_\nu (\rho U^\nu) = 0 \\ \nabla_\nu T^{\mu\nu} = 0 \end{cases}$$

$$+$$

$$\begin{cases} \partial_{[\mu} F_{\mu\alpha]} = 0 \\ \nabla_\nu F^{\mu\nu} = -J^\mu \end{cases}$$

$$\frac{dx^\mu}{d\tau} = u^\mu$$

$$\frac{du_\mu}{d\tau} = \Gamma^\nu_{\mu\sigma} u^\sigma u_\nu$$

Extensions and refinements of currently used simulation tools can be carried out:

- Enriching and improving the physical models, e.g. Resistivity in GRMHD with efficient ImEx schemes;
- Creating new methods, e.g. particle methods for SR/GR;

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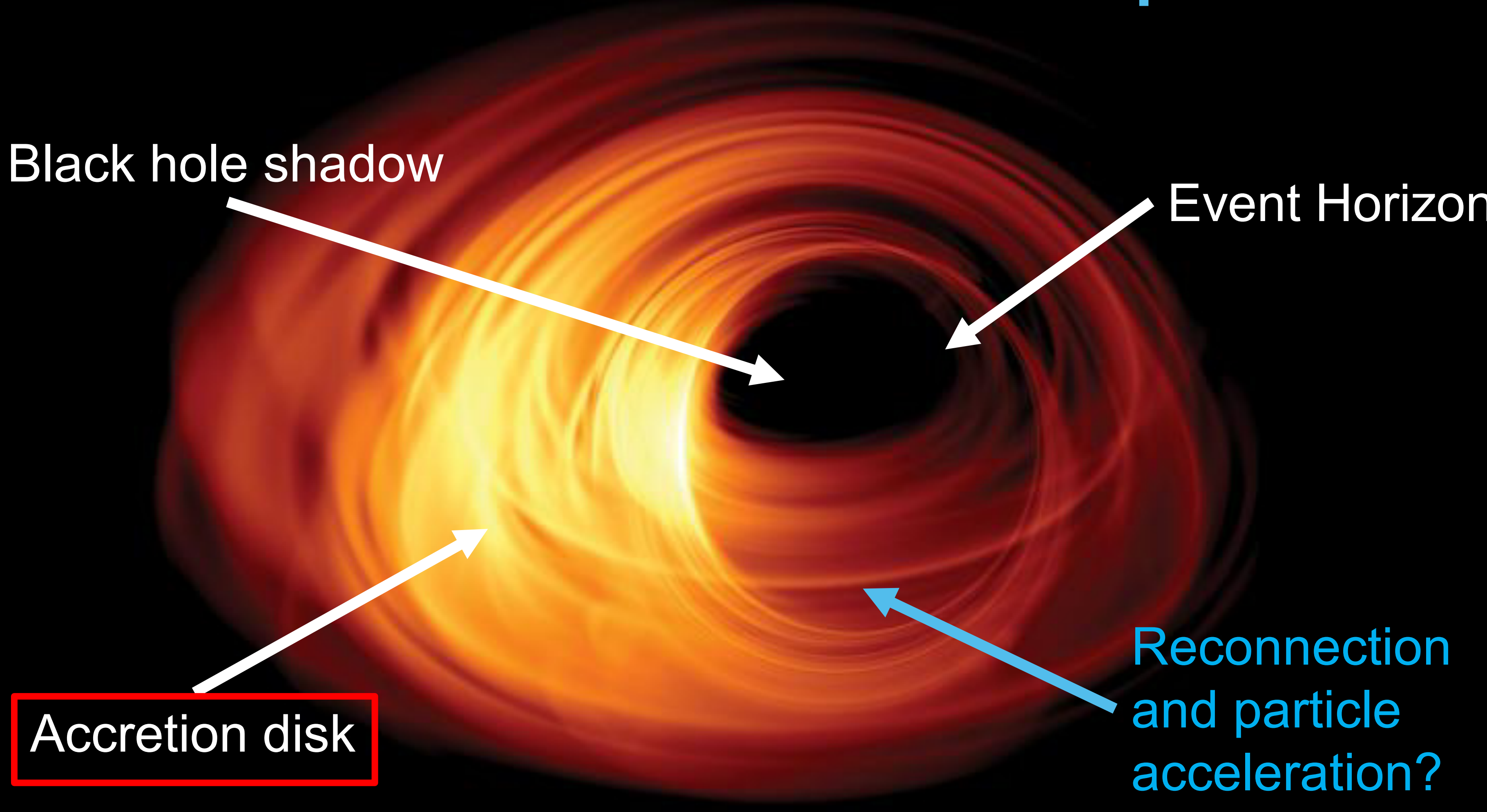
A black hole in a computer

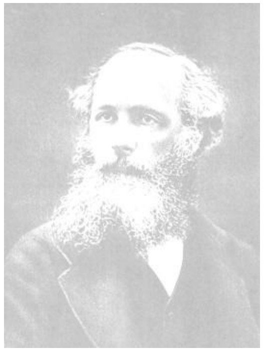
[Bronzwaer, Moscibrodzka, Davelaar, and Falcke, using HARM2D GRMHD.]

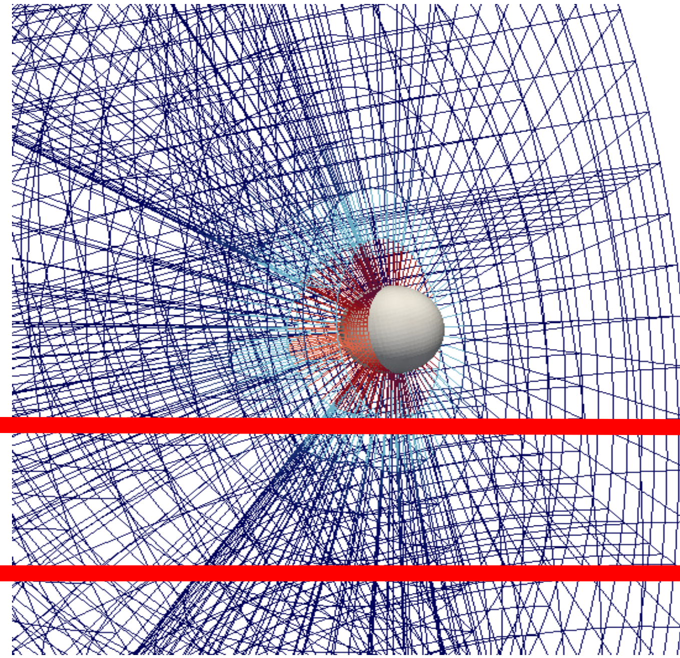
A black hole in a computer



[Bronzwaer, Moscibrodzka, Davelaar, and Falcke, using HARM2D GRMHD.]

The macro scale: General Relativistic MHD



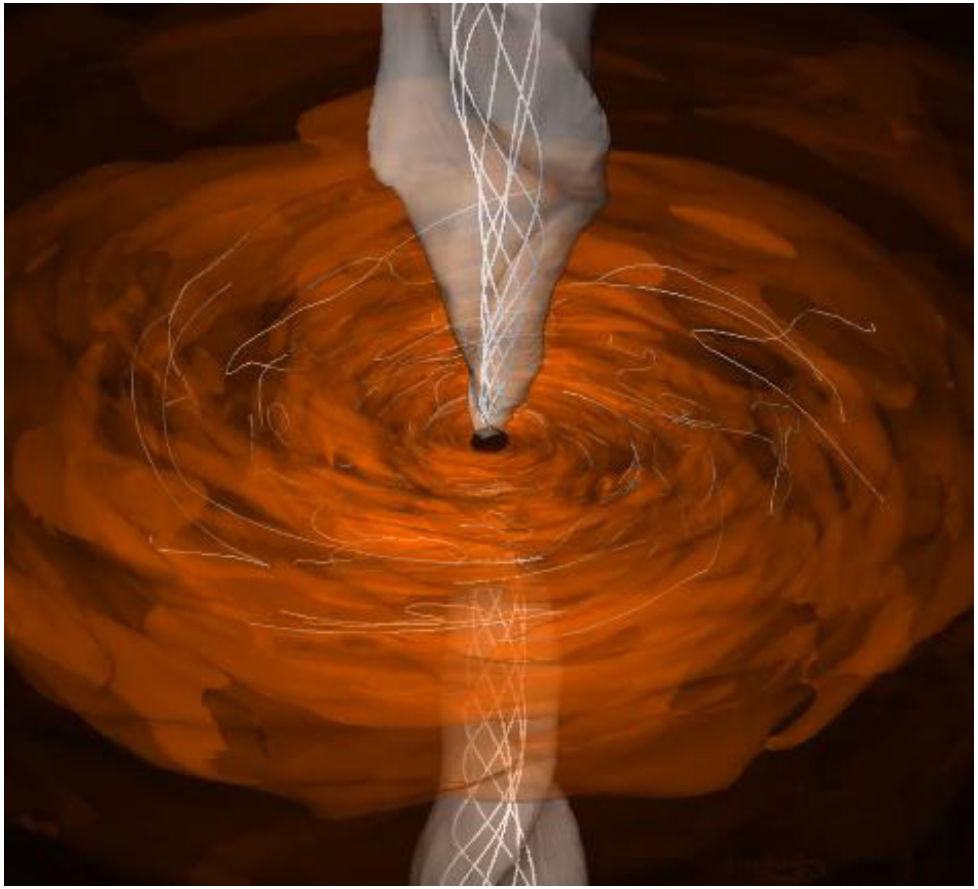


$$\partial_{[\mu}F_{\mu\alpha]} = 0$$
$$\nabla_\nu F^{\mu\nu} = -J^\mu$$

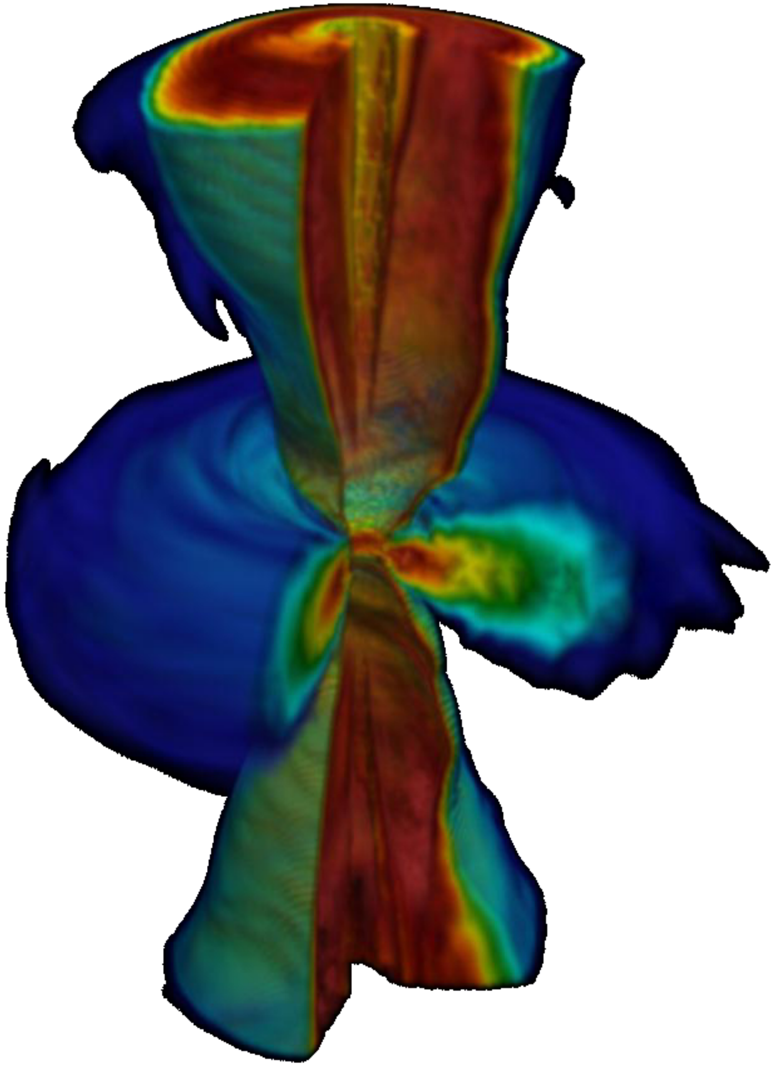
$$\nabla_\nu(\rho U^\nu) = 0$$
$$\nabla_\nu T^{\mu\nu} = 0$$

- Ignore spacetime evolution → (ideal) GRMHD for modelling the macroscopic dynamics of plasmas around black holes, e.g. accretion disks / jet formation: BHAC, ECHO, HARM, ATHENA, KORAL…
- Recently, resistivity was added to GRMHD codes → reconnection processes can be simulated

The macro scale: General Relativistic MHD



[Hotaka Shiokawa]



[Dolence, using HARM3D]

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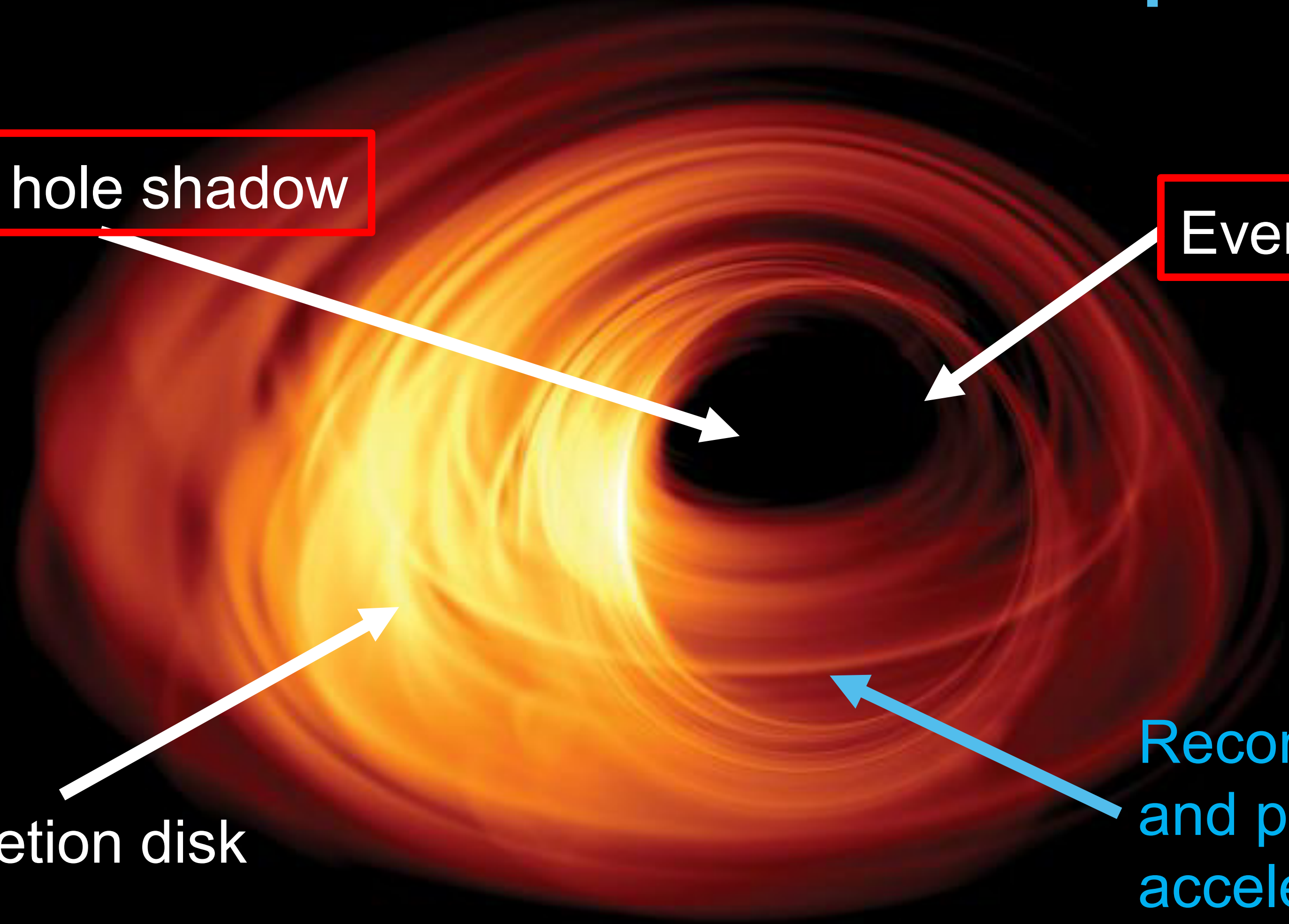
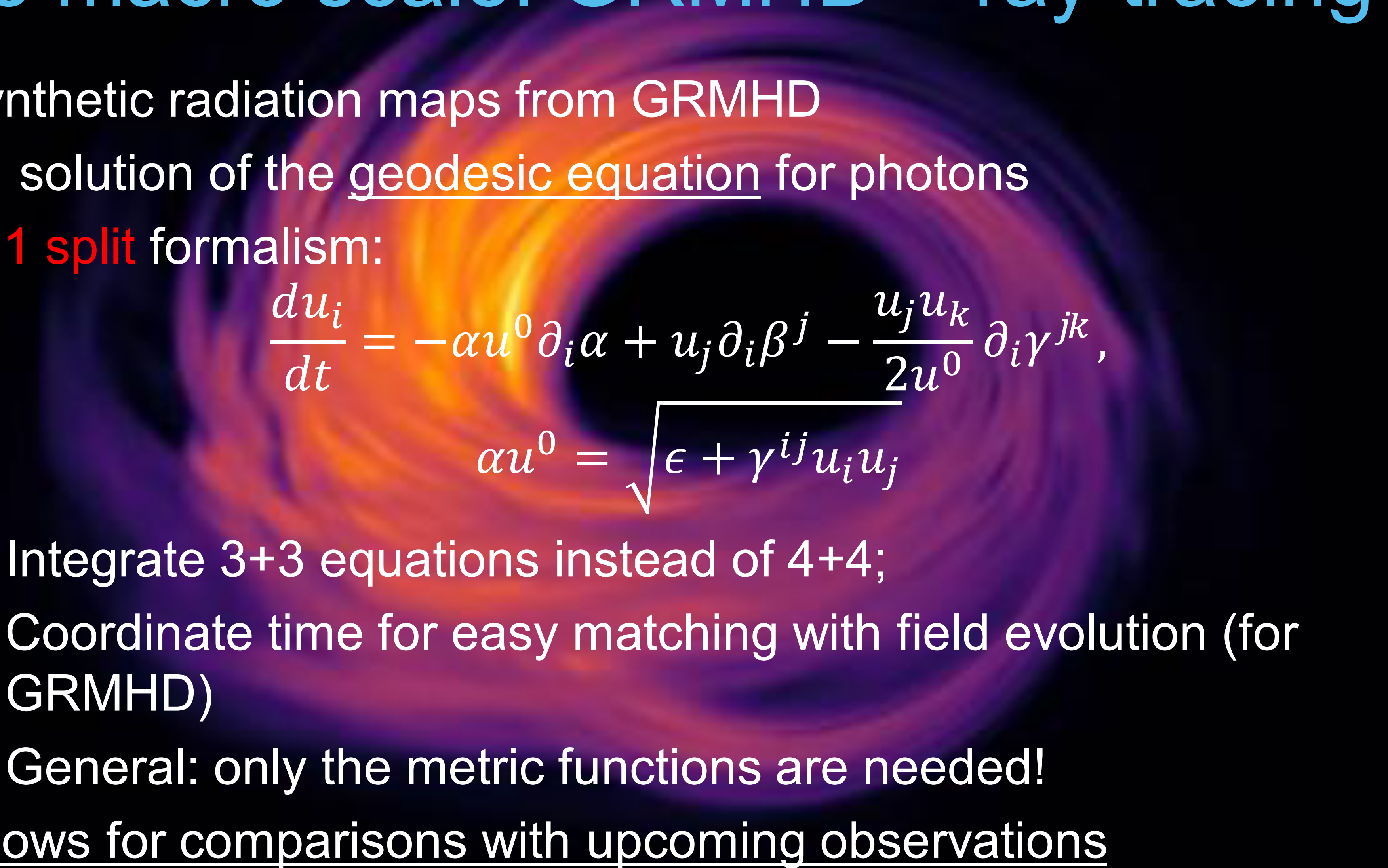
A black hole in a computer



[Bronzwaer, Moscibrodzka, Davelaar, and Falcke, using HARM2D GRMHD.]

The macro scale: GRMHD + ray-tracing



Synthetic radiation maps from GRMHD

→ solution of the geodesic equation for photons

3+1 split formalism:

$$\frac{du_i}{dt} = -\alpha u^0 \partial_i \alpha + u_j \partial_i \beta^j - \frac{u_j u_k}{2u^0} \partial_i \gamma^{jk},$$

$$\alpha u^0 = \sqrt{\epsilon + \gamma^{ij} u_i u_j}$$

- Integrate 3+3 equations instead of 4+4;
- Coordinate time for easy matching with field evolution (for GRMHD)
- General: only the metric functions are needed!

Allows for comparisons with upcoming observations

[Ziri Younsi, using BHOSS + BHAC]

Simulating geodesic motion

- 3+1 split formalism: set of 6 nonlinear, coupled, first-order differential equations for massive and massless particles

- Standard (RK4 or IMR) numerical methods introduce large errors due to non-conservation of energy around metric singularities

- Solution: an exactly (to round-off) energy-conserving numerical scheme derived from the underlying Hamiltonian

$$\frac{dx^i}{dt} = \frac{\partial H(x^l, u_l)}{\partial u_i} \qquad \frac{du_i}{dt} = -\frac{\partial H(x^l, u_l)}{\partial x^i}$$

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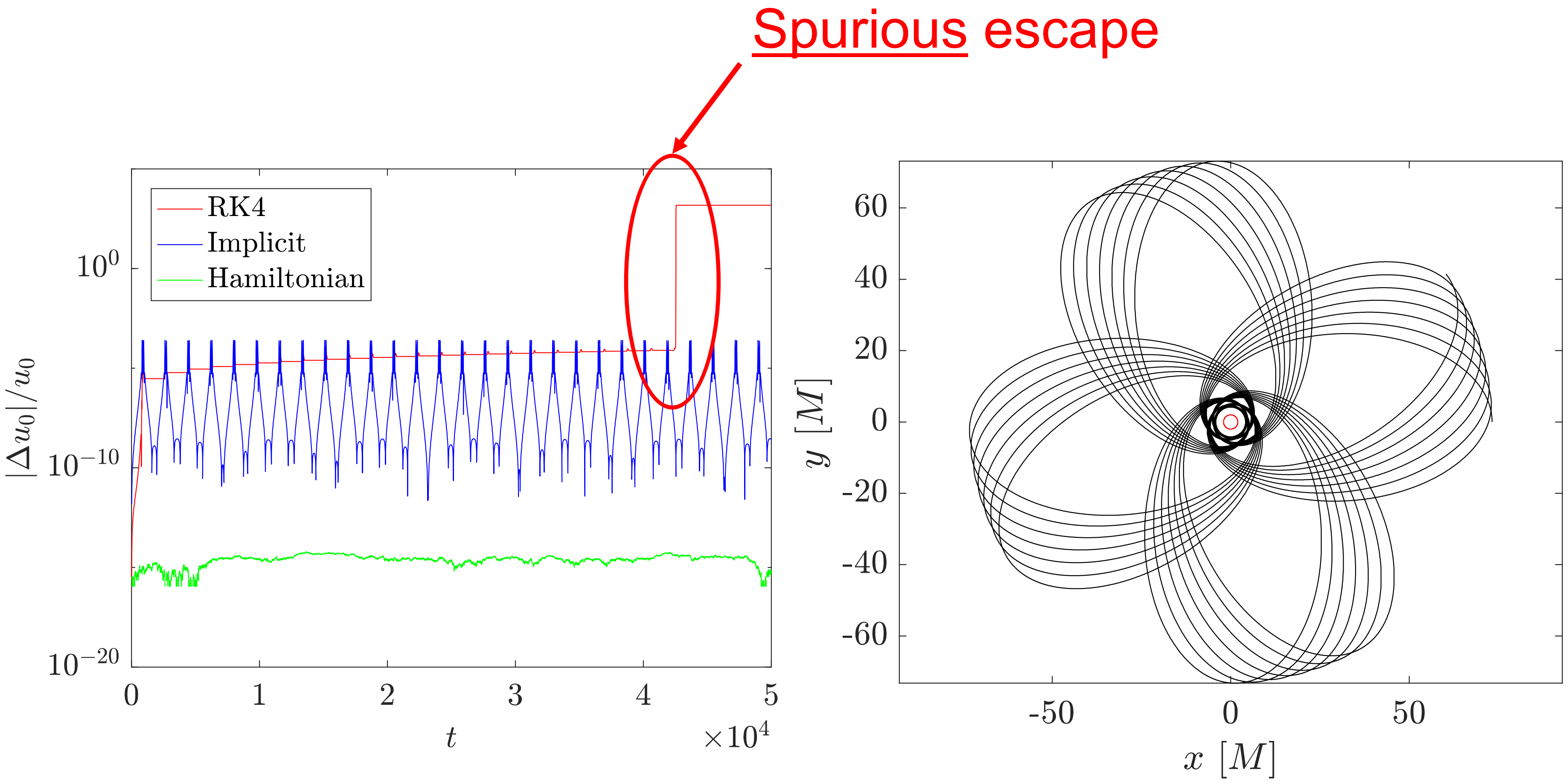
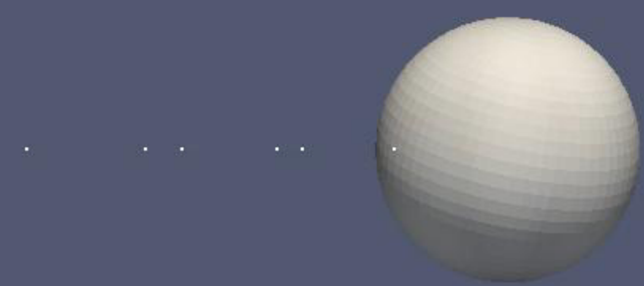
Practical effects: massive particles



Simulating geodesic motion



Generalized, energy-conserving numerical integration of geodesics in General Relativity

F. Bacchini, B. Ripperda, A. Chen, L. Sironi, submitted to ApJS

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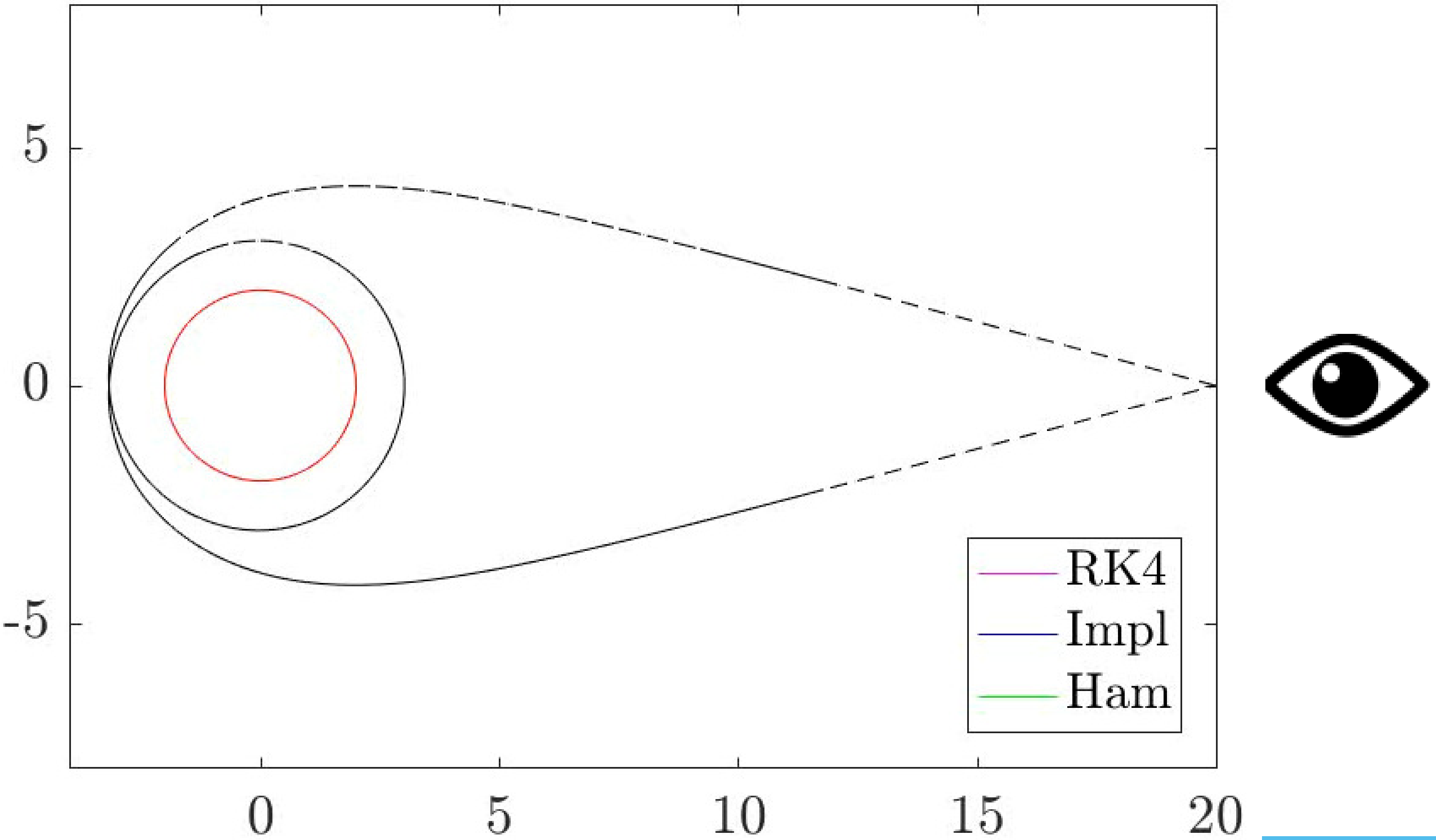
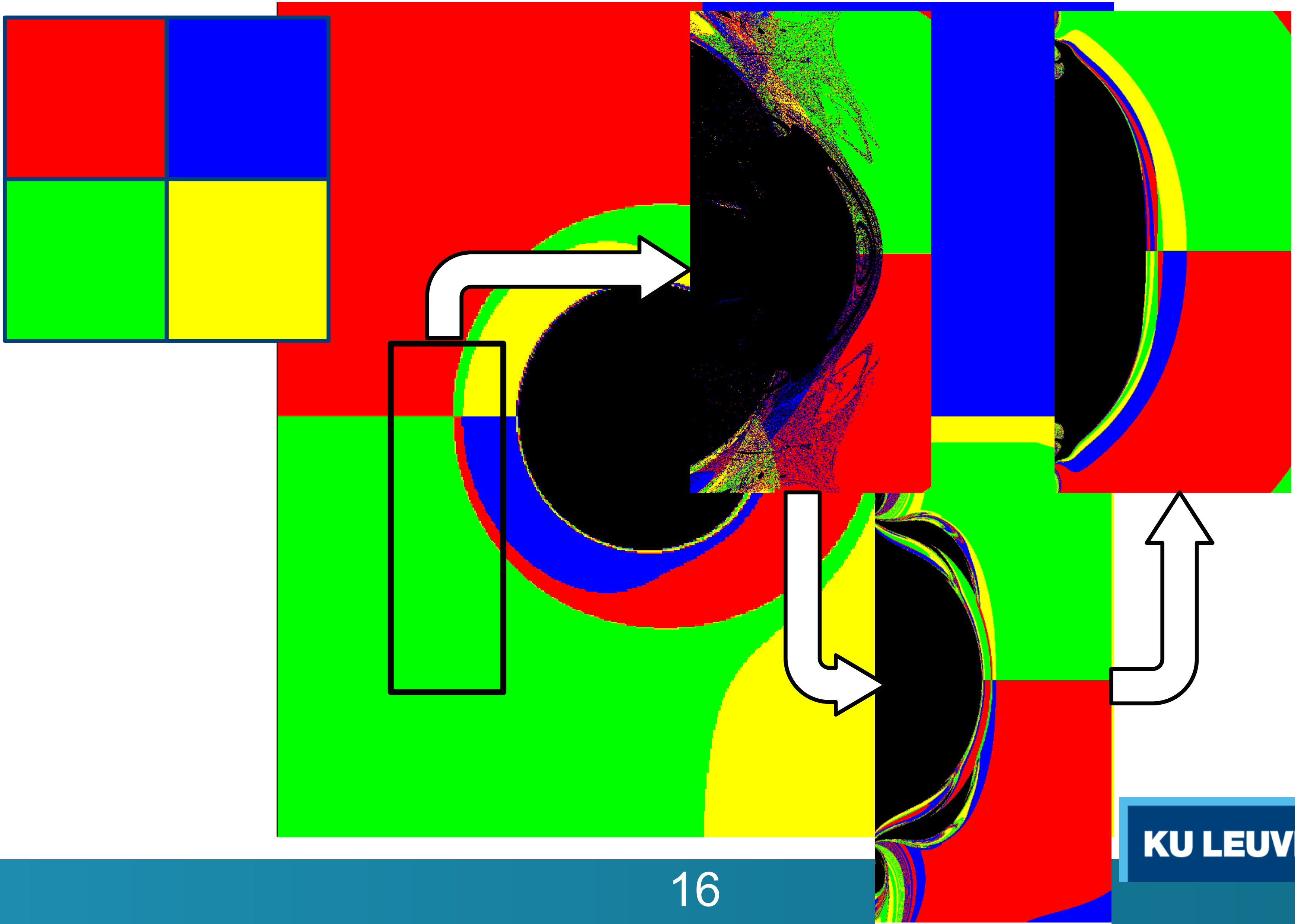
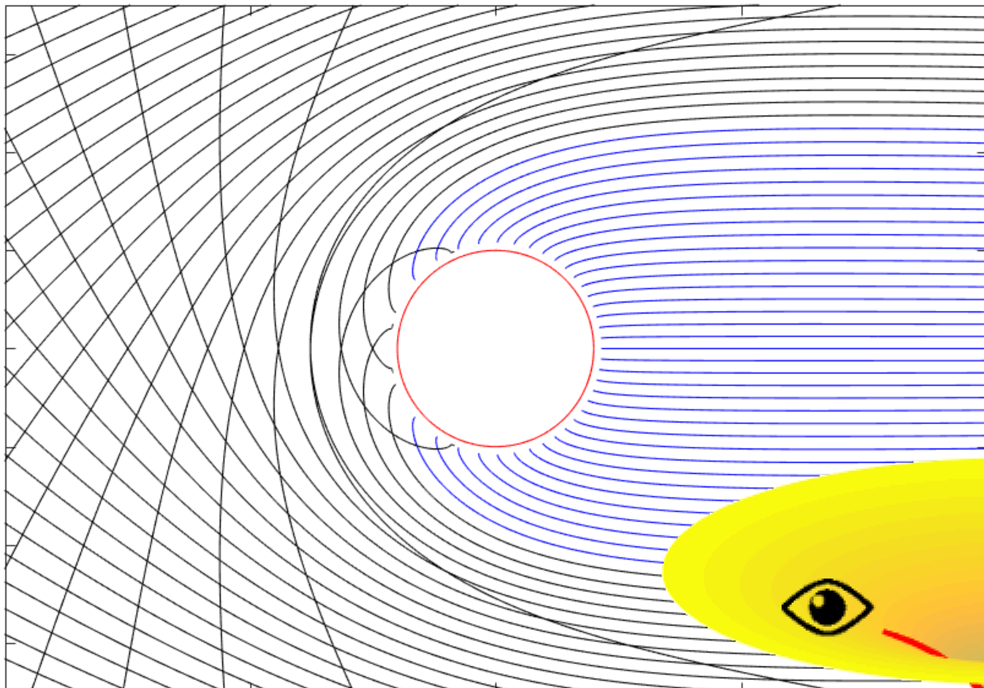
Practical effects on black hole imaging

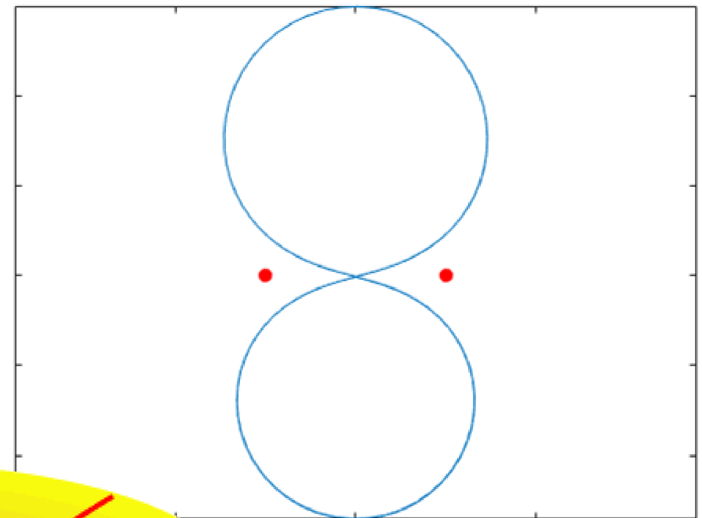


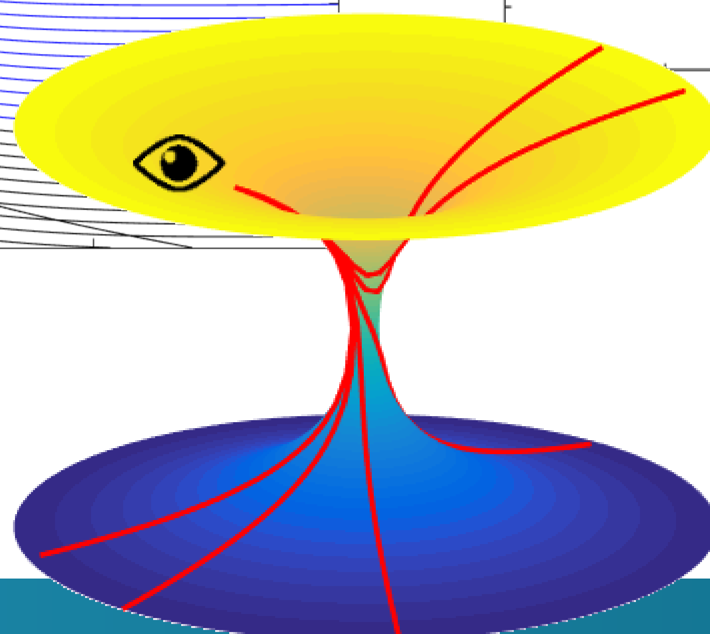
Practical effects on black hole imaging



Framework flexibility: nonstandard spacetimes







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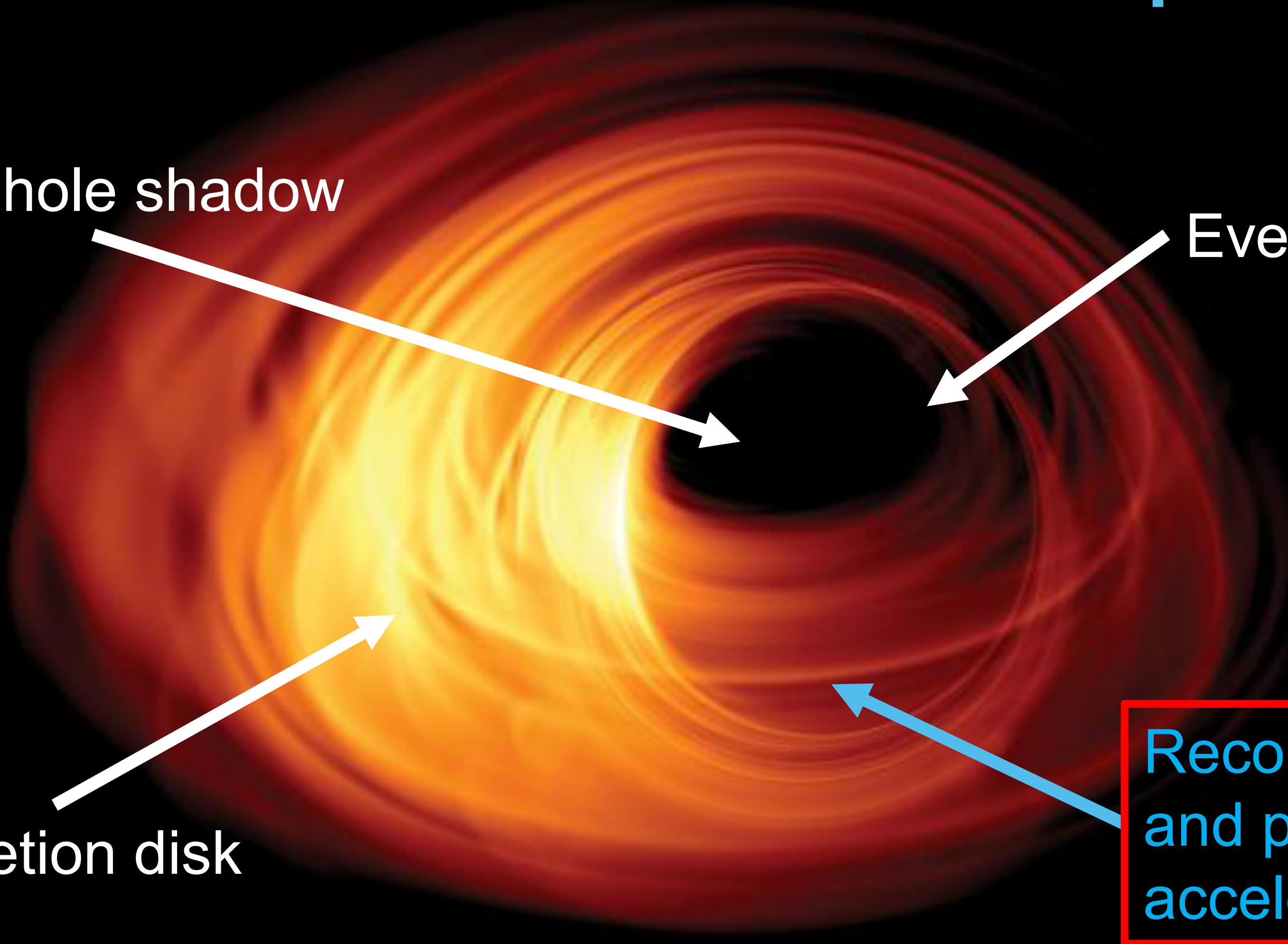
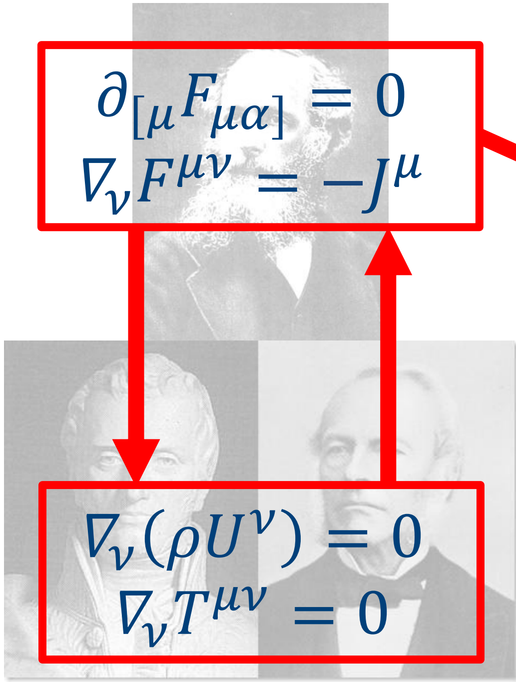
A black hole in a computer



[Bronzwaer, Moscibrodzka, Davelaar, and Falcke, using HARM2D GRMHD.]

Macro to micro: test particles in GRMHD



$$\partial_{[\mu}F_{\mu\alpha]} = 0$$
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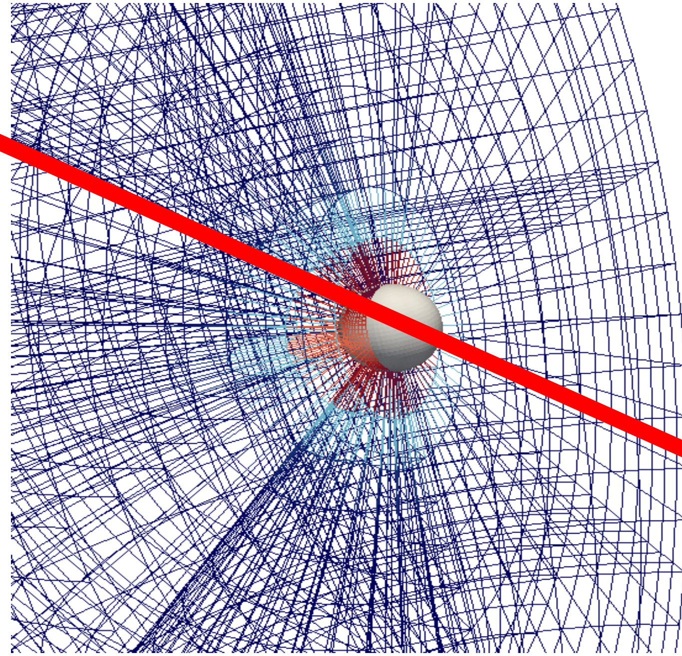
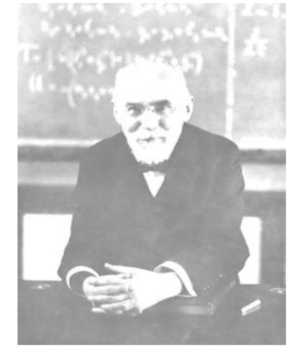
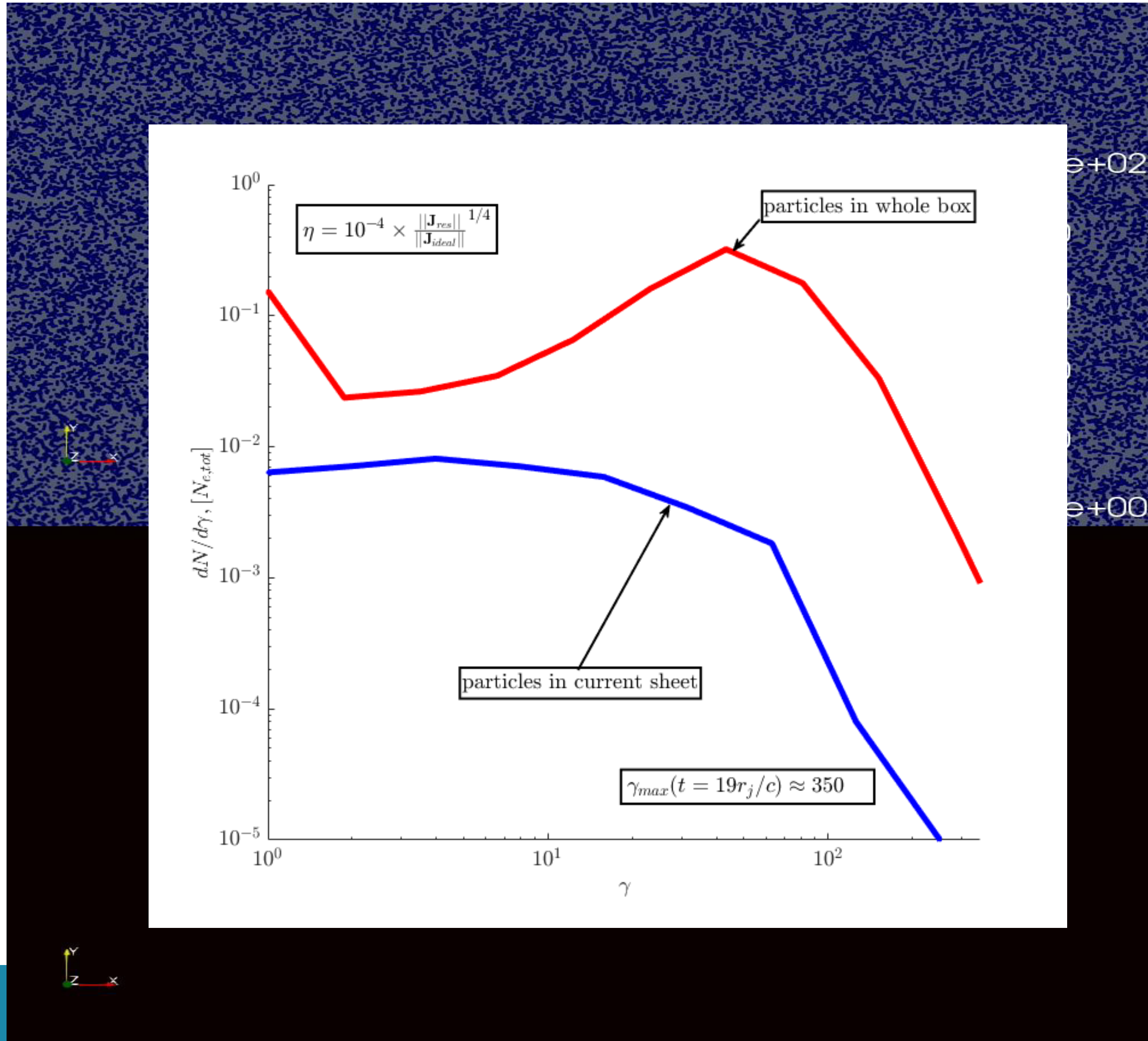




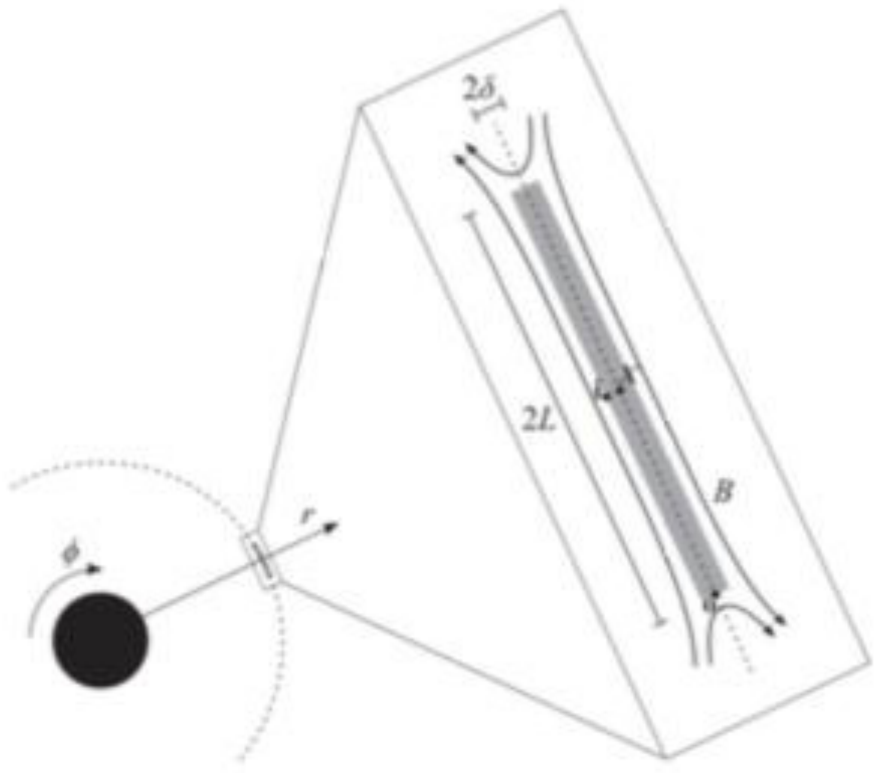
$$\frac{dx^\mu}{dt} = \frac{u^\mu}{u^0}$$
$$\frac{Du_\mu}{Dt} = F_{\mu\nu}\frac{u^\nu}{u^0}$$

- The system is not fully self-consistent, but it gives some insight into the dynamics at the particle level
- Only <u>special relativistic</u> particle movers have been thoroughly developed and studied. Geodesic motion is usually ignored!

Macro to micro:
Particle acceleration from SR reconnection



From locally flat to GRMHD



Asenjo & Comisso (PRL, 118, 055101, 2017)

- Advanced schemes including resistivity make possible to simulate MHD reconnection in curved spacetimes
- But reconnection is a microscopic process! Hence, we need particles in full GR
- A fully self-consistent description must rely on kinetic models

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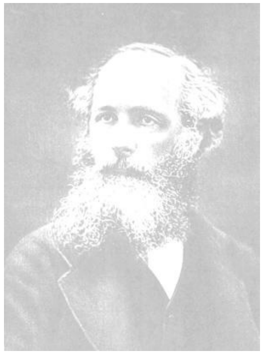
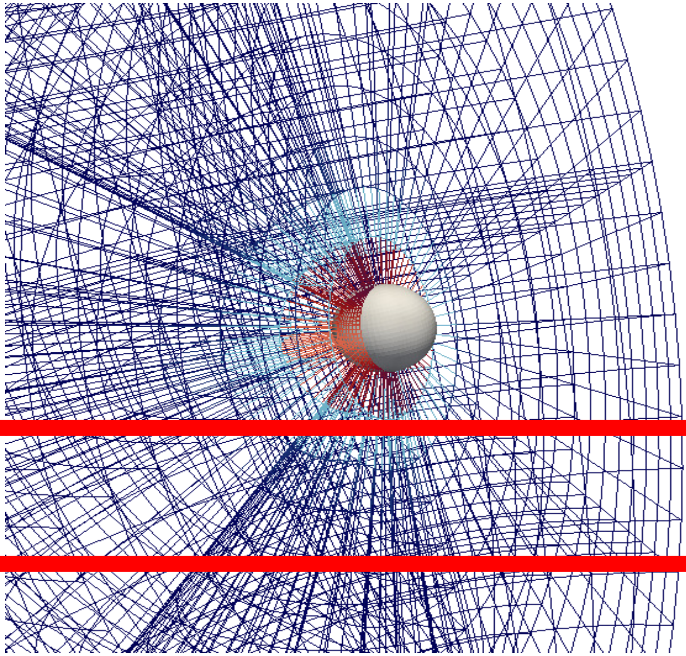
The real micro scale: GR-kinetics





$$\partial_{[\mu} F_{\mu\alpha]} = 0$$
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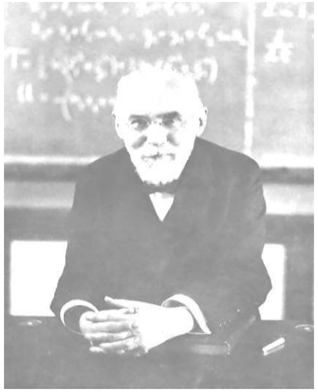
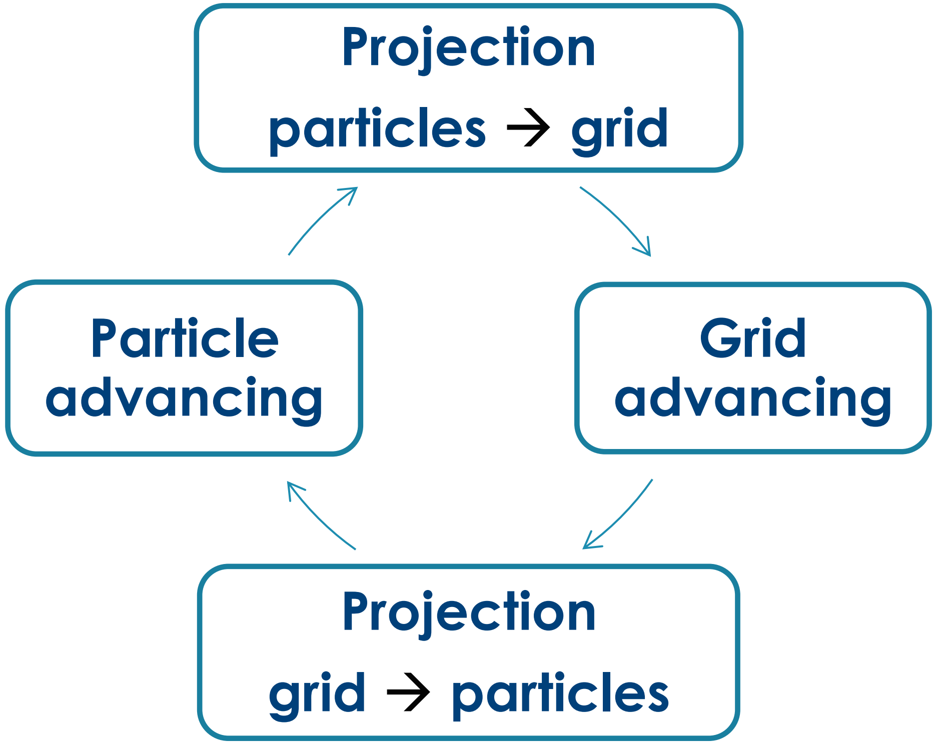
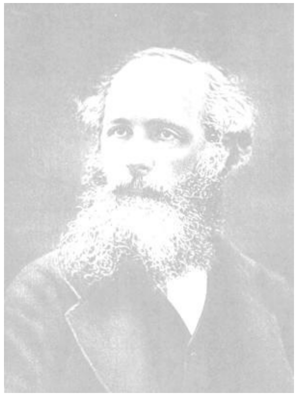
$$\frac{dx^\mu}{dt} = \frac{u^\mu}{u^0}$$
$$\frac{Du_\mu}{Dt} = F_{\mu\nu}\frac{u^\nu}{u^0}$$

- Simulating plasmas fully consistently at the particle level is physically more accurate, but also computationally more expensive
- <u>Particle-in-Cell methods</u> are designed for the task. However, there are very few and specific GR-PiC methods!

The real micro scale: Particle-in-Cell







In SR: iPiC3D, Tristan-MP, xPic (implicit). What about GR?

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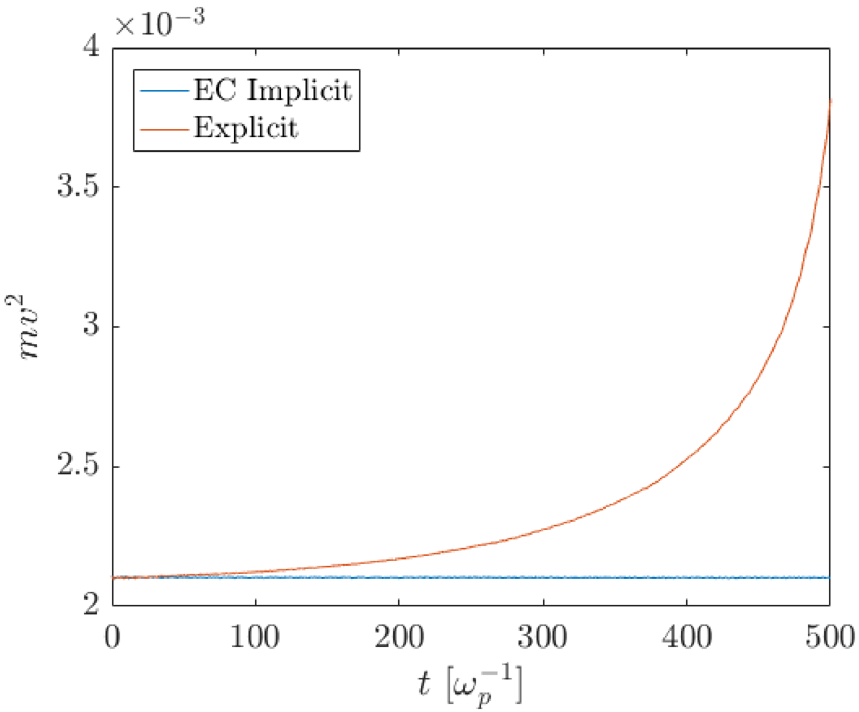
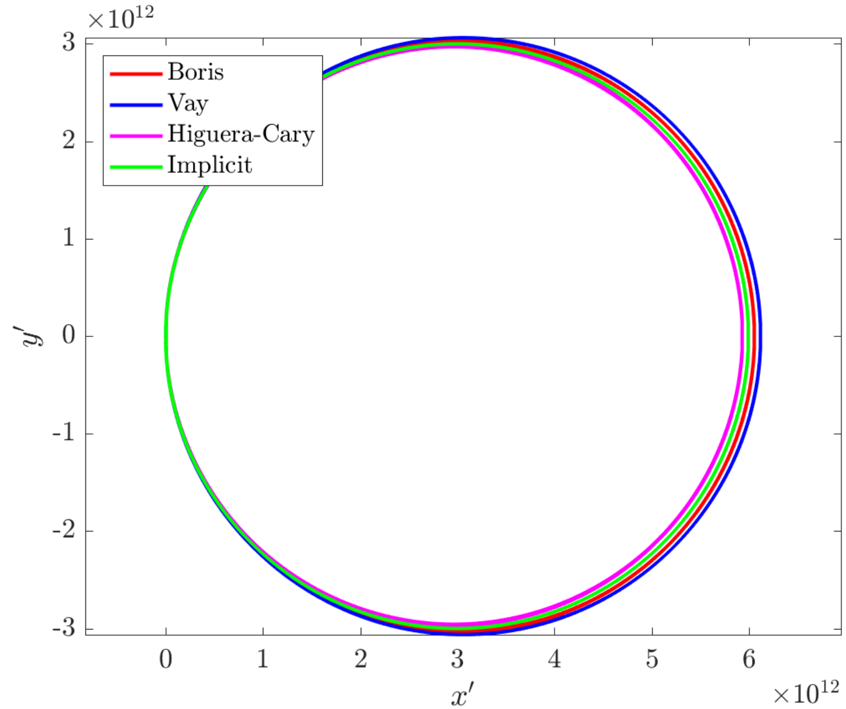
The importance of the numerical method

Energy conservation



Preservation of invariants



A comprehensive comparison of relativistic particle integrators, B. Ripperda, F. Bacchini, et al., accepted in ApJS (2018)

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Charged particle dynamics in GR

- We need to solve the full geodesic equation of motion + Lorentz force:

$$\frac{du_\mu}{d\tau} - \Gamma^{\beta}_{\mu\alpha} u_\beta u^\alpha = \frac{q}{m} F_{\mu\nu} u^\nu$$

- The method developed for the geodesic part (energy-conserving from Hamiltonian) can be easily extended to include the Lorentz force → energy-conserving charged particles movers for GR

- Energy conservation is critical for massive particles, even in SR!

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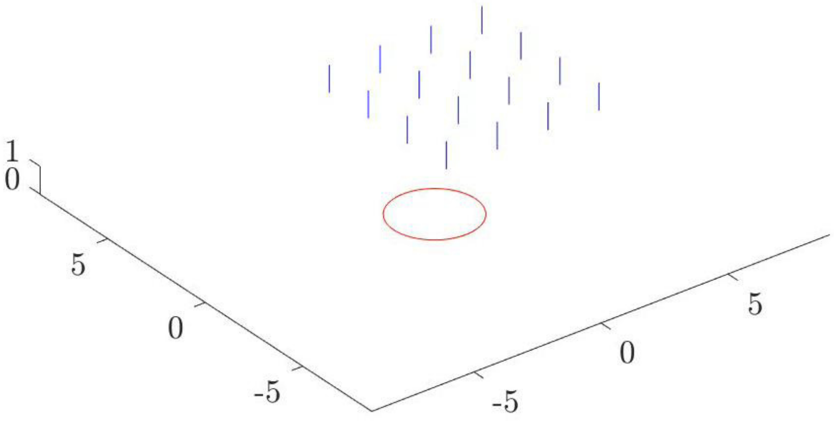
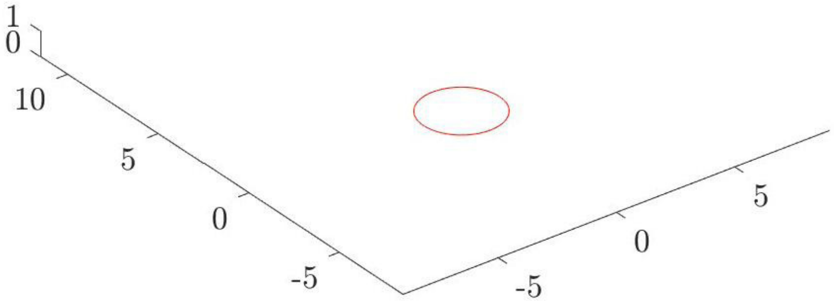
Charged particle dynamics in GR

Uniform B field (Wald solution)

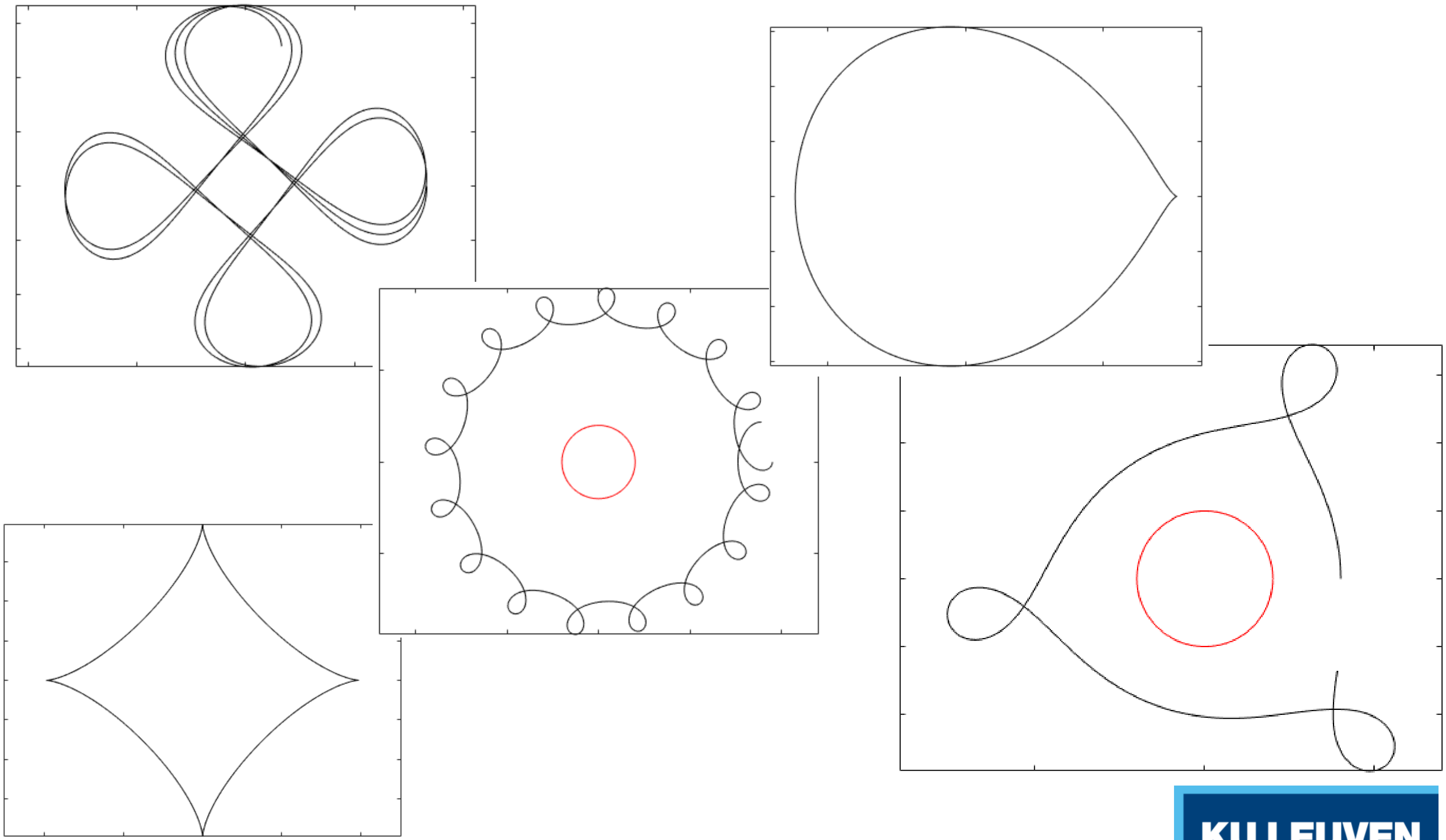


Pure geodesic motion

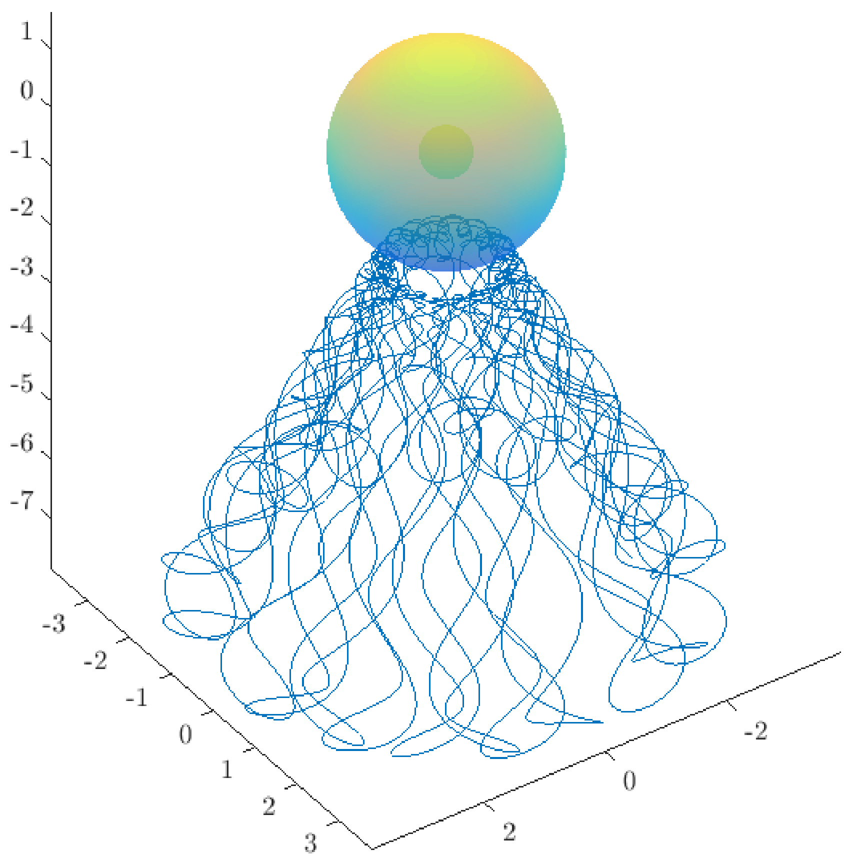


Charged particle dynamics in GR

A much richer picture in a variety of spacetimes



Towards the GR-PiC approach



- Energy-conserving movers for charged particles in GR;
- Maxwell solvers are available (already used in MHD)
- Still missing a suitable generalized infrastructure + moment gathering steps…

… but not an impossible task!

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Conclusions

- We have GRMHD simulations with resistivity
  → study accretion disks and jet launching;
- We created a versatile framework for calculating photon and massless particle geodesics
  → ray-tracing for black hole shadows
- We can easily extend to GR-Lorentz force
  → charged test particles in GRMHD
- With a "little" extra effort
  → GR-PiC codes for microscopic processes

Bonus features:

- Can handle any 3+1 split metric, only requires the spacetime functions
- Includes a new, <u>exactly energy-conserving</u> numerical scheme based on the Hamiltonian formulation

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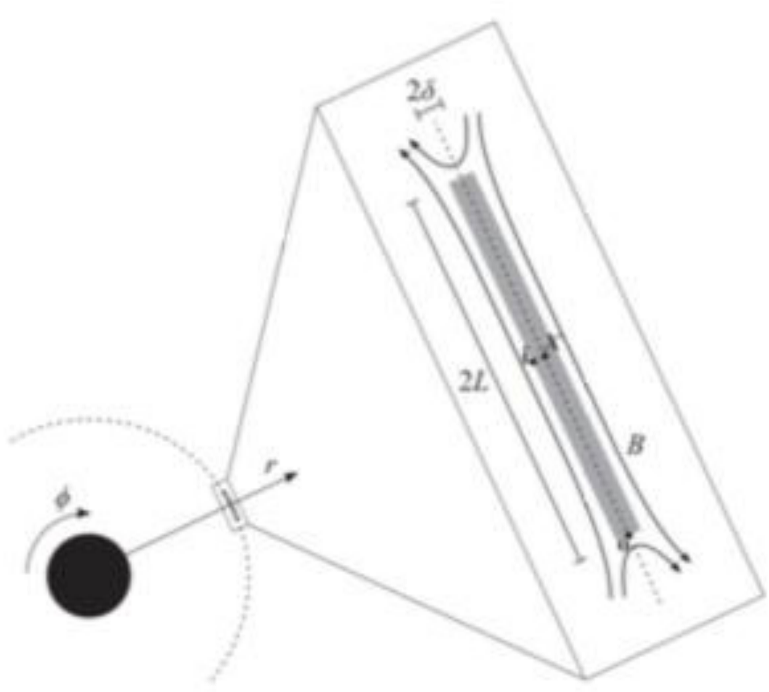
Next steps?



- Wrap up studies on GR-Lorentz particle movers
- Test charged particles in GRMHD to study particle distributions in reconnection
- Full GR-PiC (some time in the future)

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