Halo Sparsity

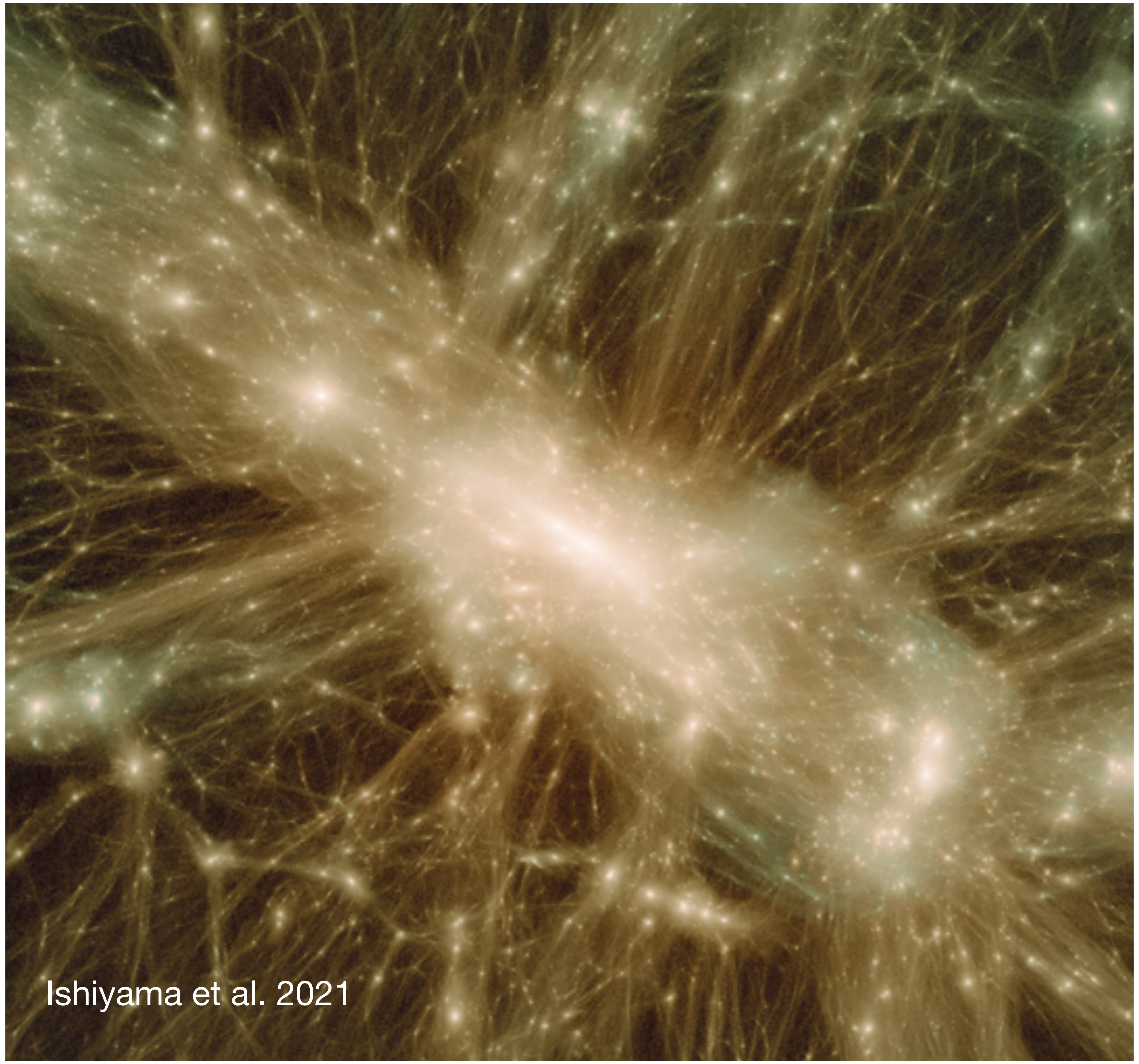
A Swiss army knife for galaxy cluster astrophysics and cosmology

Tamara R. G. Richardson
LUTH day 2022

Outline

- What is Sparsity?
- Cluster Mergers
- The Halo Mass Function
- Cosmology

What are haloes?



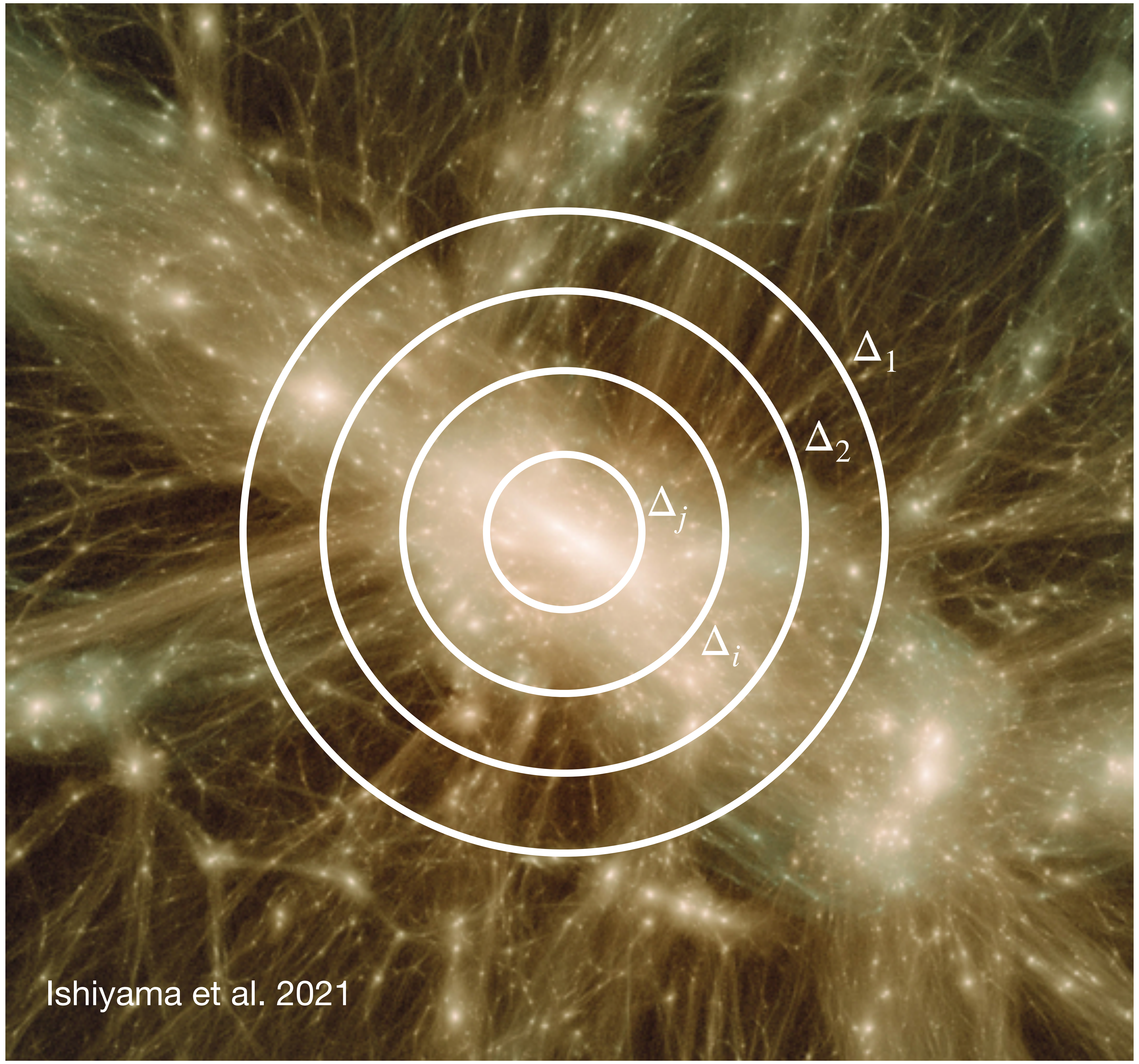
Product of **cosmological** structure formation.

Hosts of galaxies, galaxy clusters and their **astrophysical** processes.

Imprint on the internal structure of haloes?

What can it tell us about the Universe?

What is Halo Sparsity?

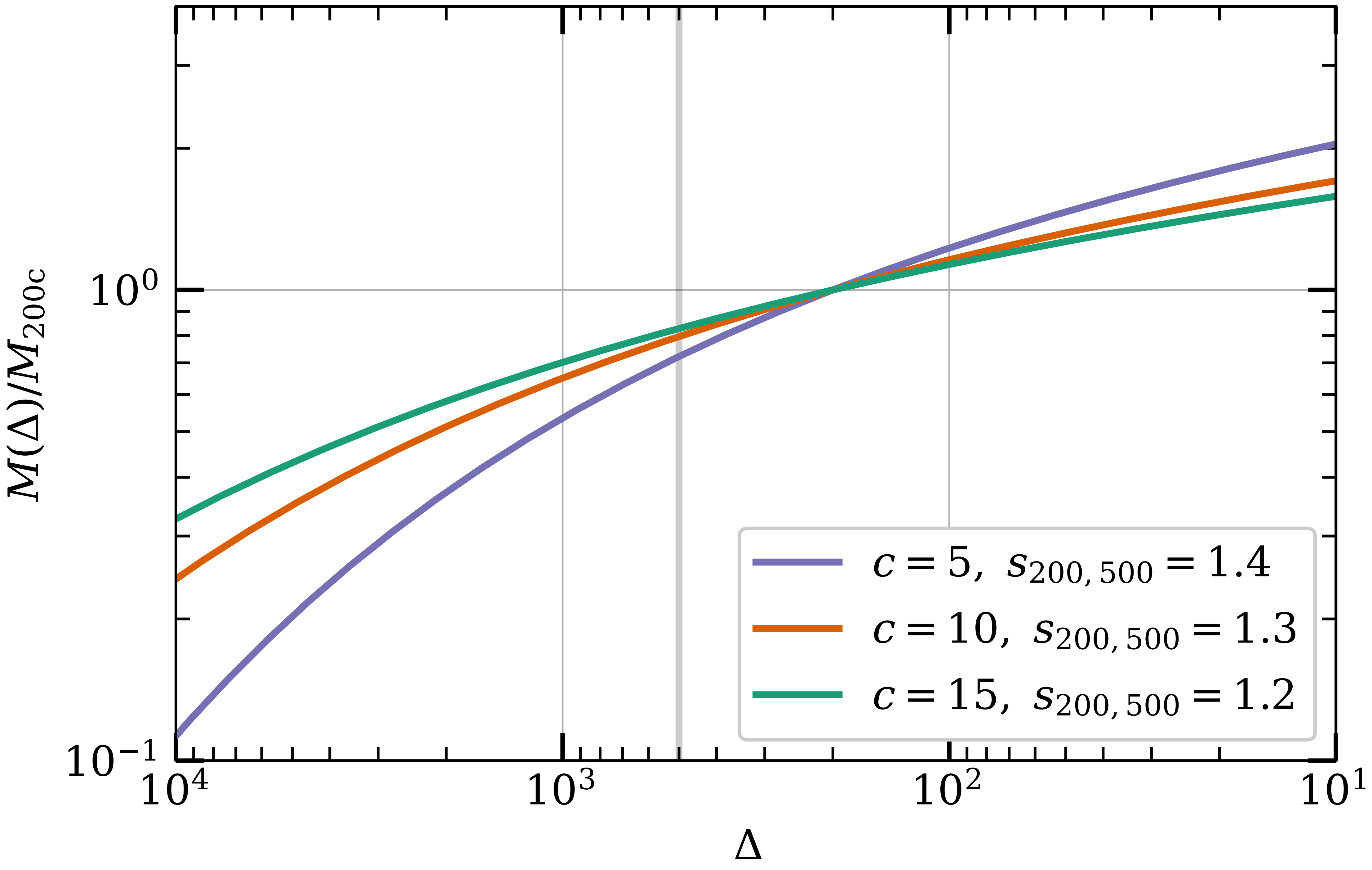


$$s_{\Delta_1,\Delta_2} = \frac{M_{\Delta_1}}{M_{\Delta_2}}$$

Non - Parametric

Astrophysics

Cosmology



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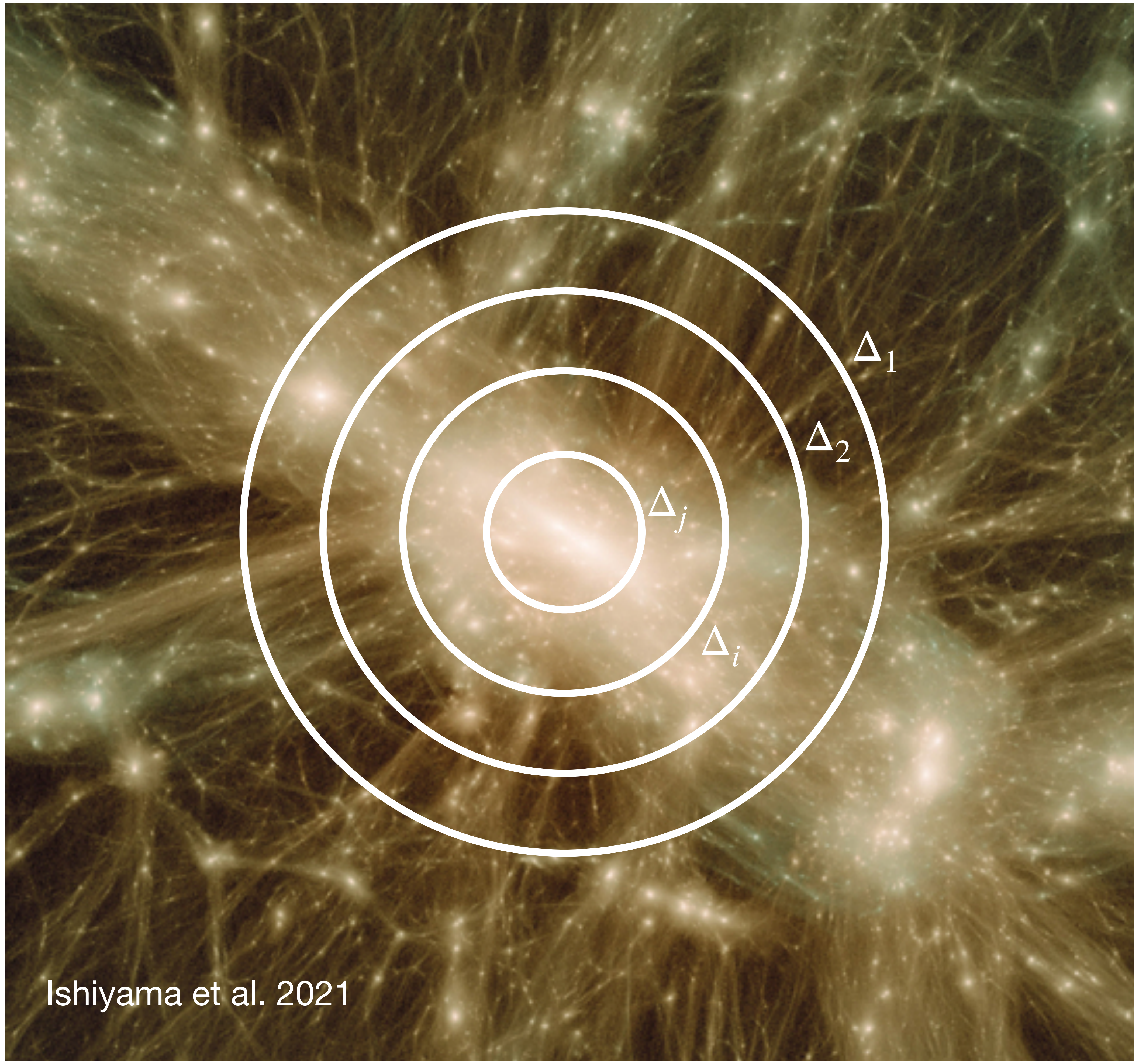

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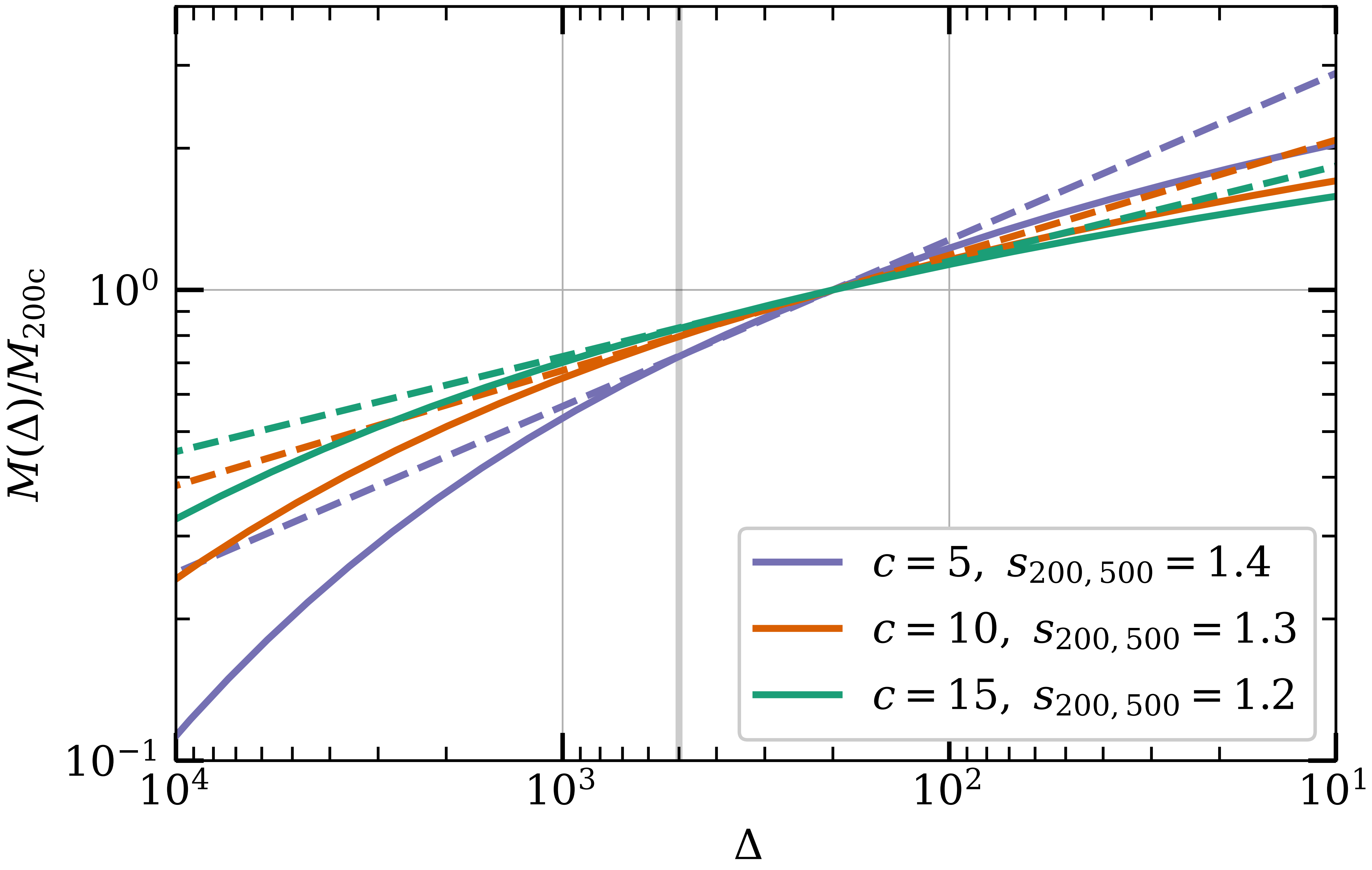

Major Mergers

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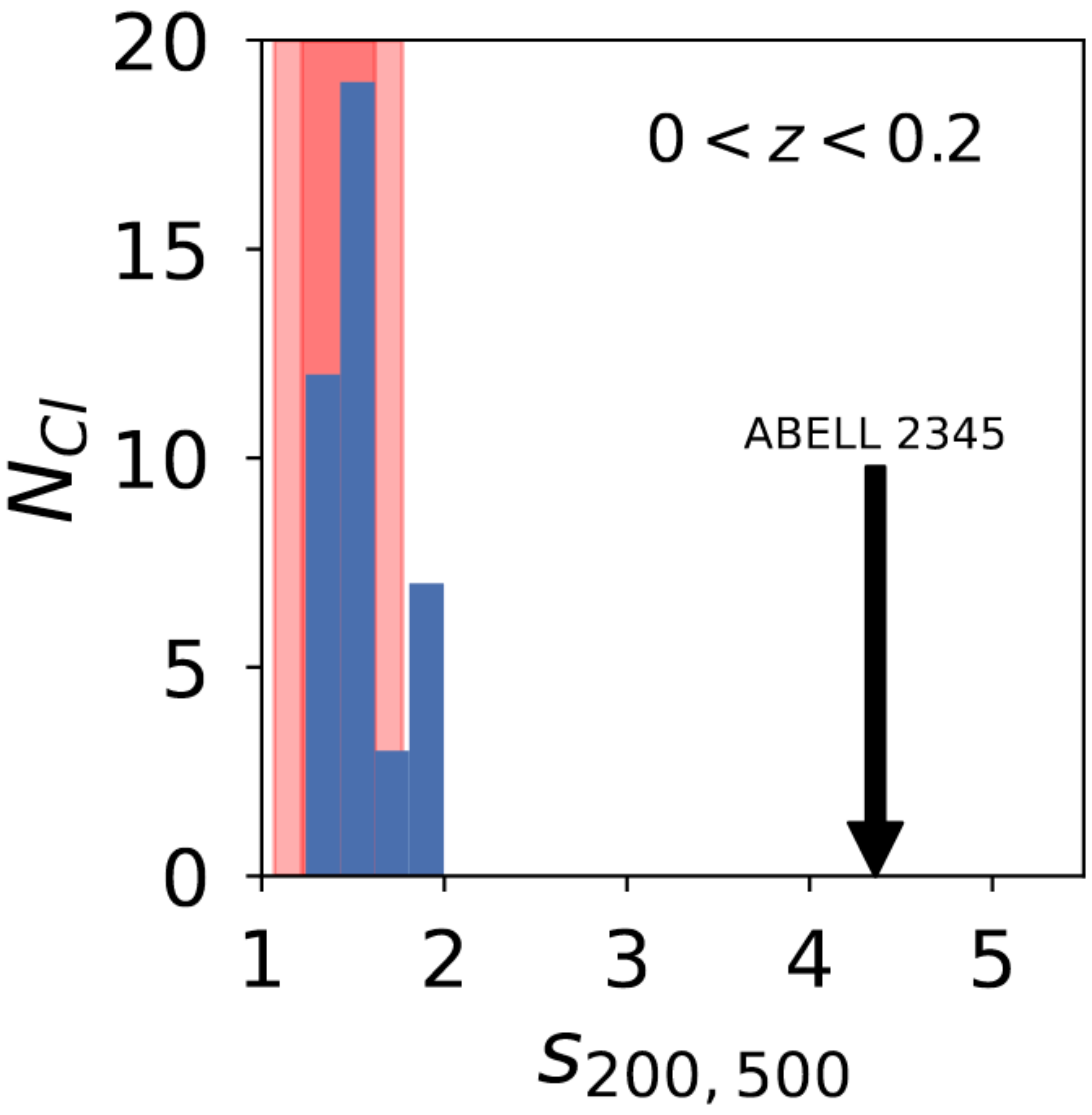
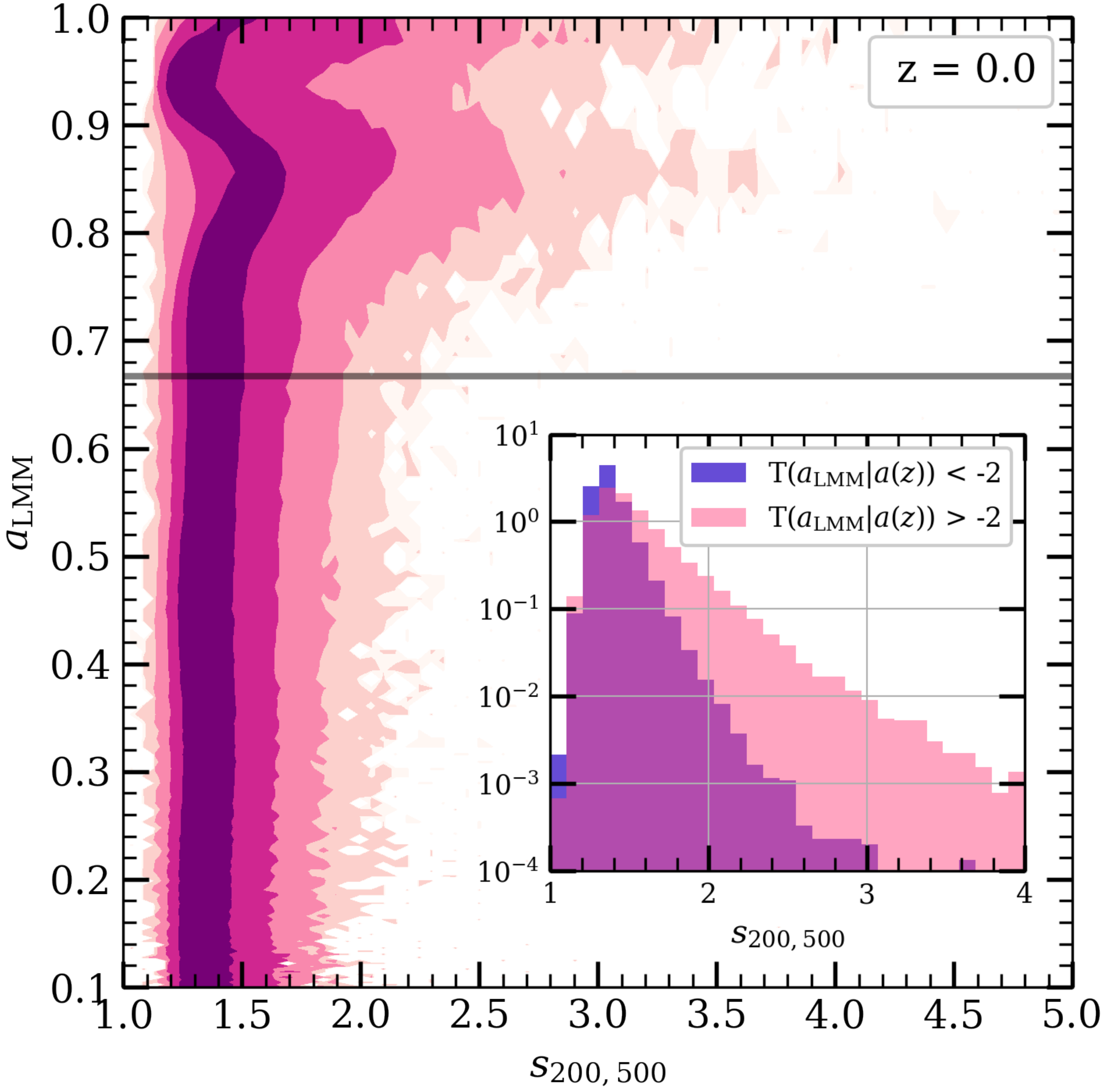


Corasaniti et al. 2021: Are clusters with **large s_{Δ_1,Δ_2} merging or unrelaxed?**

Richardson & Corasaniti 2022: **Yes.**
Sparsity reacts systematically to mergers

Major Mergers



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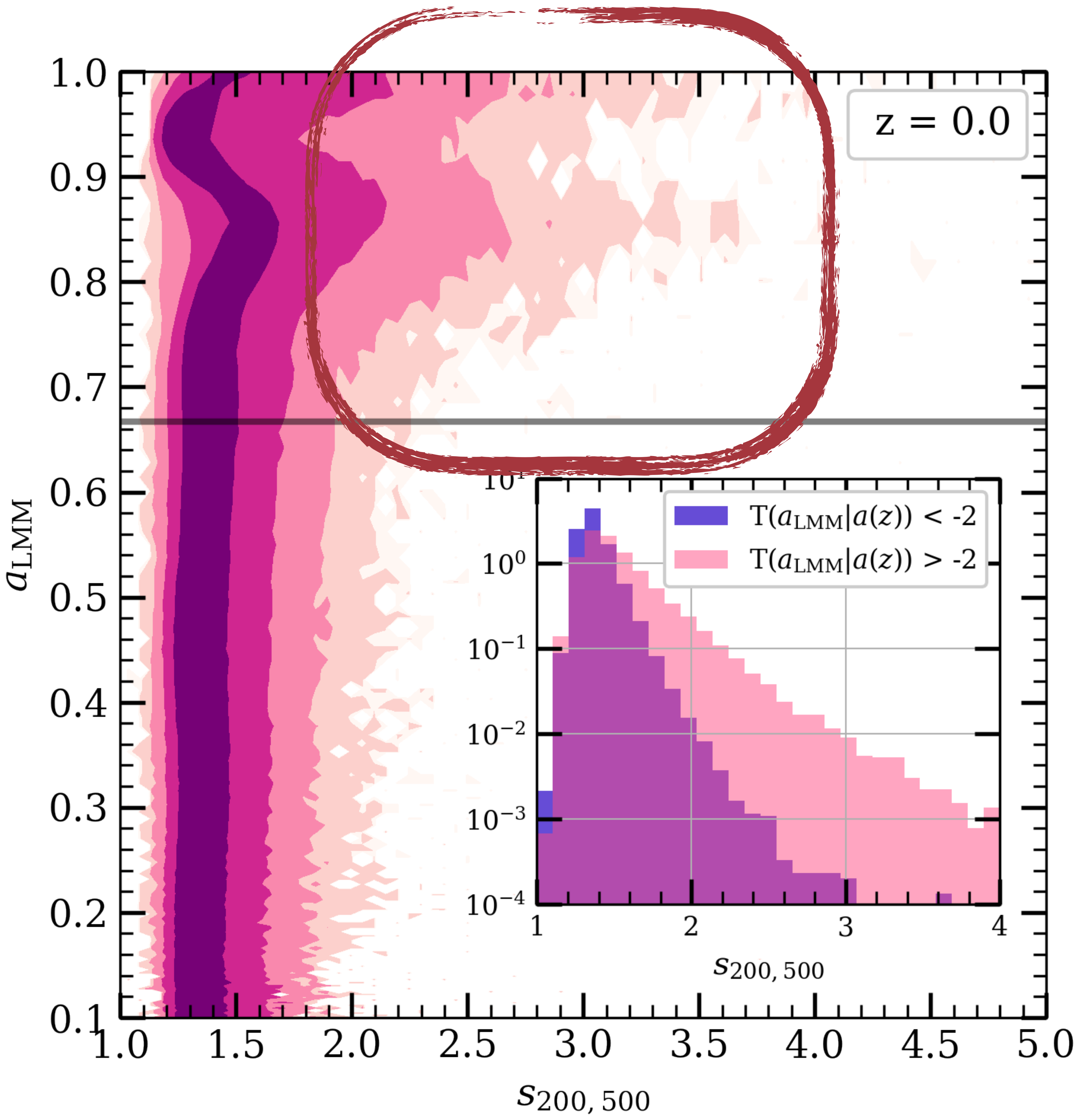

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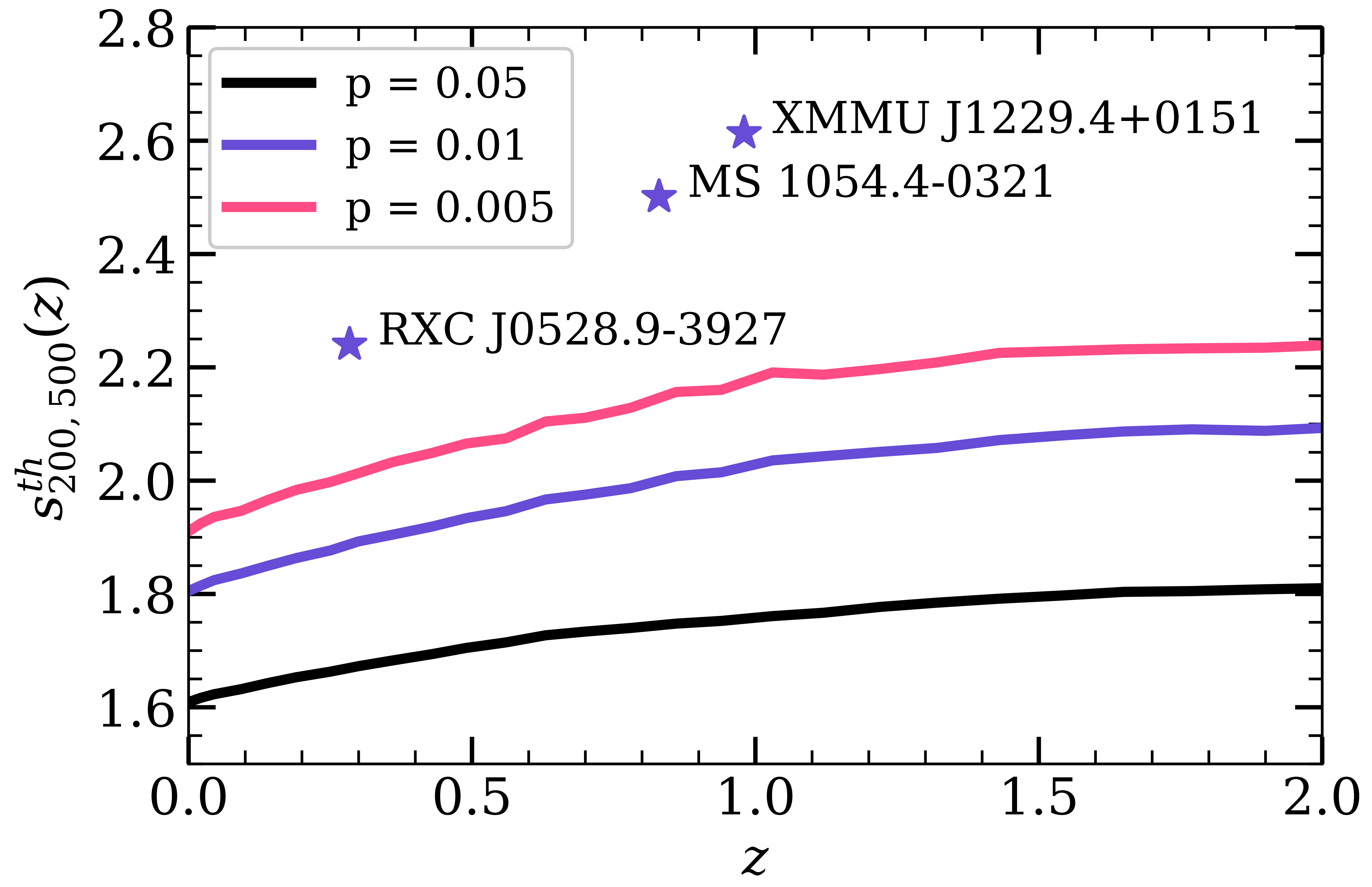
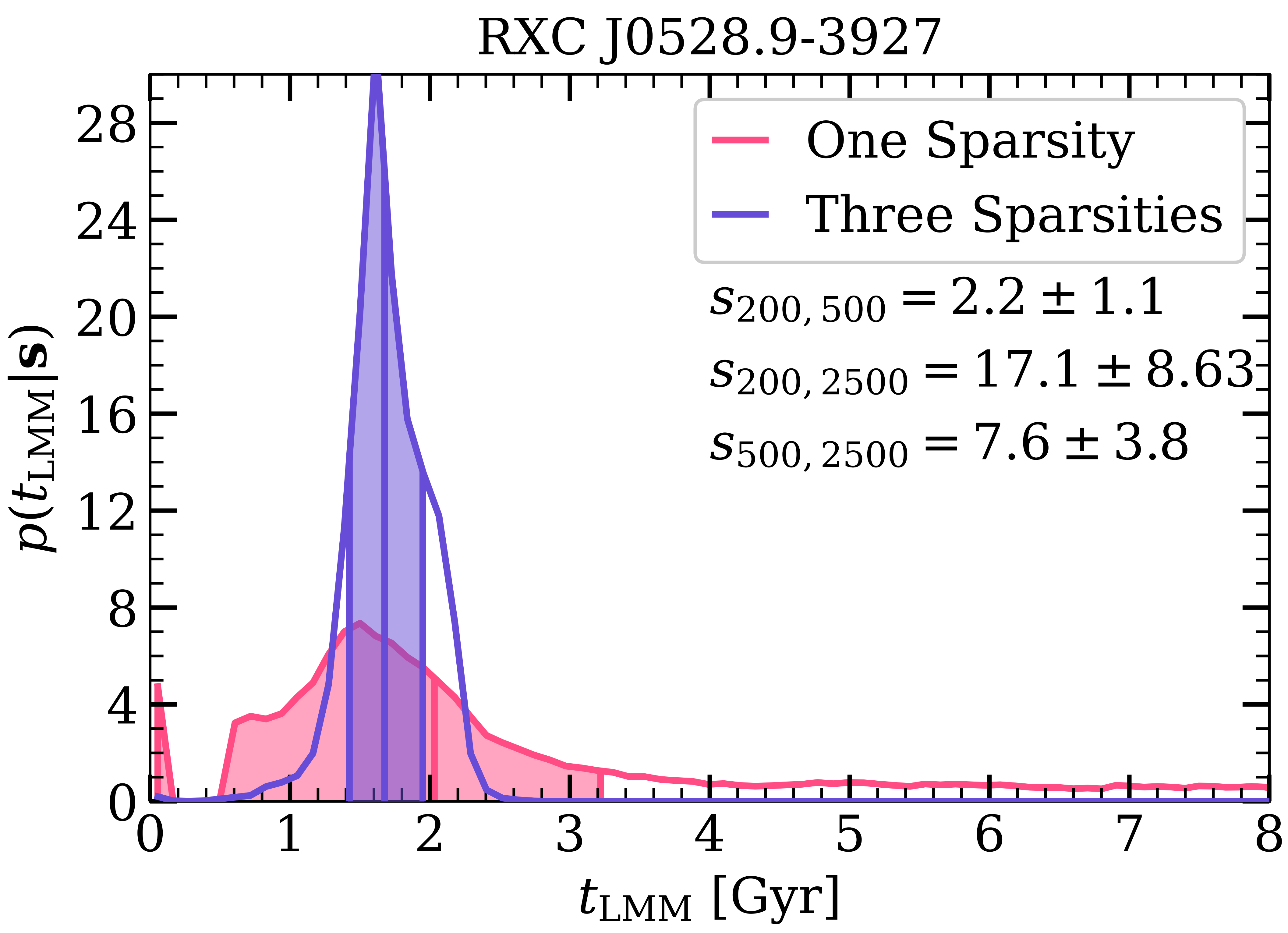
Large sparsity is very likely to be a merger

Detection with sparsity



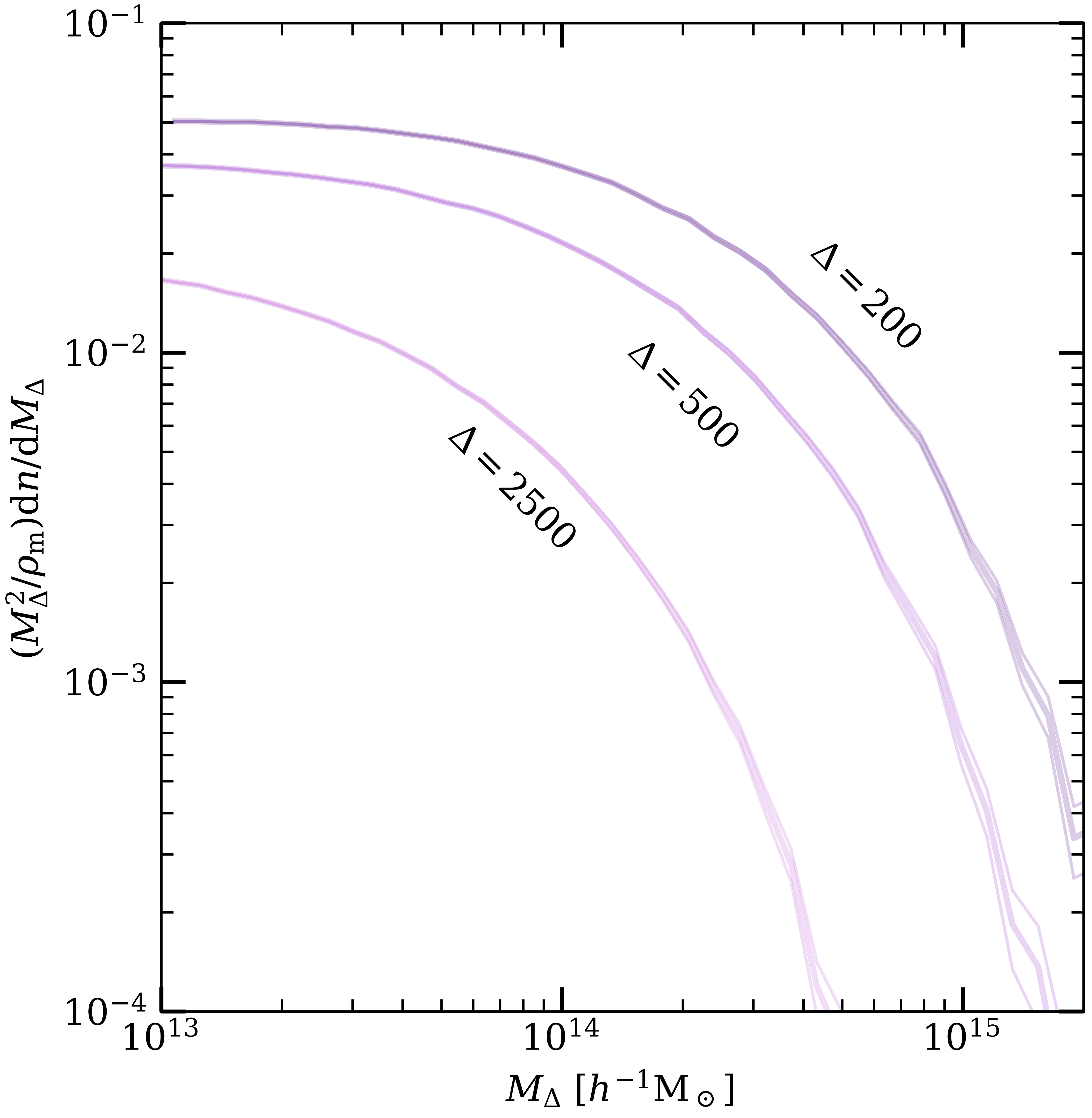


We can use the probability distribution from the simulation as a likelihood to set **thresholds** and even get a first **estimate of the merger time**.

Not super precise but can be used to **quickly** screen catalogues

The Halo Mass Function

The Halo Mass Function



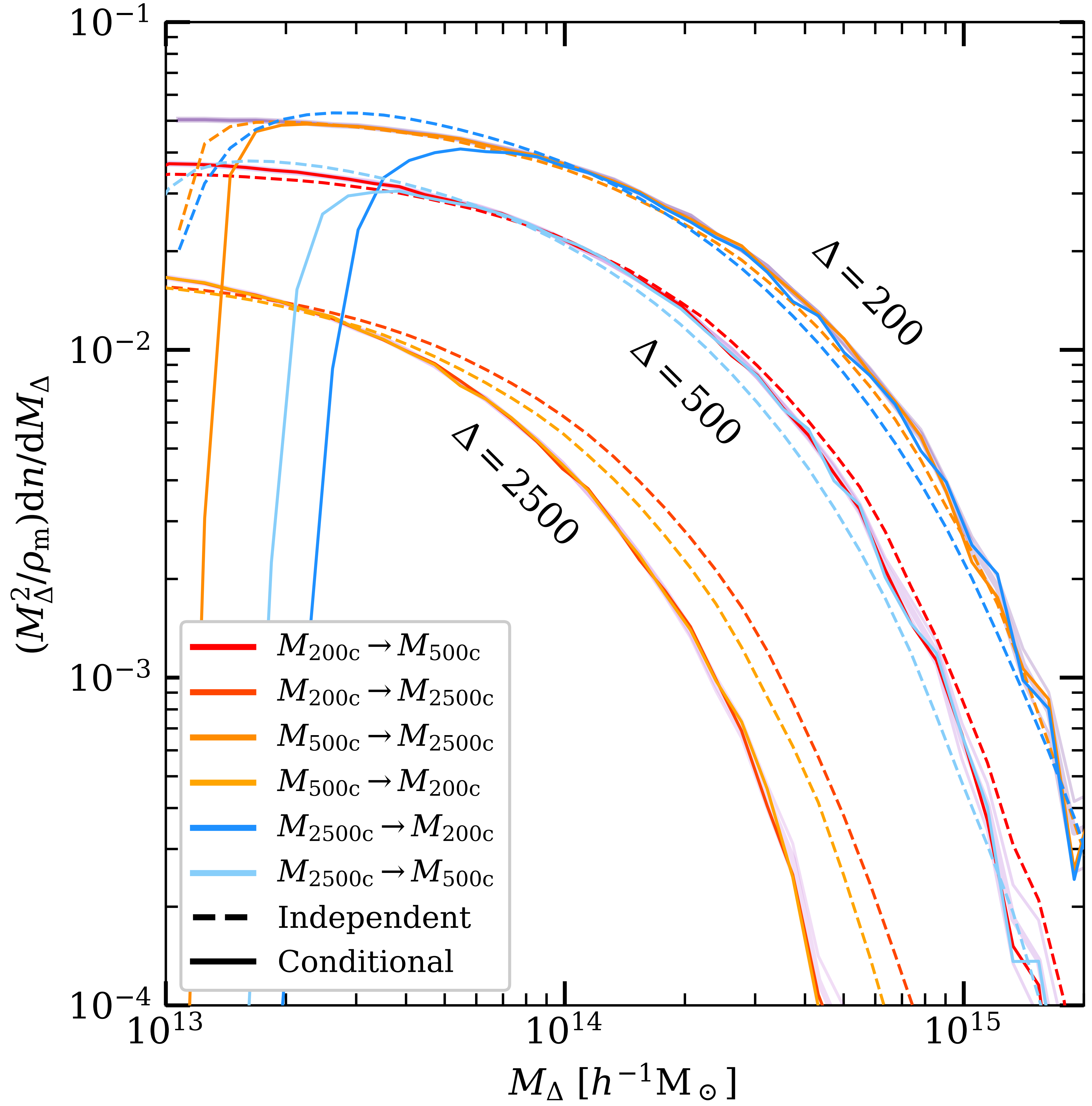
The HMF: How many haloes are expected per unit mass per unit volume.

Alternate view: The probability of finding a halo of mass $M_\Delta \in [M, M + \mathrm{d}M]$ in a volume $\mathrm{d}V$.

M_Δ is a random variable drawn from $\dfrac{\mathrm{d}n}{\mathrm{d}M_\Delta}$

$s_{\Delta_1,\Delta_2} = \dfrac{M_{\Delta_1}}{M_{\Delta_2}}$ relates **3 random variables**

Sparsity Transformations



We can relate their distribution functions:

$$M_{\Delta_1} \sim \frac{dn}{dM_{\Delta_1}}, \quad s \sim \rho_s(s\,|\,M_\Delta) \quad \text{and} \quad M_{\Delta_2} \sim \frac{dn}{dM_{\Delta_2}}$$

"Outward" $M_{\Delta_1} = s_{\Delta_1,\Delta_2} M_{\Delta_2}$

$$\frac{dn}{dM_{\Delta_1}}(M_{\Delta_1}) = \int_1^\infty \frac{1}{x} \rho_s(x\,|\,M_{\Delta_1}/x) \frac{dn}{dM_{\Delta_2}}(M_{\Delta_1}/x) dx$$

"Inward" $M_{\Delta_2} = M_{\Delta_1}/s_{\Delta_1,\Delta_2}$

$$\frac{dn}{dM_{\Delta_2}}(M_{\Delta_2}) = \int_1^\infty x\,\rho_s(x\,|\,xM_{\Delta_2}) \frac{dn}{dM_{\Delta_1}}(xM_{\Delta_2})\, dx$$

Recovering the literature

By assuming: $\rho(s_{\Delta_1,\Delta_2} | M_{\Delta_1}) = \delta_{\mathrm{D}}(s_{\Delta_1,\Delta_2} - \langle s_{\Delta_1,\Delta_2}\rangle)$

$$\frac{\mathrm{d}n}{\mathrm{d}M_{\Delta_2}} = \left[\frac{\bar{\rho}_m}{M_{\Delta_2}}\frac{\mathrm{d}\ln\sigma^{-1}}{\mathrm{d}M_{\Delta_2}}f(\sigma)\right]\frac{1}{\langle s_{\Delta_1,\Delta_2}\rangle}$$

Richardson & Corasaniti *(on arXiv today!)*

$$\frac{\mathrm{d}n}{\mathrm{d}M_{\Delta_2}} = \left[\frac{\bar{\rho}_m}{M_{\Delta_2}}\frac{\mathrm{d}\ln\sigma^{-1}}{\mathrm{d}M_{\Delta_2}}f(\sigma)\right]\frac{M_{\Delta_2}}{M_{\Delta_1}}$$

Bocquet et al. 2016
Ragagnin et al. 2021

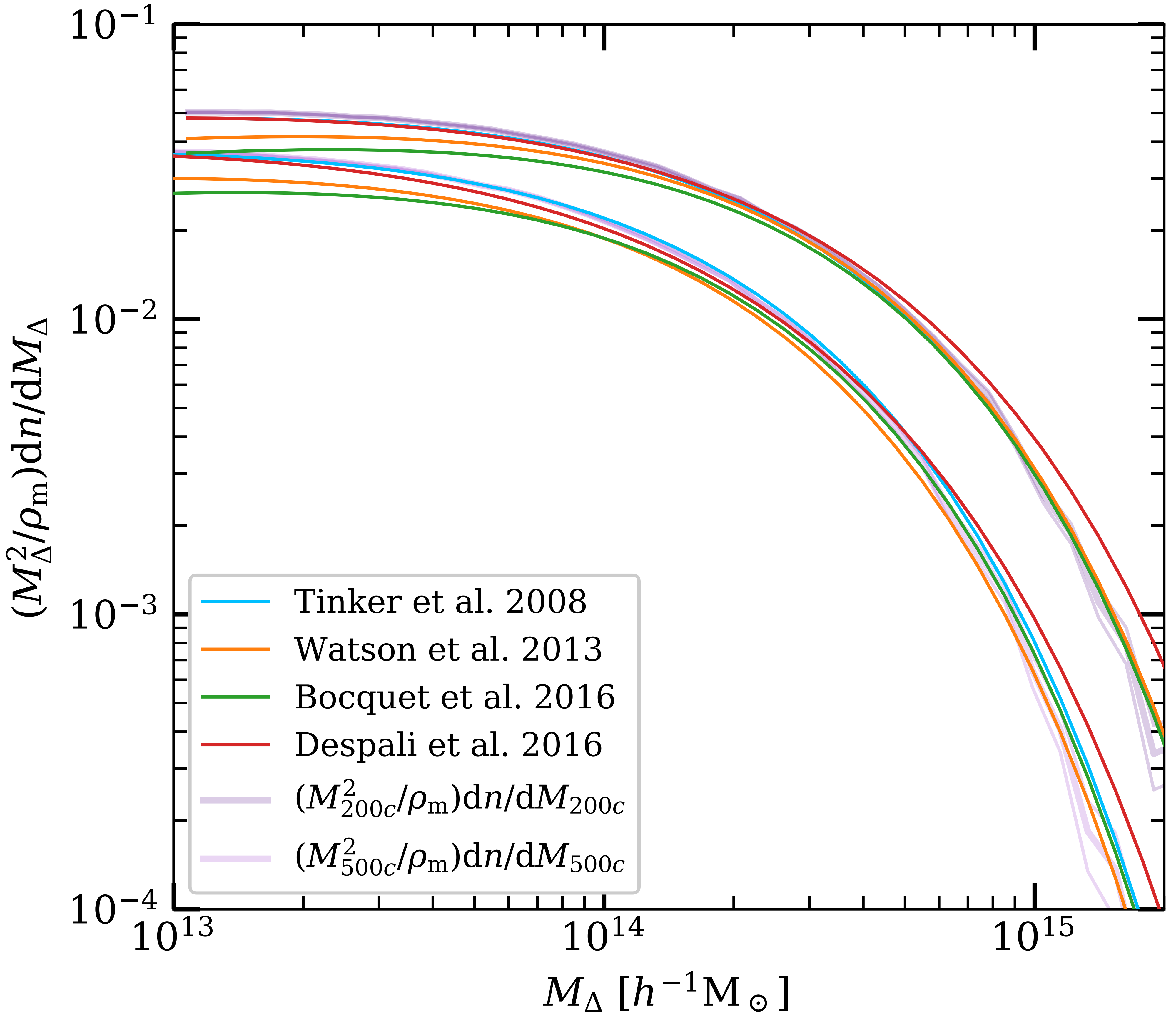
A second result from Balmès et al. 2014 can be recovered by integrating over M_{Δ_2} with the same assumption.

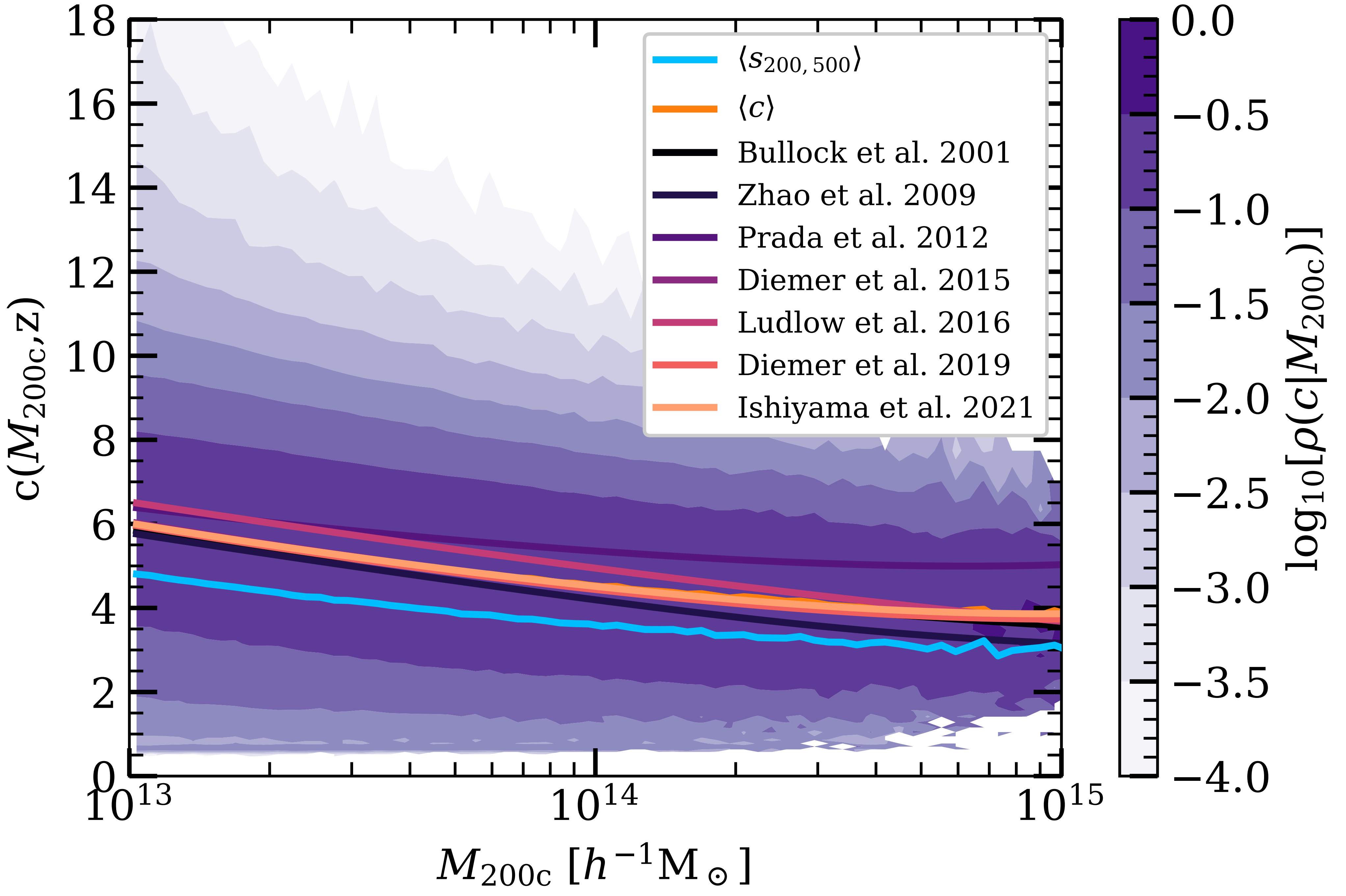
$$\int \frac{\mathrm{d}n}{\mathrm{d}M_{\Delta_2}}\mathrm{d}\ln M_{\Delta_2} = \langle s_{\Delta_1,\Delta_2}\rangle \int \frac{\mathrm{d}n}{\mathrm{d}M_{\Delta_1}}\mathrm{d}\ln M_{\Delta_1}$$

Foundational equation for sparsity constraints

Corasaniti et al. 2018, 2021, 2022

Connecting HMF and $c(M, z)$

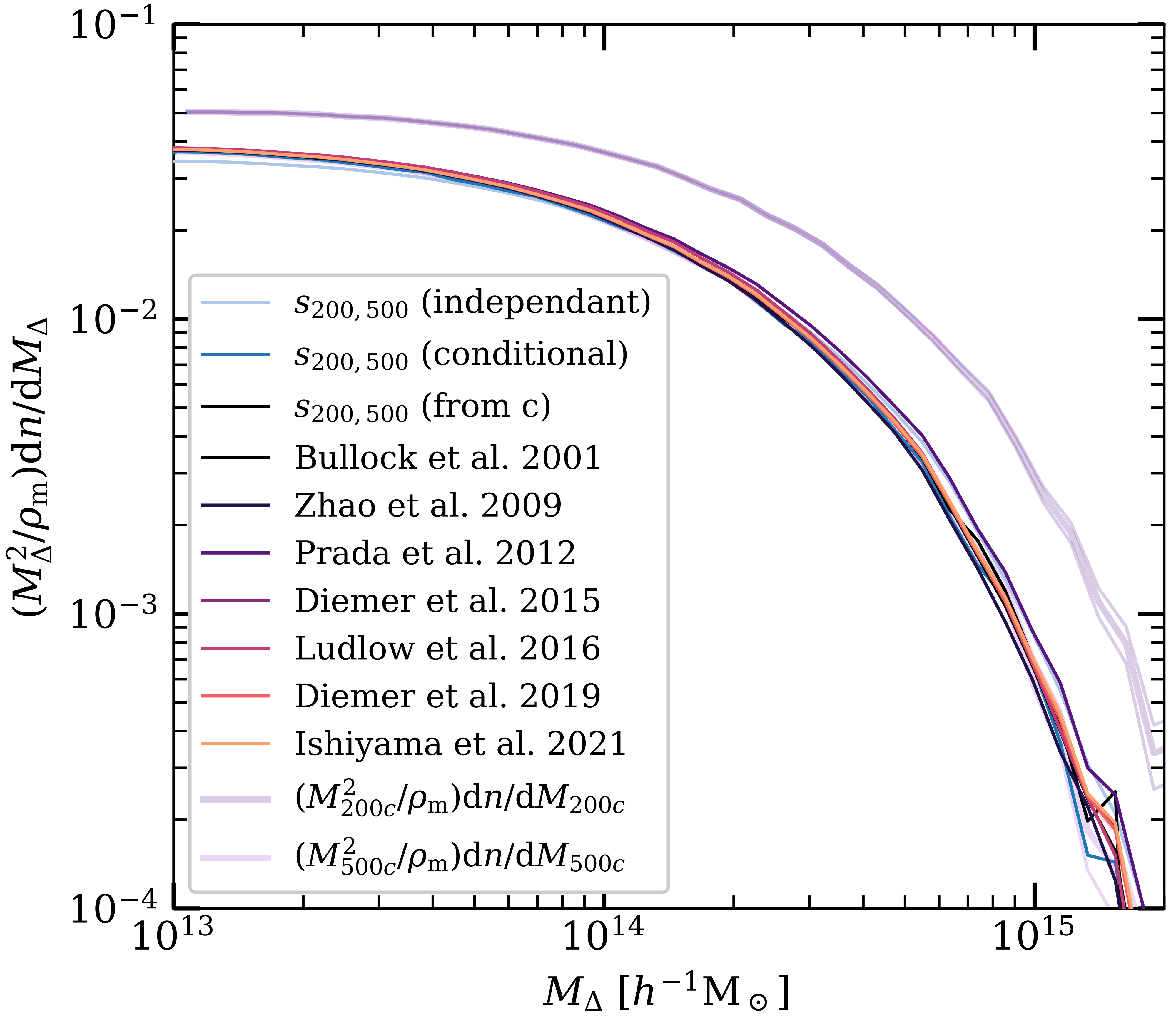


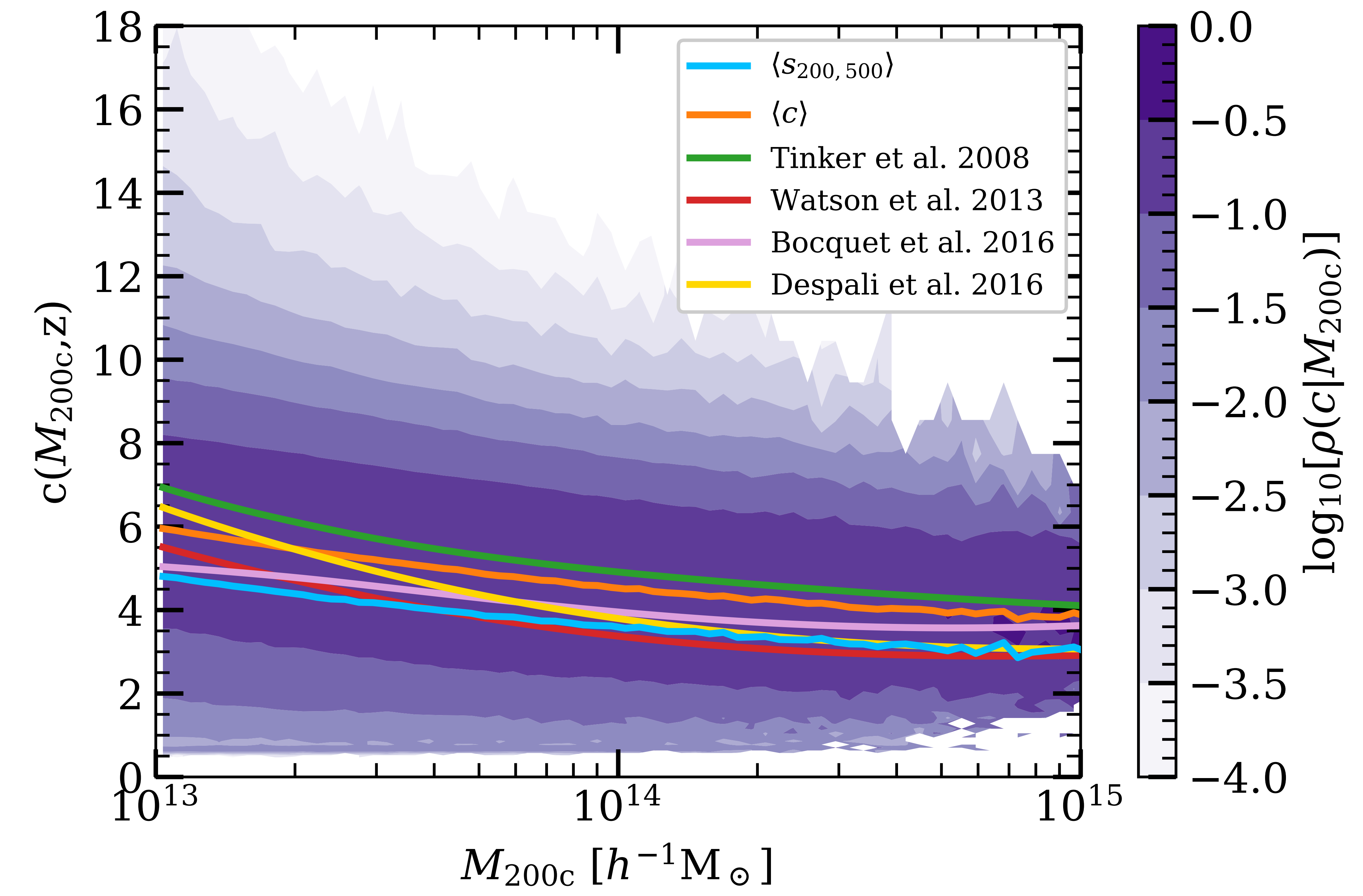


By transforming the NFW $c(M, z)$ relation into a $s_{\Delta_1,\Delta_2}(M, z)$ we can **transform the HMF**.

Inversely we can use the HMF defined for Δ_1 and Δ_2 to **predict** the $c(M, z)$

Connecting HMF and $c(M, z)$





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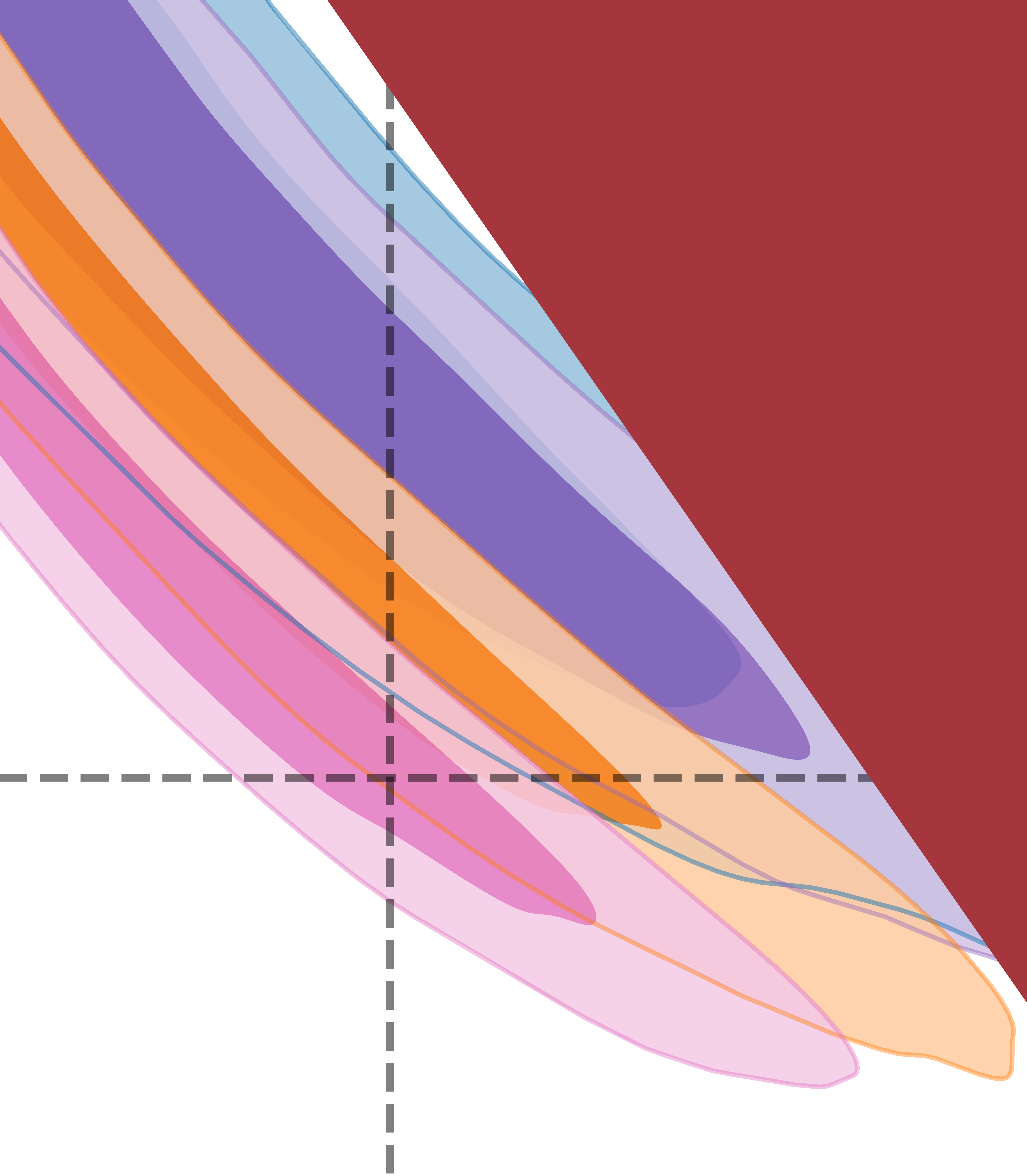


Cosmology with Sparsity

Good old fashion Cosmology

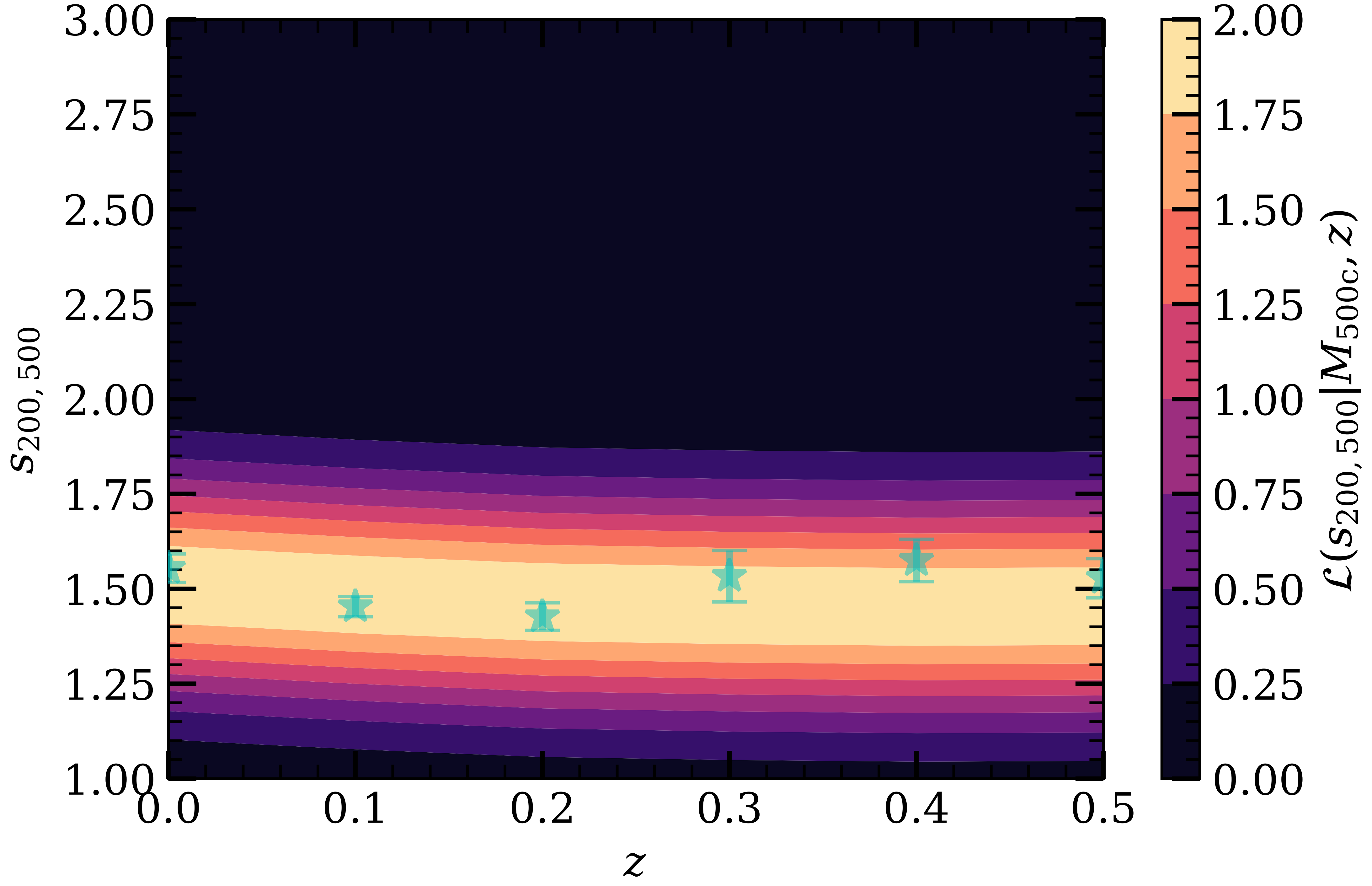
Gaussian likelihood obtained by solving,

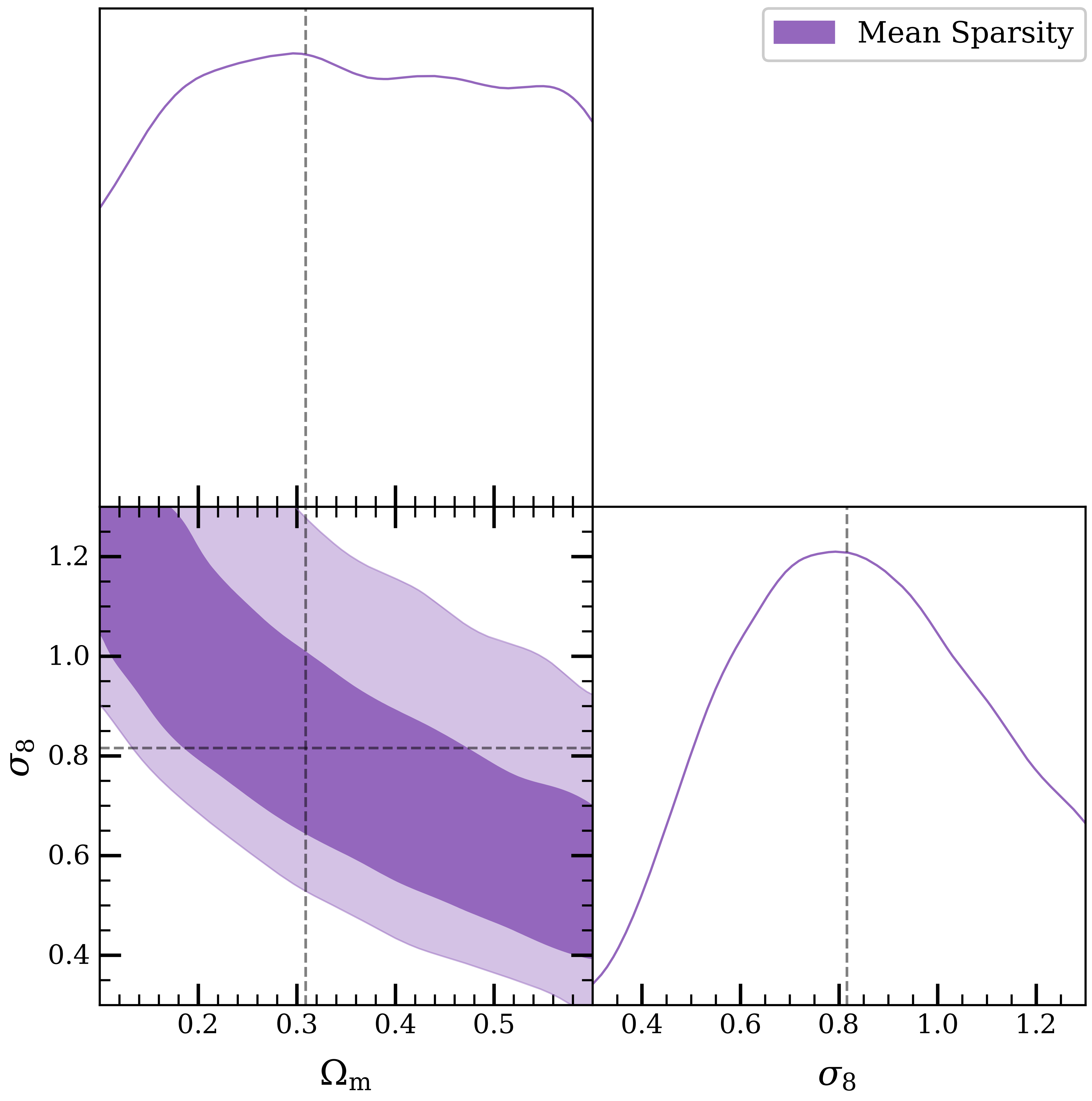
$$\int \frac{\mathrm{d}n}{\mathrm{d}M_{\Delta_2}} \mathrm{d}\ln M_{\Delta_2} = \langle s_{\Delta_1,\Delta_2} \rangle \int \frac{\mathrm{d}n}{\mathrm{d}M_{\Delta_1}} \mathrm{d}\ln M_{\Delta_1},$$

Balmès et al. 2014

for $\langle s_{\Delta_1,\Delta_2} \rangle$

Corasaniti et al. 2019, 2021, 2022

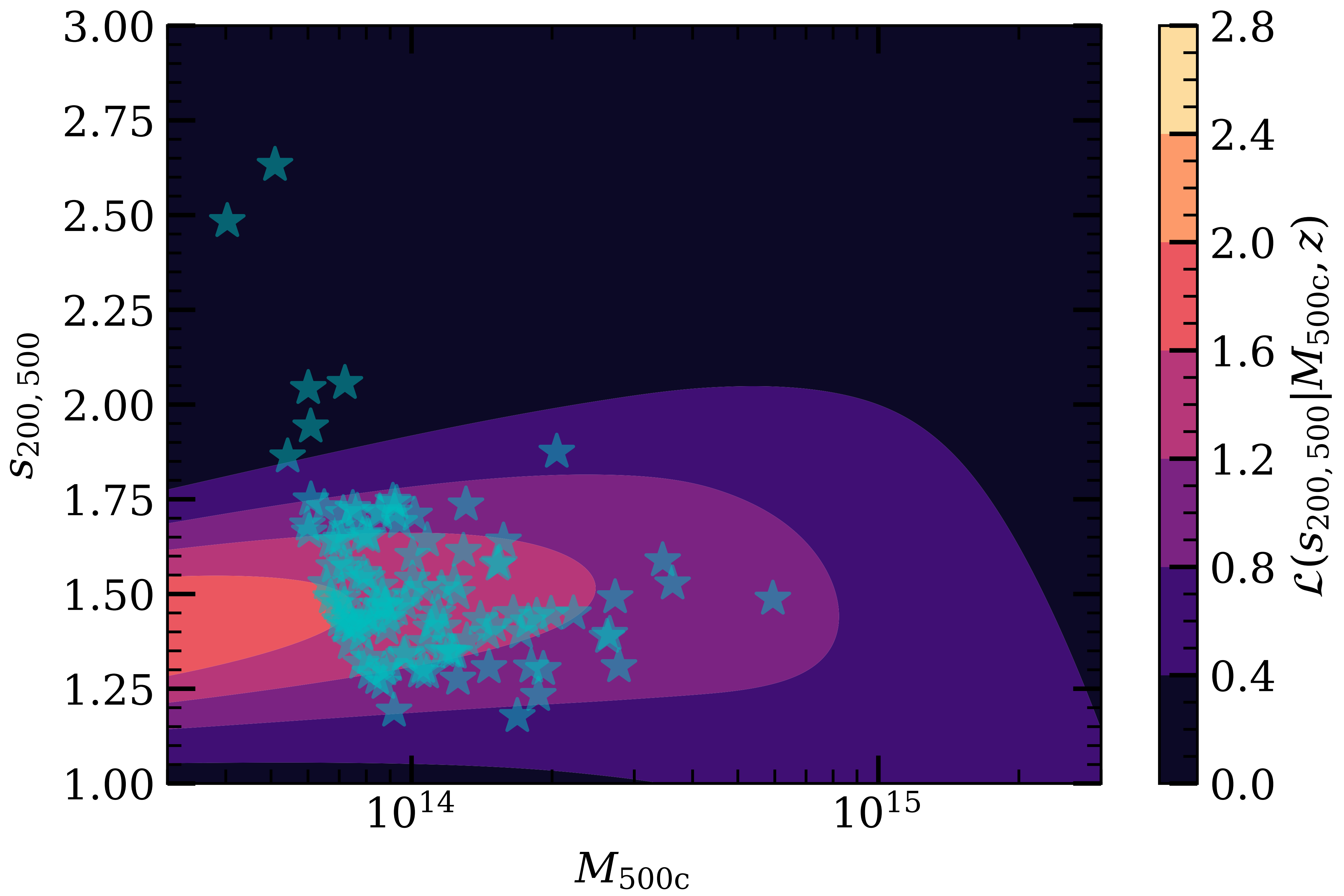


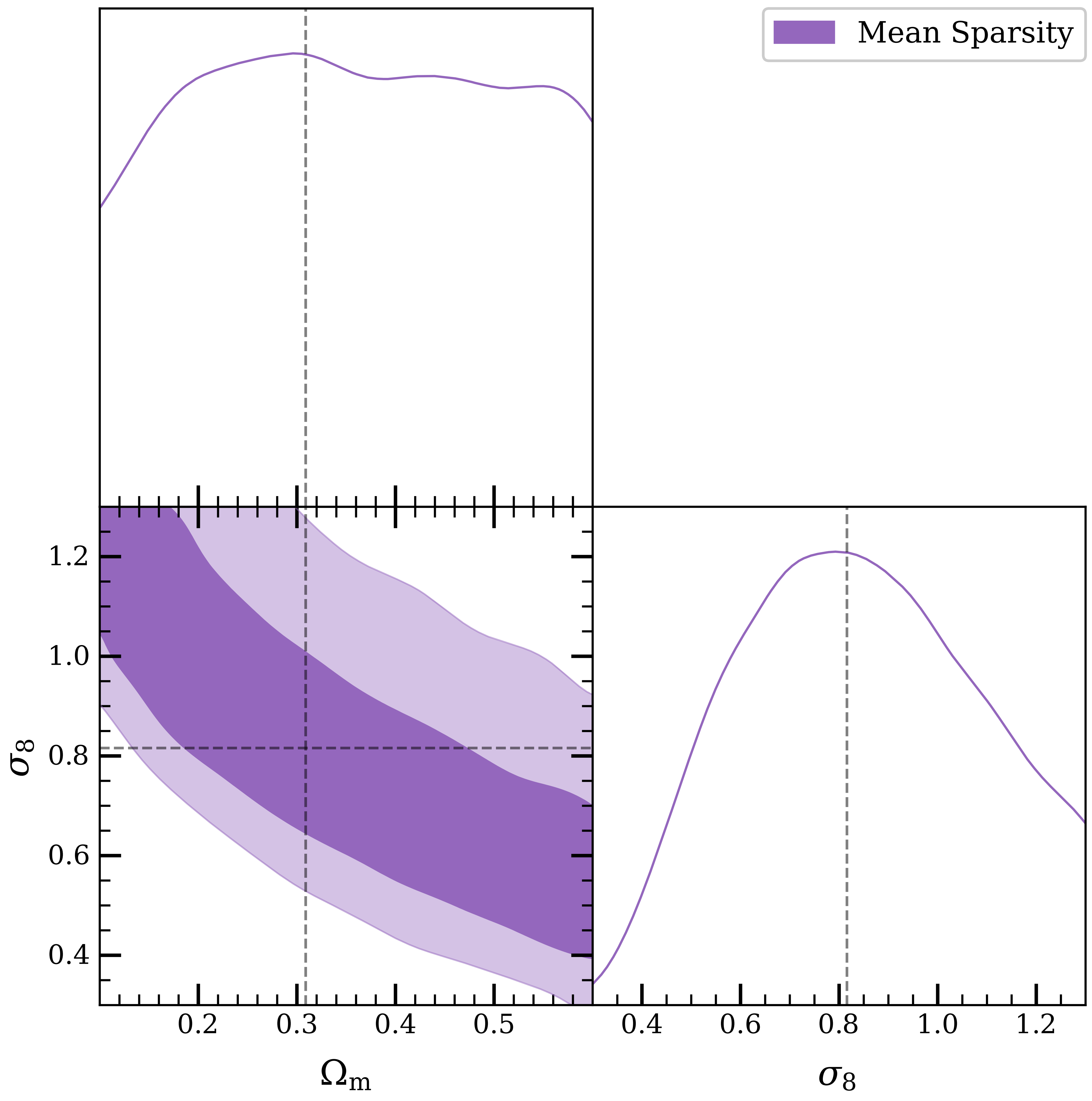


The future for sparsity

Let's do better:

Assuming the $\rho(s_{\Delta_1,\Delta_2}|M_{\Delta_2})$ is Gaussian we solve for μ and σ^2 that verify the **inward** and **outward** reconstruction.





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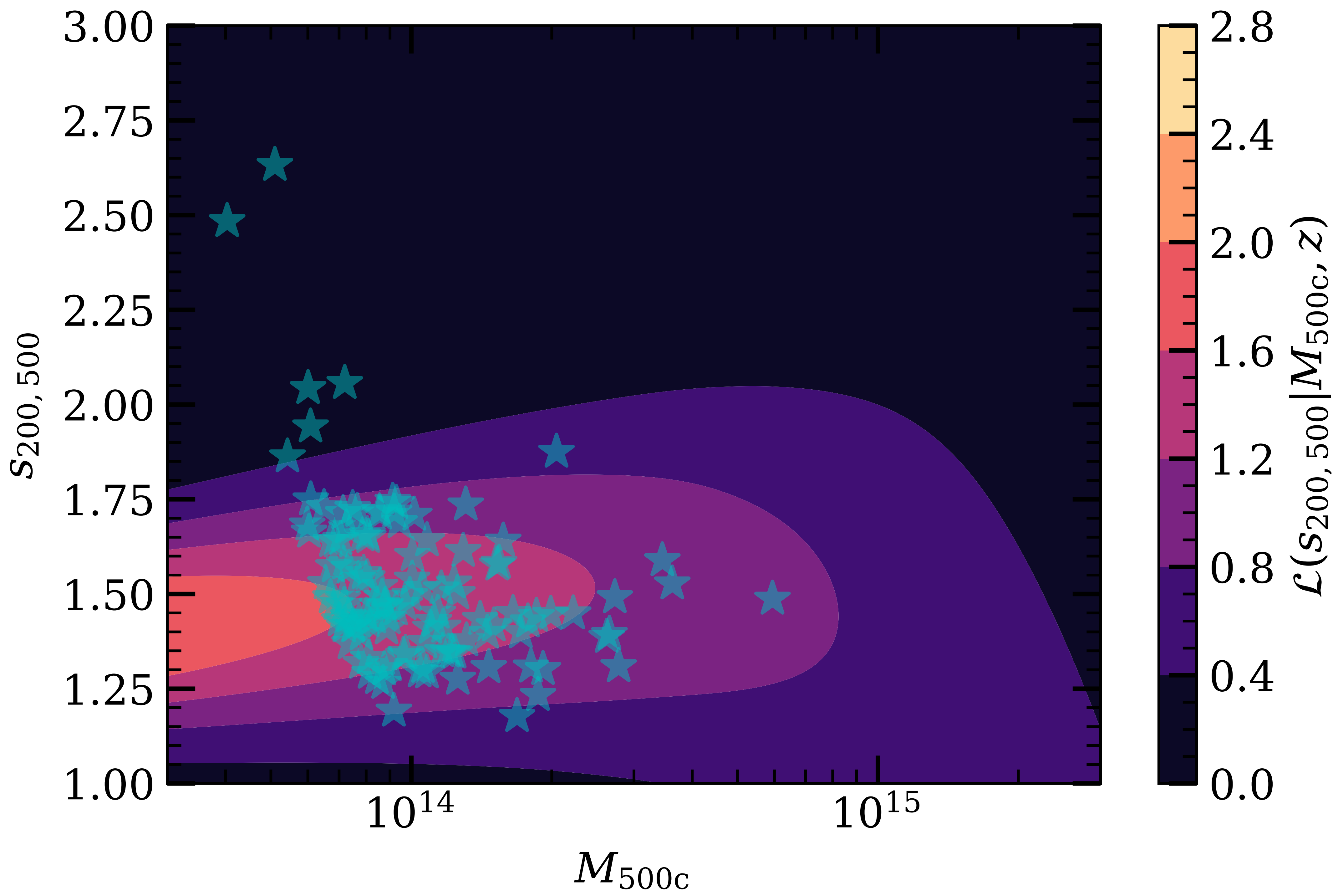
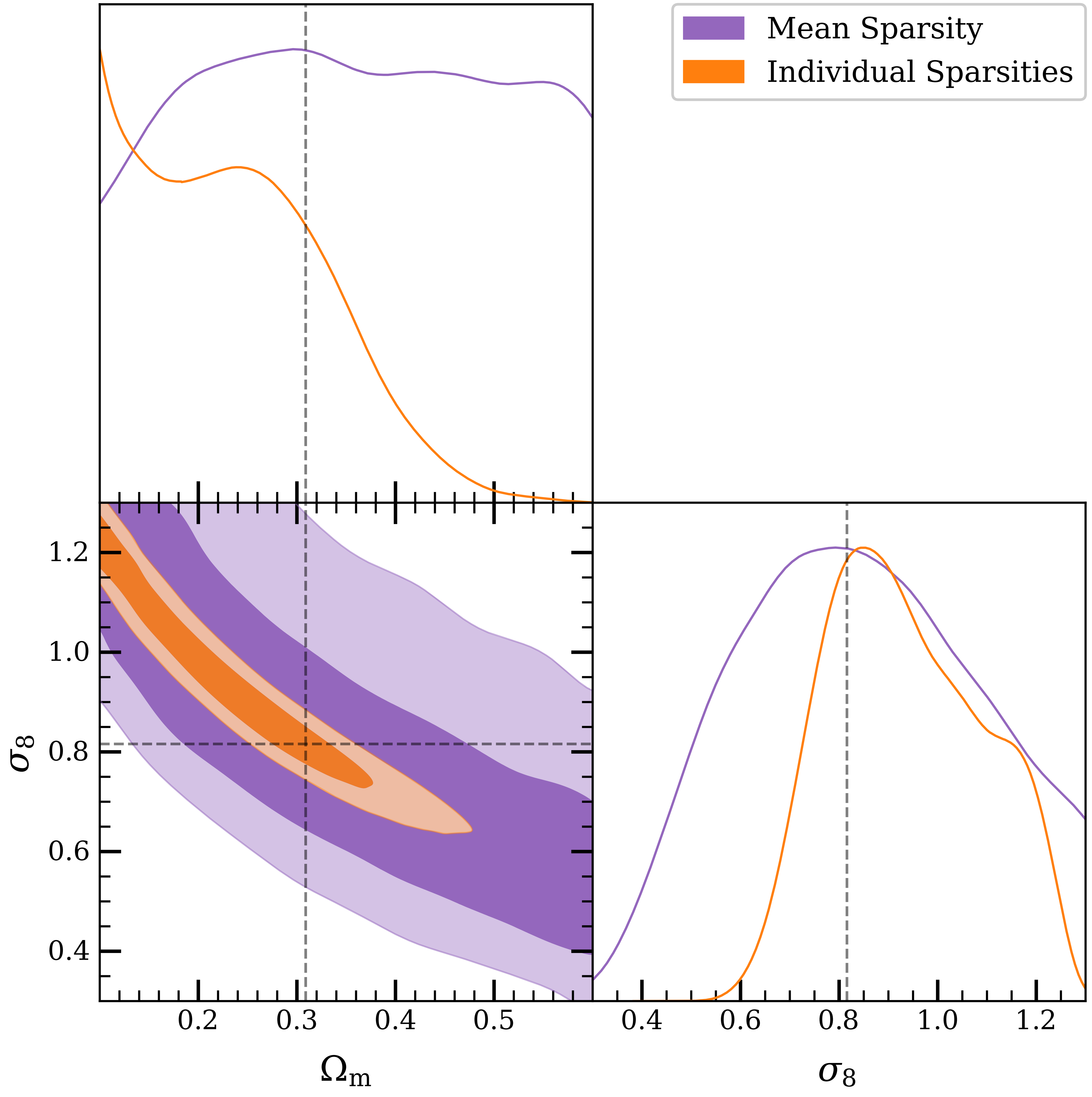
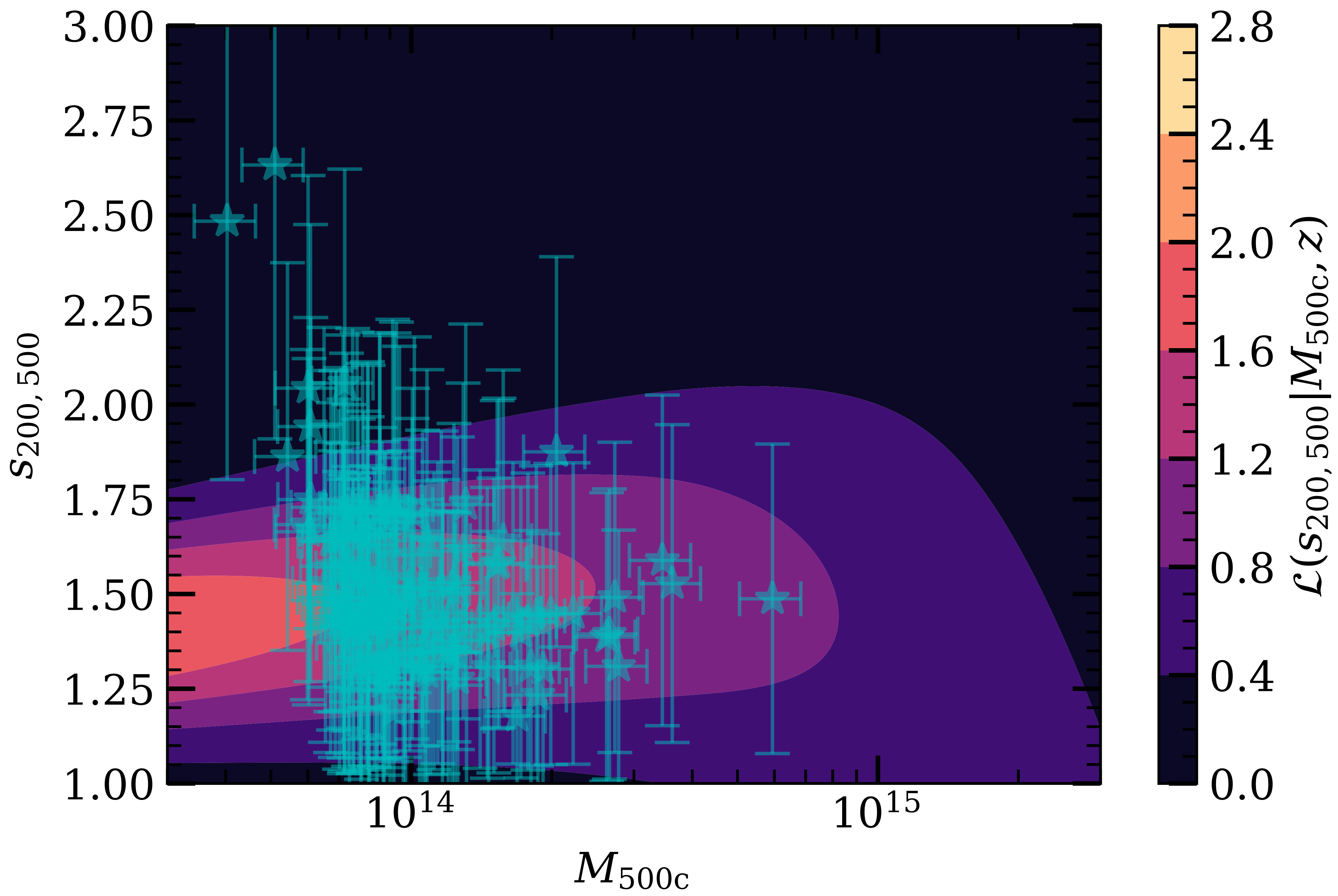


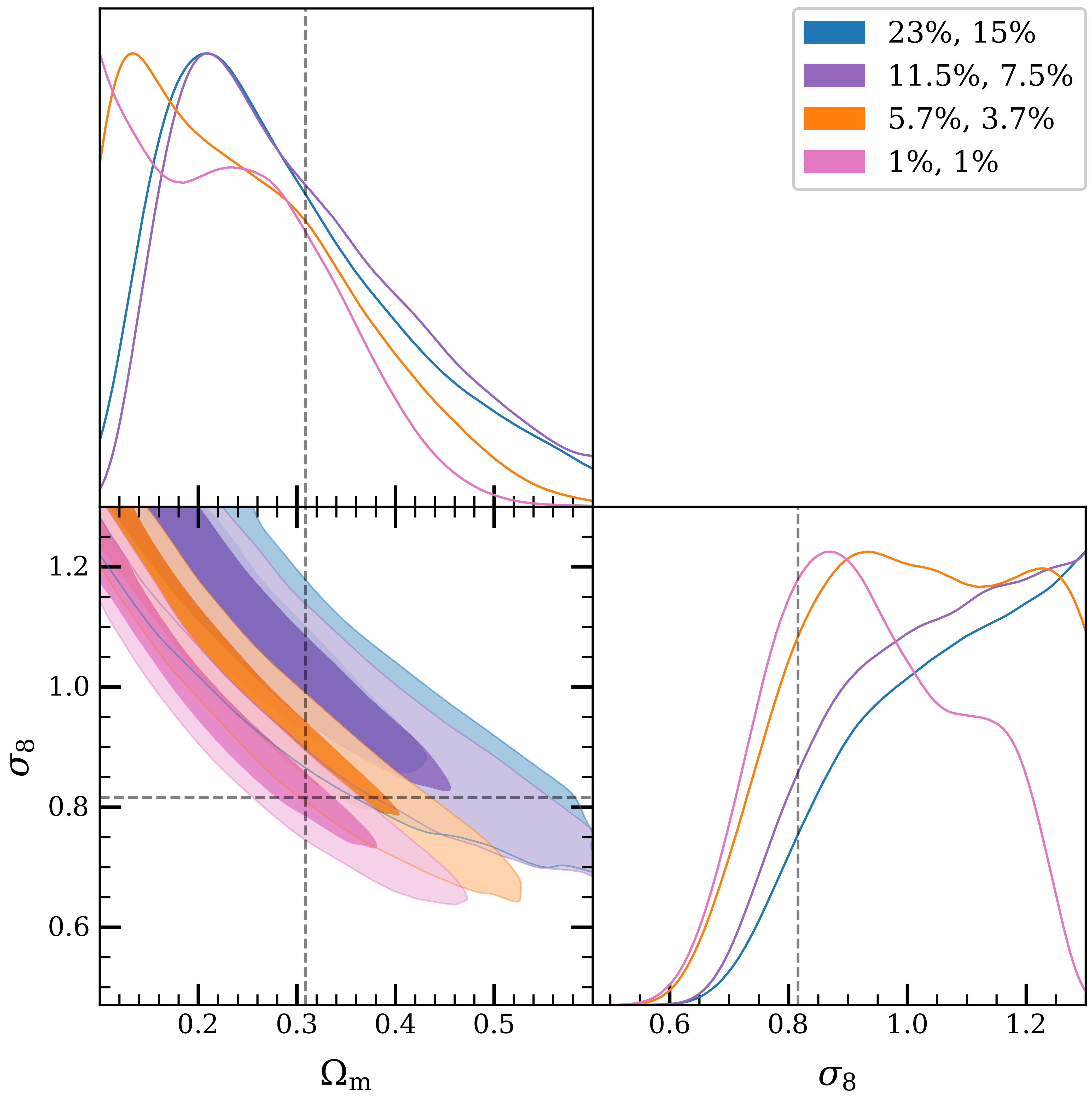


Too good to be true?

What about measurement errors?

Marginalising over realistic errors decreases the constraining power and introduces a bias towards large S_8





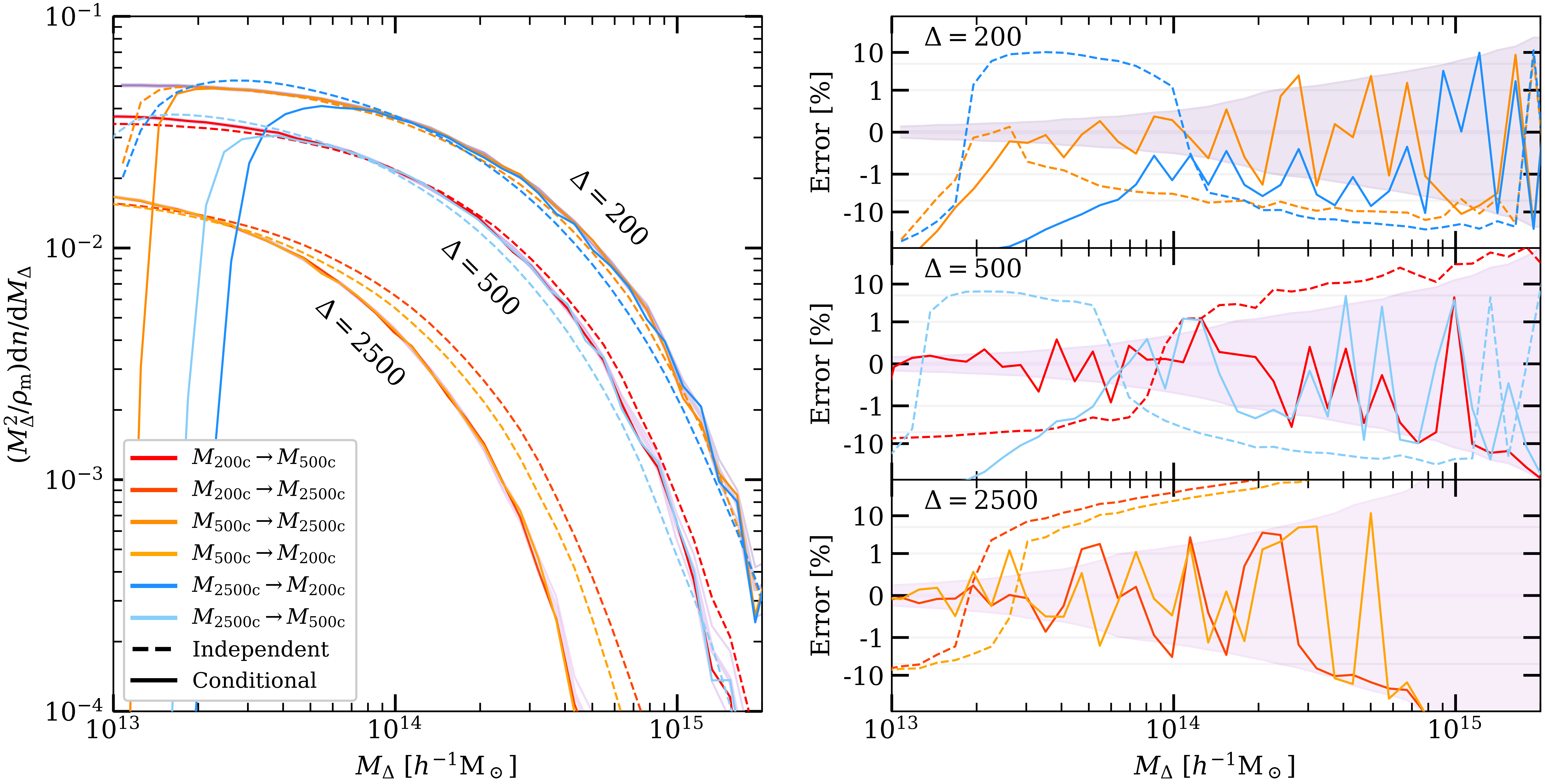
Summary

The **non-parametric nature** and **simple definition** of halo sparsity make it ideal to study the relationship between haloes and cosmological background.

Indeed we've seen that sparsity can:

- **Detect** galaxy cluster major mergers
- **Recast** the Halo Mass Function from one mass definition to another
- **Constrain** cosmology using a restricted sample of cluster observations

Sparsity Transformations



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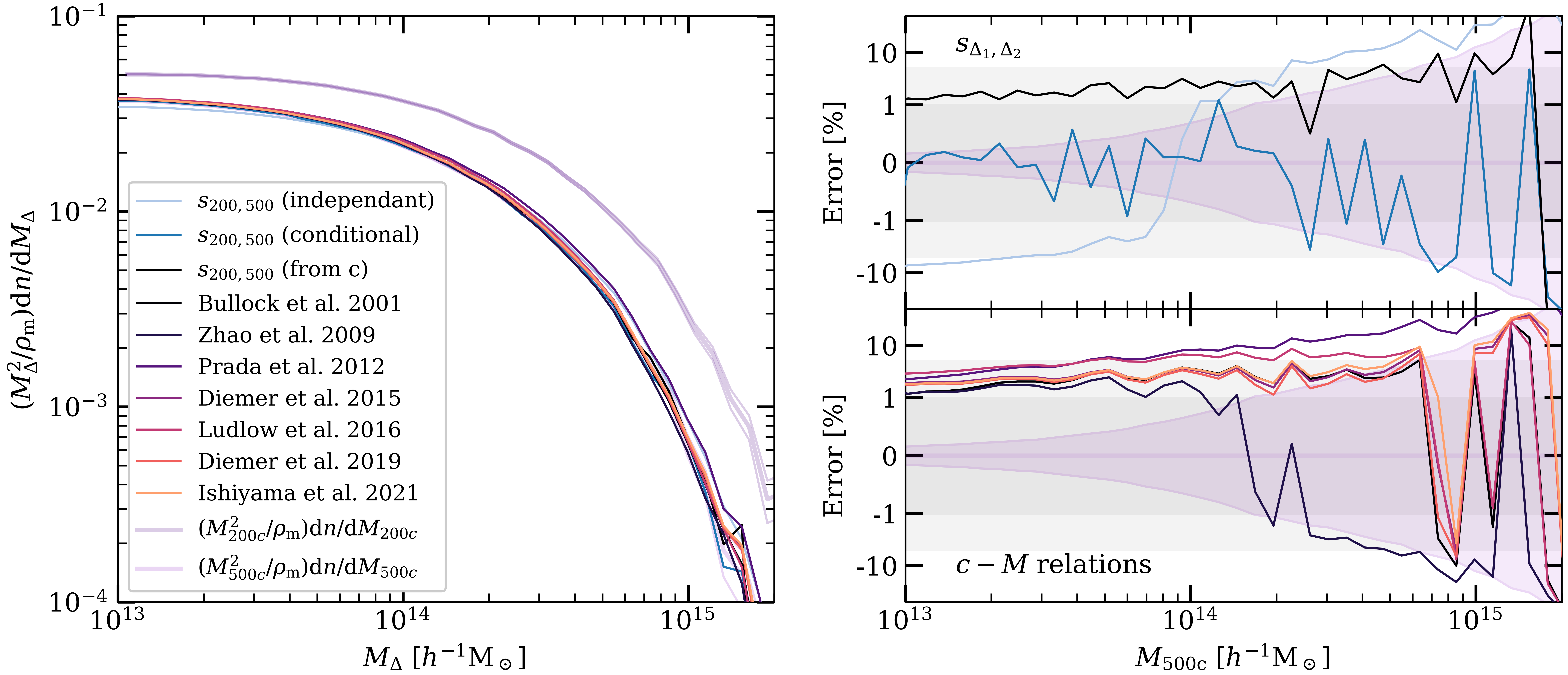

Constraints with Concentrations