Radiative transfer : the diffusion approximation

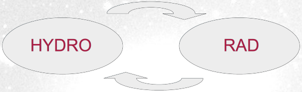
Océane Saincir

Journée des étudiants du LUTH

LUTH, july, 5th, 2016

Radiation hydrodynamics

- Supersonic, hypersonic flows
  - Compression of matter
  - High temperature
  - Photon emission
- Radiation : change the dynamics and morphology of flows



Stellar jets ©NASA, ESA, & M. Livio

SNR ©Digitized sky Survey, ESA/ESO/NASA

HADES 2D

- Numerical code for radiation hydrodynamics models simulations
  - Finite volume approach
  - Parallelized code written in Fortran 90

- Computation of physical quantities as a function of time

Hydrodynamic quantities :

- Density : ρ
- Velocity : U
- Energy : E

Radiative quantities :

- Radiative energy : E_R
- Radiative flux F_R
- Radiative pressure : P_R

Radiation hydrodynamics equations

Euler equations with M1 multigroup model coupling :

Coupling: Hydro

$$\partial_t \rho + \nabla \cdot (\rho \mathsf{u}) = 0,$$

$$\partial_t (\rho \mathsf{u}) + \nabla \cdot (\rho (\mathsf{u} \otimes \mathsf{u}) + p\mathbb{I}) = \sum_{g=1}^{\mathcal{G}} \mathsf{S}_g,$$

$$\partial_t E + \nabla \cdot (\mathsf{u}(E+p)) = \sum_{g=1}^{\mathcal{G}} c\, S_g^0,$$

Coupling: Rad

$$\partial_t E_{R_g} + \nabla \cdot \mathsf{F}_{R_g} = -c\, S_g^0,$$

$$\partial_t \left(c^{-2} \mathsf{F}_{R_g}\right) + \nabla \cdot \mathsf{P}_{R_g} = -\mathsf{S}_g, \qquad g = 1, \ldots, \mathcal{G}.$$

Turpault, PhD thesis 03 ; Lowrie et al., JQRST 01 ; Mihalas & Auer, JQRST 01

M1 multigroup model

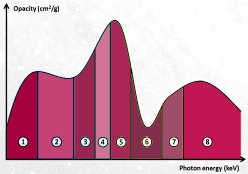
Source terms (TLE) :

$$S^0 = \kappa_P \left(E_R - a_R T^4\right),$$
$$\mathsf{S} = \kappa_R\ \mathsf{F}_R/c,$$

with resp. κ_P and κ_R Planck and Rosseland mean opacities :

$$\kappa_P = \frac{\int_\nu \kappa(\nu) B(\nu,T)\ \mathrm{d}\nu}{\int_\nu B(\nu,T)\ \mathrm{d}\nu}, \qquad \kappa_R^{-1} = \frac{\int_\nu \chi^{-1}(\nu)\partial_T B(\nu,T)\ \mathrm{d}\nu}{\int_\nu \partial_T B(\nu,T)\ \mathrm{d}\nu}.$$

- Generally, strong fluctuation of opacities
- Segmentation of frequencies ν in $\mathcal{G}$ groups



- Opacity calculations in HADES : use of tables of opacities which are results from atomic physics calculations. Interpolation on the grid (ρ, T)

Diffusion approximation

λ : mean free path of a photon L : characteristic length of the phenomena

- Optically very thick medium : $\lambda \ll L$
  - Fluid opaque to photons, local radiative phenomena, diffusion approximation

Radiative quantities :

$$E_R = a_R T^4 \qquad \mathrm{F}_R = -\frac{1}{3}\frac{c}{\kappa_R}\nabla E_R \qquad \mathrm{P}_R = \frac{1}{3}E_R\mathbb{I}$$

Equation of state for perfect gas : $T = m(\gamma - 1)/k_B(E/\rho - ||\mathrm{u}||^2/2)$

Equations in the diffusion approximation :

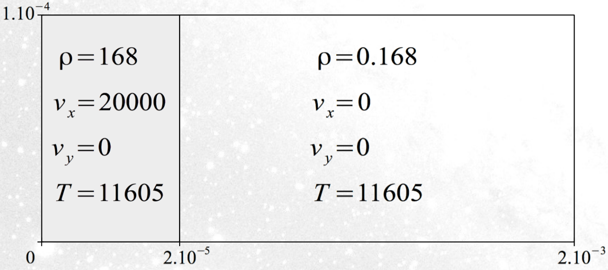
$$\partial_t \rho + \nabla \cdot (\rho \mathrm{u}) = 0,$$

$$\partial_t (\rho \mathrm{u}) + \nabla \cdot (\rho (\mathrm{u} \otimes \mathrm{u}) + p\mathbb{I}) = -\frac{1}{3}\nabla E_R,$$

$$\partial_t E + \nabla \cdot (\mathrm{u}(E + p)) = -\partial_t E_R + \frac{1}{3}\frac{c}{\kappa_R}\Delta E_R - \frac{4}{3}\nabla \cdot (\mathrm{u}E_R)\,.$$

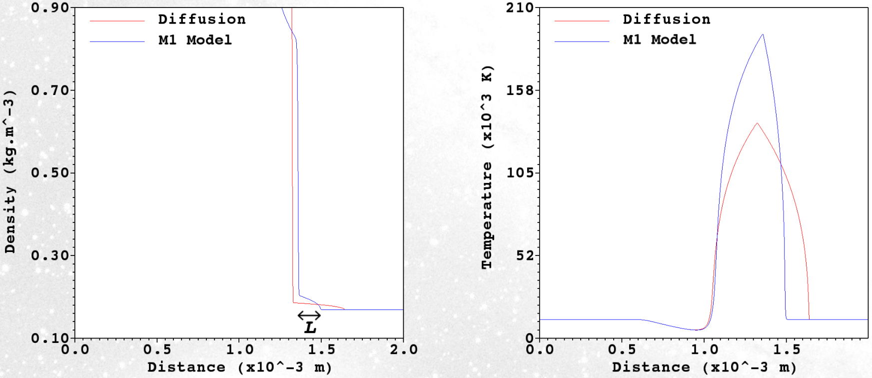
Stationary radiative shock

- Comparison between diffusion approximation and the M1 model according to the mean free path of photons :



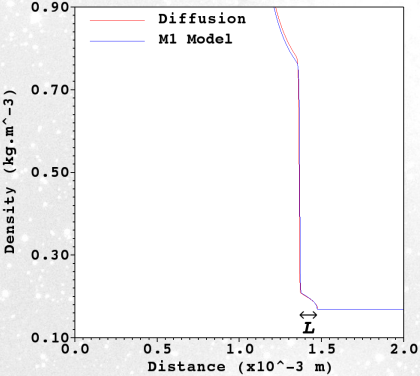
- $M_x = 2000$, $M_y = 10$ and $t_f = 4.10^{-8}$ s
- Mach number $M \approx 8$

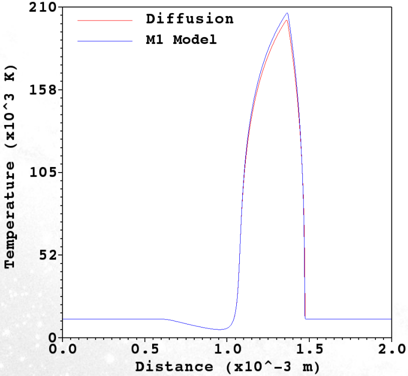
λ = 10 microns



$$\frac{\lambda}{L} \approx 0.07$$

λ = 1 micron



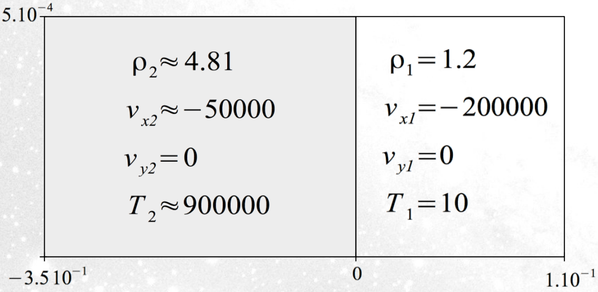


$$\frac{\lambda}{L} \approx 0.01$$

Computation time : M1 multigroup model : 12 h, diffusion approximation : 35 min

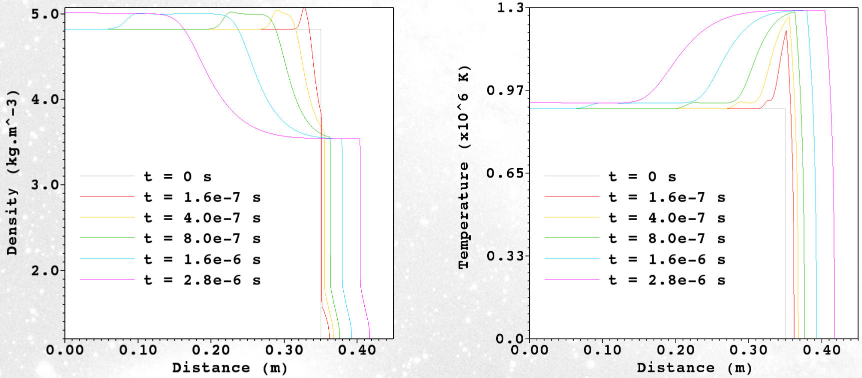
Stationary radiative shock in shock frame

- Instead λ constant, we chose $\sigma = 1/(\rho\lambda) = 4.10^3$ m^2.kg^{-1} :



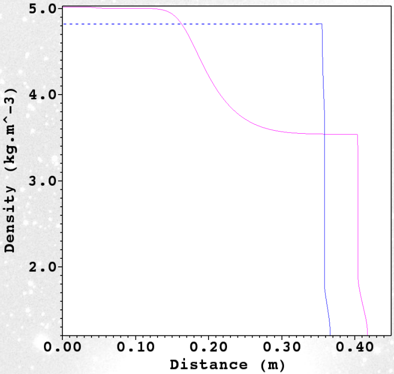
- $M_x = 4500$ and $M_y = 5$
- Mach number $M \approx 540$

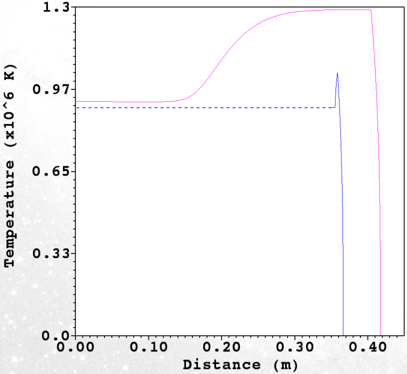
Stationary radiative shock in shock frame



Stationary radiative shock in shock frame

- In the literature : computation are not lead in the post-choc medium (Dirichlet boundary condition)





- Peak for temperature

Conclusion

- Radiative transfer : expensive simulations
- Diffusion approximation : simpler description of transfer
- Lower computation times
- Useful to prepare simulations with the M1 multigroup model

Thank you for your attention !