Spinning black holes fall in Love

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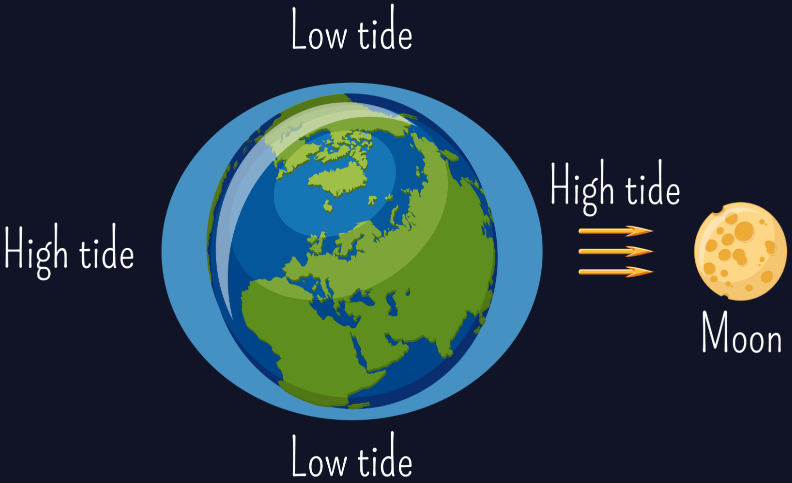
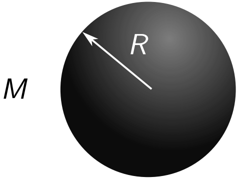
TIDES



Newtonian theory of Love numbers



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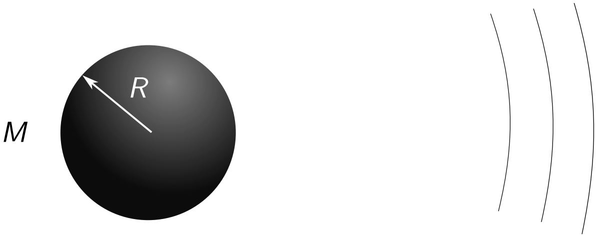

$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(0)$$

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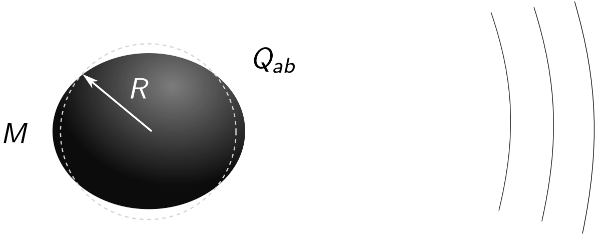

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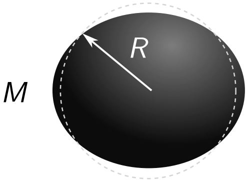

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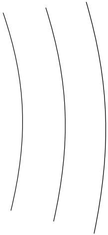

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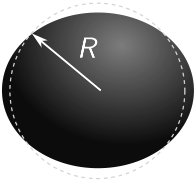

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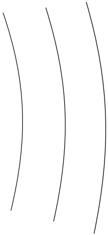

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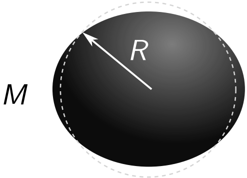

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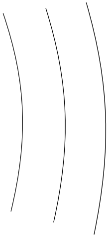

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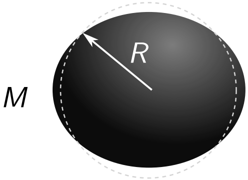

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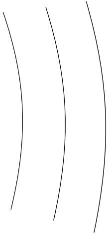

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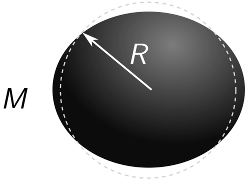

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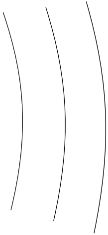

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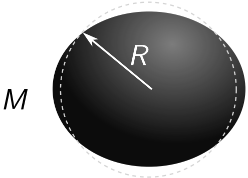

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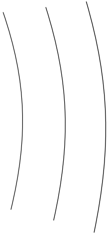

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$S = \chi M^2$

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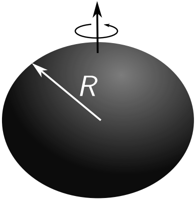

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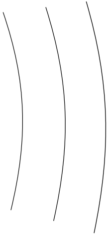

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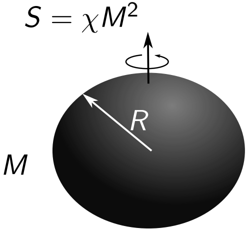

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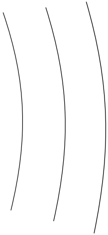

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$$k_{\ell m} = k_\ell^{(0)} + i m \chi\, k_\ell^{(1)} + O(\chi^2)$$

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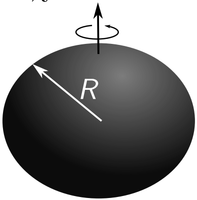

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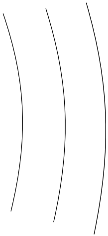

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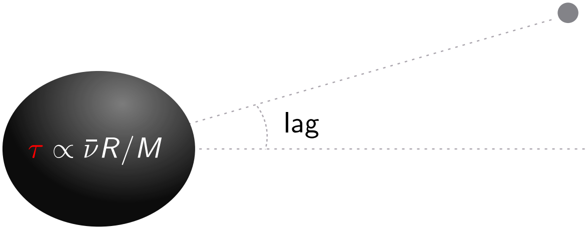
Tidal Love numbers $k_{\ell m}$ $\longleftrightarrow$ **body's internal structure**

Tidal dissipation: lag, heating and torquing



$$Q_{ab}(t) = -\frac{2}{3} k_2 R^5 \left[\mathcal{E}_{ab}(t) - \tau \dot{\mathcal{E}}_{ab}(t) + \cdots \right]$$
$$= -\frac{2}{3} k_2 R^5 \left[\mathcal{E}_{ab}(t - \tau) + \cdots \right]$$

Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_{a_1\cdots a_\ell} \propto \left[C_{0a_10a_2;a_3\cdots a_\ell}\right]^{\mathsf{STF}}, \quad \mathcal{B}_{a_1\cdots a_\ell} \propto \left[\varepsilon_{a_1bc} C_{a_20bc;a_3\cdots a_\ell}\right]^{\mathsf{STF}}$$

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- Metric and Geroch-Hansen multipole moments:

$$g_{\alpha\beta} = \mathring{g}_{\alpha\beta} + \underbrace{h^{\mathsf{tidal}}_{\alpha\beta}}_{\sim r^\ell} + \underbrace{h^{\mathsf{resp}}_{\alpha\beta}}_{\sim r^{-(\ell+1)}} \quad \longrightarrow \quad \begin{cases} M_{\ell m} = \mathring{M}_{\ell m} + \delta M_{\ell m} \\ S_{\ell m} = \mathring{S}_{\ell m} + \delta S_{\ell m} \end{cases}$$

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- *Four* families of tidal deformability parameters:

$$\lambda^{M\mathcal{E}}_{\ell m} \equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell m}} \qquad \lambda^{S\mathcal{B}}_{\ell m} \equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell m}}$$

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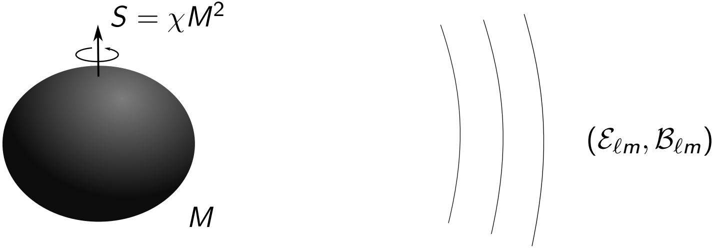
Investigating Kerr's Love



$$(\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}) \to \psi_0 \to \Psi \to h_{\alpha\beta} \to (M_{\ell m}, S_{\ell m}) \to \lambda_{\ell m}^{M/S,\mathcal{E}/\mathcal{B}}$$

Metric reconstruction through the Hertz potential Ψ

Love numbers of a Kerr black hole

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- The associated dimensionless tidal Love numbers are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} \doteq -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} \doteq 0$$

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- For a dimensionless black hole spin $\chi = 0.1$ this gives

$$|k_{2,\pm 2}| \simeq 2 \times 10^{-3} \quad \longrightarrow \quad \text{black holes are } \textbf{“rigid”}$$

Love tensor of a Kerr black hole

- For a nonspinning compact body we have the proportionality relations

$$\delta M_{ab} = \lambda_2^{\text{el}} \mathcal{E}_{ab} \quad \text{and} \quad \delta S_{ab} = \lambda_2^{\text{mag}} \mathcal{B}_{ab}$$

- For a spinning black hole we have the more general tensorial relations

$$\delta M_{ab} = \lambda_{abcd} \mathcal{E}_{cd} \quad \text{and} \quad \delta S_{ab} = \lambda_{abcd} \mathcal{B}_{cd}$$

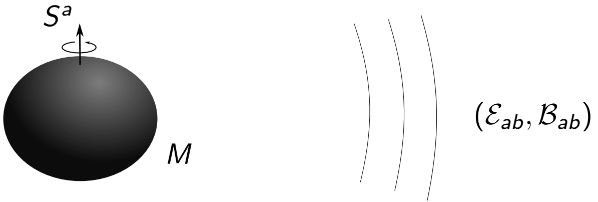
- To linear order in the black hole spin vector S^a we find

$$\delta M_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{E}^d{}_{(a} \varepsilon_{b)cd}$$

$$\delta S_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{B}^d{}_{(a} \varepsilon_{b)cd}$$

Tidal torquing of a spinning black hole

[Thorne & Hartle 1980; Poisson 2004]



- An arbitrary spinning body interacting with a tidal environment suffers a tidal torquing:

$$\langle \dot{S}^a \rangle = -\varepsilon^{abc} \langle M_{bd} \mathcal{E}^d{}_c + S_{bd} \mathcal{B}^d{}_c \rangle$$

- Applied to a spinning black hole this yields

$$\langle \dot{S} \rangle \doteq -\frac{8}{45} M^5 \chi \left[2\langle \mathcal{E}^{ab} \mathcal{E}_{ab} \rangle - 3 \langle \mathcal{E}_{ab} s^b \mathcal{E}^{ac} s_c \rangle + (\mathcal{E} \to \mathcal{B}) \right]$$

Summary

- Love numbers of Kerr black holes do not vanish in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- Kerr black holes deform like any other self-gravitating body, despite being particularly “rigid” compact objects
- This is closely related to the phenomenon of tidal torquing
- New black hole test of the Kerr-like nature of the massive compact objects at the center of galaxies?

Spinning black holes fall in Love!