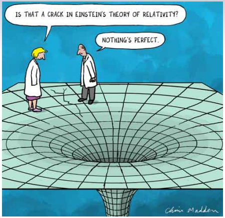
MODIFY GRAVITY



A classification of "healthy" higher-order scalar-tensor theories

Karim Noui

Fédération Denis Poisson, Tours

AstroParticules et Cosmologie, Paris 7

Einstein gravity : from 1915 to 2015...

1915 : Gravity results from deformations of space-time

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G_N \, T_{\mu\nu}$$

2015 : First detection of gravitational waves by LIGO
Total agreement with theoretical predictions

Gravity : a success story BUT...

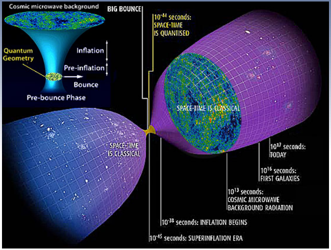
A beautiful theory with a happy end...

- Gravity is Lorentzian geometry
- It gives a beautiful picture of (almost) full history of universe
- Agreements with observations : Planck, LIGO, Microscope...

... But open issues in “extreme” regimes

- Very short (Planck) scale : singularities ?
  - Big bang singularity at the origin of the universe
  - Black hole singularity behind the horizon
  - $\Longrightarrow$ Breakdown of the theory ? Need of quantization ?
- Very large (cosmological) scale : dark energy ?
  - Accelerated expansion of the universe leads to troubles
  - Signature of a modification of gravity laws ?

Quantum gravity : just a word



- Modification of gravity at Planck scale : $\ell_P = 10^{-35} m$
  - Gravity is no more classical geometry
  - Two main approaches : String vs. Loop
  - Loop : discretization of space

$$\text{Area} \propto \ell_p^2 \sum_{j=0}^{\infty} n_j \sqrt{j(j+1)}$$

Modification of Einstein gravity at small distances

$$\left(\frac{\dot{V}}{V}\right)^2 = 24\pi G_N \,\rho \left(1 - \frac{\rho}{\rho_{max}}\right) \Longrightarrow \rho < \rho_{max}$$

ρ cannot diverge and then V reaches a minimum with no more initial singularity

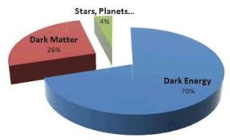
Outlook

Alternative models for dark energy ?

Let's modify gravity

- The cosmological constant problem
- How to modify gravity ?
- Scalar-Tensor theories : examples and issues
- Classification : DHOST theories
- Theoretical and observational constraints
- Discussion

Cosmological constant problem



Evidence for the ΛCDM Model

- Homogeneous and isotropic universe : FLRW space-time
- Fluctuations about FLRW background related to structures
- Best fit model : 70 % of energy density is dark energy
- Dark energy has negative pressure $\Longrightarrow$ Repulsive strength

The cosmological constant Λ

- General relativity has only two coupling constants : G_N and Λ
- Λ "simple" explanation of dark energy : $\rho_\Lambda \approx 10^{-29}$ g.cm^{-3}
- From particles physics : $\rho_{vac} \approx 10^{74}$ GeV$^4 > 10^{120}\rho_\Lambda$
- Extreme fine tuning : cosmological constant problem
- Alternative models for dark energy : numerous attempts

Robustness of gravity

Uniqueness of gravity + cosmological constant

- Hyp.1 : Space-time is of dimension 4 (+ symmetries)
- Hyp.2 : Gravity is described by a metric only $g_{\mu\nu}$
- Hyp.3 : Euler-Lagrange equations are local and second order
- Lovelock theorem : Einstein gravity + Cosmological constant

No much room available...

... Drop out one of the hypothesis above with :

- Explain dark energy with eventually self-tuning mechanism
- Theoretical/Experimental constraints
- No modifications at "small" scales (screening mechanism)

Try to modify gravity

I like the idea that space-time is 4-dimensional

However, we assume that

- Gravity comes with a scalar field ϕ : a fifth force which is expected to be responsible for dark energy $\Longrightarrow$ Scalar-Tensor theories
- Equations of motion are not necessarily of second order

Motivations

- Adding a scalar is the simplest case, but there are more complicated scenarii (bi-gravity, vectors,...)
- Higher order equations because the dynamics of gravity is governed by an action with second order derivatives : $\partial_\mu \partial_\nu g_{\rho\sigma} \rightarrow \partial_\mu \partial_\nu \phi$

Many examples in the market

"Standard" scalar-tensor theories

- Simplest extension of GR with a scalar field

$$S[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}[R + Z(\phi)\phi_\mu\phi^\mu - V(\phi)]\,, \quad \phi_\mu \equiv \partial_\mu\phi$$

- K-essence field to account for dark energy

$$S[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}[R + K(\phi, X)]\,, \quad X \equiv \phi_\mu\phi^\mu$$

Higher-Order scalar-tensor theories

- DGP theory or cubic Galileon (inspired from brane world) with higher-order terms in the action...

$$S[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}[R + \alpha\, X\Box\phi]\,, \quad \Box\phi \equiv \nabla_\mu\partial^\mu\phi$$

Horndeski theories

"Viable or safe" Higher derivative Scalar-Tensor theories

Horndeski (1974)

Most general scalar-tensor action leading to at most second order Euler-Lagrange equations for the scalar field and the metric

« Generalized Galileons »

- Deffayet & al (2011) rediscovered Horndeski result
- Combination of the four Lagrangians (with $X = \phi^{\mu}\phi_{\mu}$)

$$
\begin{aligned}
L_2^{\mathrm{H}} &\equiv G_2(\phi, X)\,, \qquad L_3^{\mathrm{H}} \equiv G_3(\phi, X)\,\Box\phi\,,\\
L_4^{\mathrm{H}} &\equiv G_4(\phi, X)\,{}^{(4)}R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})\\
L_5^{\mathrm{H}} &\equiv G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\times\\
&\quad\times(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})
\end{aligned}
$$

Beyond Horndeski

New Higher derivative "safe" Scalar-Tensor actions which do not belong to Horndeski class but propagate 3 DOF

The proposal of Gleyzes & al (2014) : Beyond Horndeski

$$L_4^{\mathrm{bH}} \equiv F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$
$$L_5^{\mathrm{bH}} \equiv F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}\ ,$$

with higher order equations of motion !

Many other examples exist in none of previous classes !

- Khronometric theories : invariance under $\phi \to \tilde{\phi}(\phi)$
- Mimetic gravity : with an extra conformal invariance $g_{\mu\nu} \to \Omega g_{\mu\nu}$
- Chern-Simons gravity (with parity breaking)

Need of a classification

Are these theories all physically viable ?
Are they equivalent up to symmetries or fields redefinition ?

Theoretical constraints

- Higher-Order Scalar-Tensor theories with 3 DOF : one scalar only
- Cosmology : avoid cosmological instabilities and ghosts

Observational constraints

Obtain an effective description to parametrize (consistent) deviations from gravity that we could observe in the future :
$\longrightarrow$ speed of GW, structure of stars, BH hairs and emission of GW, large scale structures, equivalence principle...
$\Longrightarrow$ Testing many aspects of gravity

Classical mechanics : a single variable

Dynamics of a point like particle $q(t)$

$$S[q(t)] = \int dt \left(\dot{q}^2 - \omega^2 q^2 + \alpha \ddot{q}^2\right) \implies \ddot{q} + \omega^2 q - \alpha \ddddot{q} = 0\,.$$

Degrees of Freedom and Ostrogradski ghost

- One needs 4 initial conditions : $q(0)$, $\dot{q}(0)$, $\ddot{q}(0)$ and $\dddot{q}(0)$
- Hence, two degrees of freedom : one is necessarily a ghost !

Hamiltonian unbounded from below

$$S_{eq}[q, X] = \int dt \left(\dot{q}^2 - \omega^2 q^2 - 2\alpha[\dot{X}\dot{q} + \frac{1}{2}X^2]\right) \implies X = \ddot{q}$$

Kinetic energy : $T = \dot{q}^2 - 2\alpha \dot{X}\dot{q} = (\dot{q} - \alpha\dot{X})^2 - \alpha^2 \dot{X}^2$

Classical mechanics : a coupled system

Dynamics of two point like particles $q(t)$ and $x(t)$

$$S[q,x] = \int dt \, \left(\dot{q}^2 - \omega^2 q^2 + \alpha \ddot{q}^2 + \alpha\beta^2 \dot{x}^2 + 2\alpha\beta\dot{x}\ddot{q} - V(x)\right) \, .$$

$$\text{EOM} : \ddot{q} + \omega^2 q - \alpha \ddddot{q} + \alpha\beta \dddot{x} = 0 \text{ and } \beta\ddot{x} - \tfrac{1}{2}V'(x) + \dddot{q} = 0 \quad .$$

How many degrees of Freedom ?

- It is not clear at all from the Lagrangian and and the EOM
- EOM are higher derivatives $\Longrightarrow$ Ostrogradski ghost ?
- Change variables $X \equiv \beta x + \dot{q}$

$$S[q,x] = \int dt \, \left(\dot{q}^2 - \omega^2 q^2 + \alpha \dot{X}^2 - V(x)\right)$$

Not always as easy, eg. $L = \dot{x}^2/(1+\ddot{q}) + F(\dot{q}, q, x)$

Evading Ostrogradski instability

Not easy to see from the Lagrangian nor from the EOM.

$$L(\ddot{q}, \dot{q}, q; \dot{x}, x) \Longrightarrow L_{eq} = L(\dot{Q}, Q, \phi; \dot{x}, x) + \pi(\dot{q} - Q)$$

Kinetic matrix and degeneracy

$$K = \begin{pmatrix} L_{\dot{Q}\dot{Q}} & L_{\dot{x}\dot{Q}} \\ L_{\dot{x}\dot{Q}} & L_{\dot{x}\dot{x}} \end{pmatrix} \quad \text{with} \quad L_{XY} = \frac{\partial^2}{\partial X \partial Y} L_{eq} \,.$$

- K is degenerate iff there is a primary constraint of type ($K \neq 0$)

$$\Psi = P - F(p) \approx 0 \quad \Longleftrightarrow \quad \frac{\partial L_{eq}}{\partial \dot{Q}} - F(\frac{\partial L_{eq}}{\partial \dot{x}}) \approx 0$$

- Hamiltonian analysis : det $K = 0 \Longleftrightarrow$ No-Ostrogradski ghost
- Euler-Lagrange equation are higher order but reduce to 2nd order !

Quartic Scalar-Tensor Theories

The covariance is a key ingredient which enables us classifying more explicitly Scalar-Tensor theories

Covariant quartic Lagrangians

$$S[g_{\mu\nu},\phi] \equiv \int \sqrt{-g}\,(f\,\mathcal{R} + C^{\mu\nu\rho\sigma}\,\nabla_\mu\nabla_\nu\phi\,\nabla_\rho\nabla_\sigma\phi)$$

where $f = f(X,\phi)$ and $C^{\mu\nu\rho\sigma}$ depend on ϕ and ϕ_μ only

$$\begin{aligned} C^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} \;&=\; \alpha_1\,\phi_{\mu\nu}\phi^{\mu\nu} + \alpha_2\,(\Box\phi)^2 + \alpha_3\,\Box\phi(\phi_{\mu\nu}\phi^\mu\phi^\nu) \\ &+\; \alpha_4\,\phi_{\mu\nu}\phi^{\nu\rho}\phi^\mu\phi_\rho + \alpha_5\,(\phi_{\mu\nu}\phi^\mu\phi^\nu)^2 \end{aligned}$$

with $\alpha_i = \alpha_i(X,\phi)$ and $X = \phi_\mu\phi^\mu$.

We can go further (up to cubic order), but for simplicity we restrict our presentation to quartic theories.

Hamiltonian decomposition

Equivalent action in terms of a vector field A^μ

$$S[g_{\mu\nu}, \phi; A_\mu, \lambda^\mu] \equiv \int \sqrt{|g|}\,(f\,\mathcal{R} + C^{\mu\nu\rho\sigma} A_{\mu\nu}\, A_{\rho\sigma}) + \lambda^\mu(\phi_\mu - A_\mu)$$

ADM decomposition

- Metric : $g_{\mu\nu} \to ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
- Vector field : $A^\mu \to (A_*, A_i)$ with $A_* = (A_0 - N^i A_i)/N$

(3+1) action and Kinetic term

$$S = \int \mathcal{A}\dot{A}_*^2 + 2\mathcal{B}^{ij}\dot{A}_* K_{ij} + \mathcal{K}^{ij,kl} K_{ij} K_{kl} + \dots$$

with $K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$ the second fundamental form.

Kinetic matrix and degeneracy

Velocities are K_{ij} and $\dot{A}_*$ only. No $\dot{N}$ neither $\dot{N}^i$ in the Lagrangian !

Kinetic matrix (7 dimensions) : it mixes K_{ij} and $\dot{A}_*$

$$K = \begin{pmatrix} \mathcal{A} & \mathcal{B}^{kl} \\ \mathcal{B}^{ij} & \mathcal{K}^{ij,kl} \end{pmatrix}$$

Degeneracy conditions

- No Ghost $\Longrightarrow$ Det $\mathcal{K} = 0$
- We solved the condition [Det $\mathcal{K}$=0] and classified ghost free theories
- Degenerate Higher Order Scalar Tensor (DHOST) theories

DHOST theories

We have obtained a full classification of Degenerate Higher-Order Scalar-Tensor theories

- Up to metric redefinition : $g_{\mu\nu} \to A(\phi, X) g_{\mu\nu} + B(\phi, X)\phi_\mu \phi_\nu$
- We recovered all known theories and found new theories
- What about higher-derivatives of pure metric theories ?

$\Longrightarrow$ Landscape of scalar-tensor theories without Ostrogradski ghost

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{|g|}[F_0 + F_1 \Box\phi + F_2 R + \alpha_1 \phi_{\mu\nu}\phi^{\mu\nu} + \alpha_2(\Box\phi)^2 + \alpha_3 \Box\phi\phi^\mu\phi^\nu\phi_{\mu\nu} + \alpha_4(\phi_{\mu\nu}\phi^\nu)^2 + \alpha_5(\phi^\mu\phi^\nu\phi_{\mu\nu})^2]$$

$F_n(\phi, X)$ are free functions and $\alpha_n(\phi, X)$ are not independent...

Cosmology

Background solutions : FLRW space-time

- One finds solutions compatible with observations
- Self-tuning mechanisms for the cosmological constant

Cosmological stability (at linear level) : EFT approach

- General analysis of scalar and tensor perturbations about FLRW

$$
\begin{aligned}
S^{(2)}[\zeta] &= \frac{1}{2}\int d^3x\, dt\, a^3 \left[\mathcal{A}_s \dot{\zeta}^2 + \mathcal{B}_s \frac{(\partial_i \zeta)^2}{a^2}\right] \\
S^{(2)}[\gamma] &= \frac{1}{8}\int d^3x\, dt\, a^3 M^2 \left[\dot{\gamma}_{ij}^2 - (1+\alpha_T)\frac{(\partial_k \gamma_{ij})^2}{a^2}\right]
\end{aligned}
$$

- Stability conditions : $\mathcal{A}_s > 0$, $c_s^2 \equiv -\mathcal{B}_s/\mathcal{A}_s > 0$, $M^2 > 0$ and $c_T \equiv 1 + \alpha_T > 0$

DHOST theories after GW170817

Constraint on the speed of gravitational waves : $\alpha_T < 10^{-15}$

Assuming $\alpha_T = 0$ holds exactly, this implies

- No viable DHOST theories beyond quadratic order (in $\phi_{\mu\nu}$)
- The very famous Horndeski theories do not satisfy this constraint
- New DHOST theories remain with $\alpha_1 = 0$

$$\begin{aligned} S[g_{\mu\nu},\phi] \;\; = \;\; & \int d^4x\sqrt{|g|}[F_0(\phi,X) + F_1(\phi,X)\Box\phi + F_2(\phi,X)R \\ & +\alpha_3\Box\phi\phi^\mu\phi^\nu\phi_{\mu\nu} + \alpha_4(\phi_{\mu\nu}\phi^\nu)^2 + \alpha_5(\phi_{\mu\nu}\phi^\mu\phi^\nu)^2 \end{aligned}$$

where F_n are free functions and $\alpha_n(\phi,X)$ are not independent...

Gravitation inside matter

Quasi-static approximation on scales $r \ll H^{-1}$

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)dx^2 \,, \quad \phi = \phi_0(t) + \chi(r)$$

Modified Gravitation laws in spherical body with density $\rho(r)$

$$\begin{aligned} \frac{d\Phi}{dr} &= \frac{G_N M(r)}{r^2} + \Xi_1 G_N M''(r) \\ \frac{d\Psi}{dr} &= \frac{G_N M(r)}{r^2} + \Xi_2 \frac{G_N M'(r)}{r} + \Xi_3 G_N M''(r) \end{aligned}$$

with $(8\pi G_N)^{-1} \equiv 2F_2(1+\Xi_0)$.

Observational constraints

From stars, Hulse-Taylor binary pulsar...

Conclusion

DHOST theories provide a very general framework to describe scalar-tensor theories with higher derivatives

Parametrize modifications of gravity

- Cosmological scales (with speed of GW)
- Effect on LSS formation
- Astrophysical scales : breaking of Vainshtein mechanism
- Black Holes : Hairy BH and modification of GW form
- Quantum gravity scale : breaking of Lorentz invariance...

We hope to see modifications of gravity... And we have built a powerful frame to model these modifications...