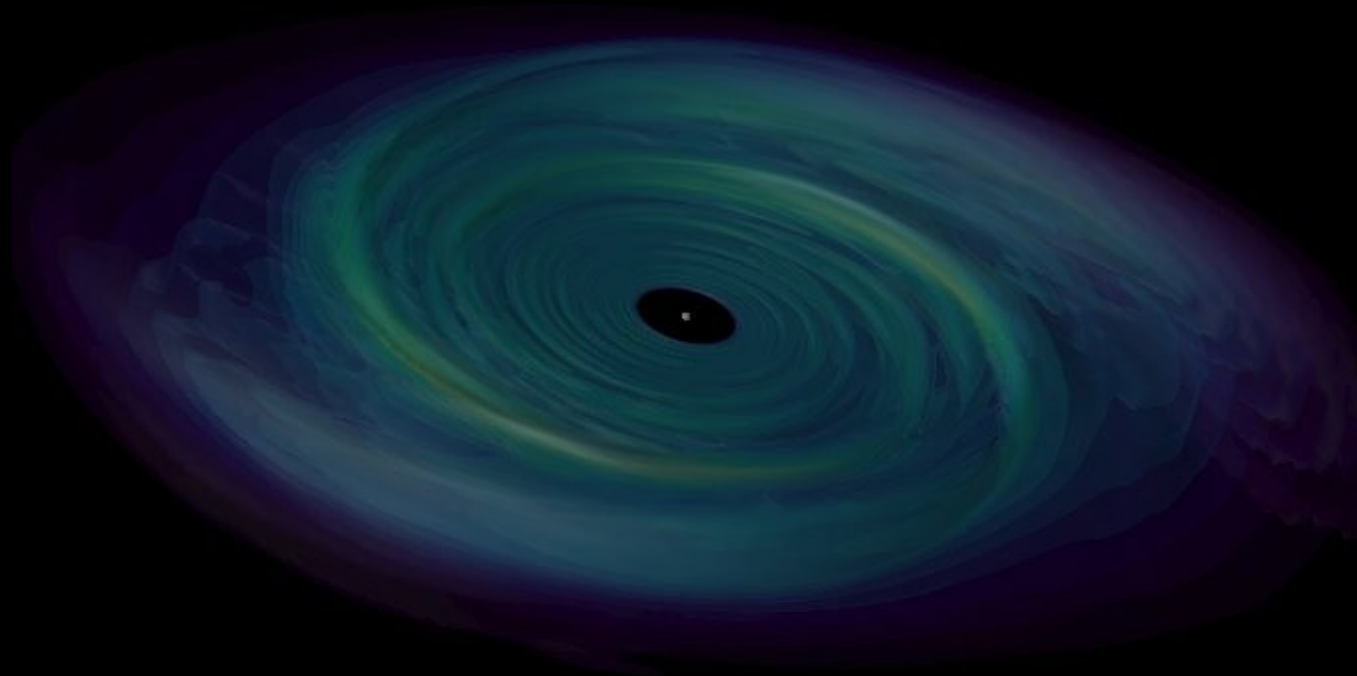


Accretion discs around black holes



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Collaborators: E. Gourgoulhon, F. Vincent, T. Paumard – M. Abramowicz, A. Sadowski, M. Bursa, R. Narayan, R. Remillard, J. McClintock, J. Steiner - C. Done, M. Middleton

Content

1. Introduction

2. Standard **THIN DISC** model of accretion

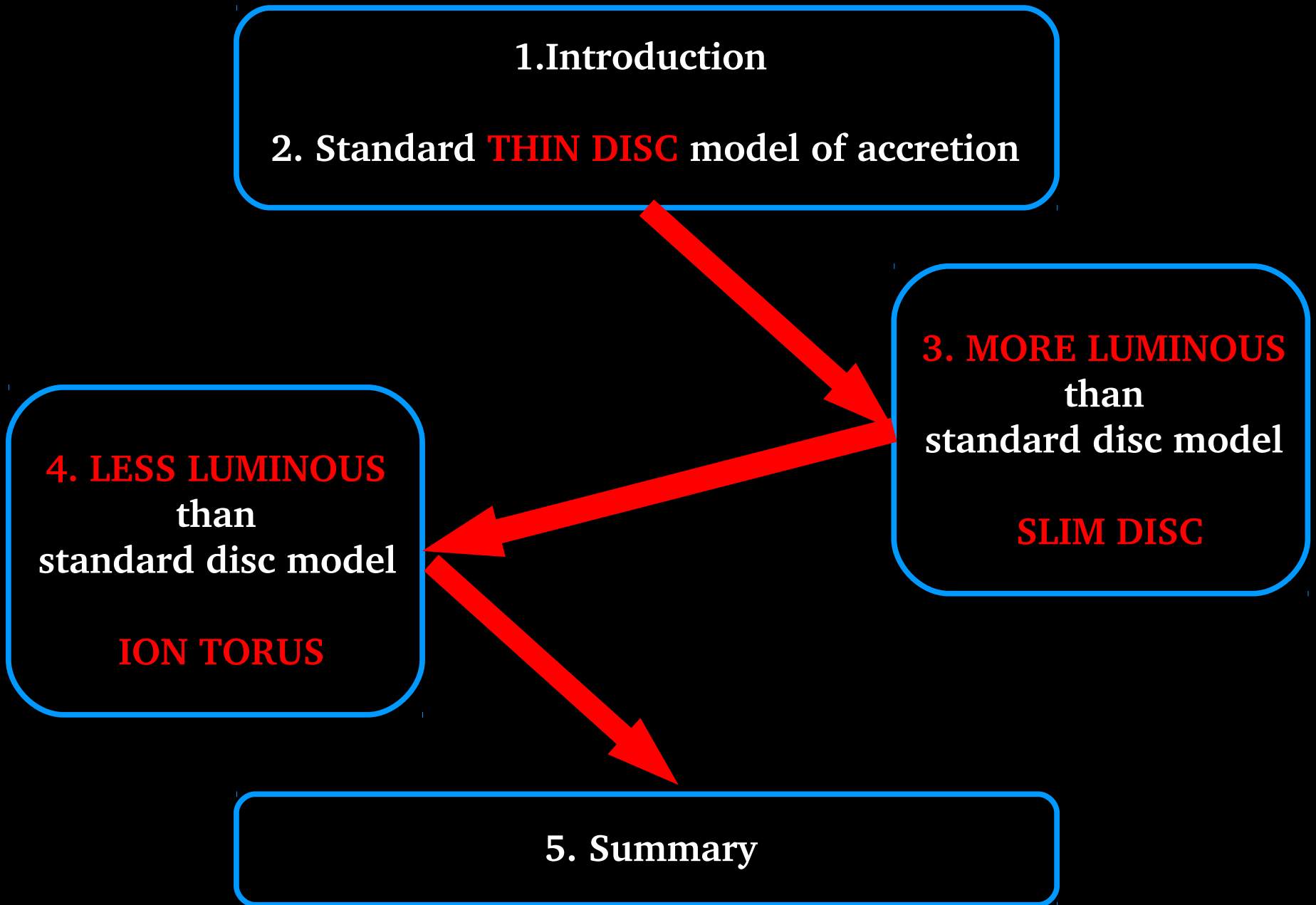
3. **MORE LUMINOUS**
than
standard disc model

SLIM DISC

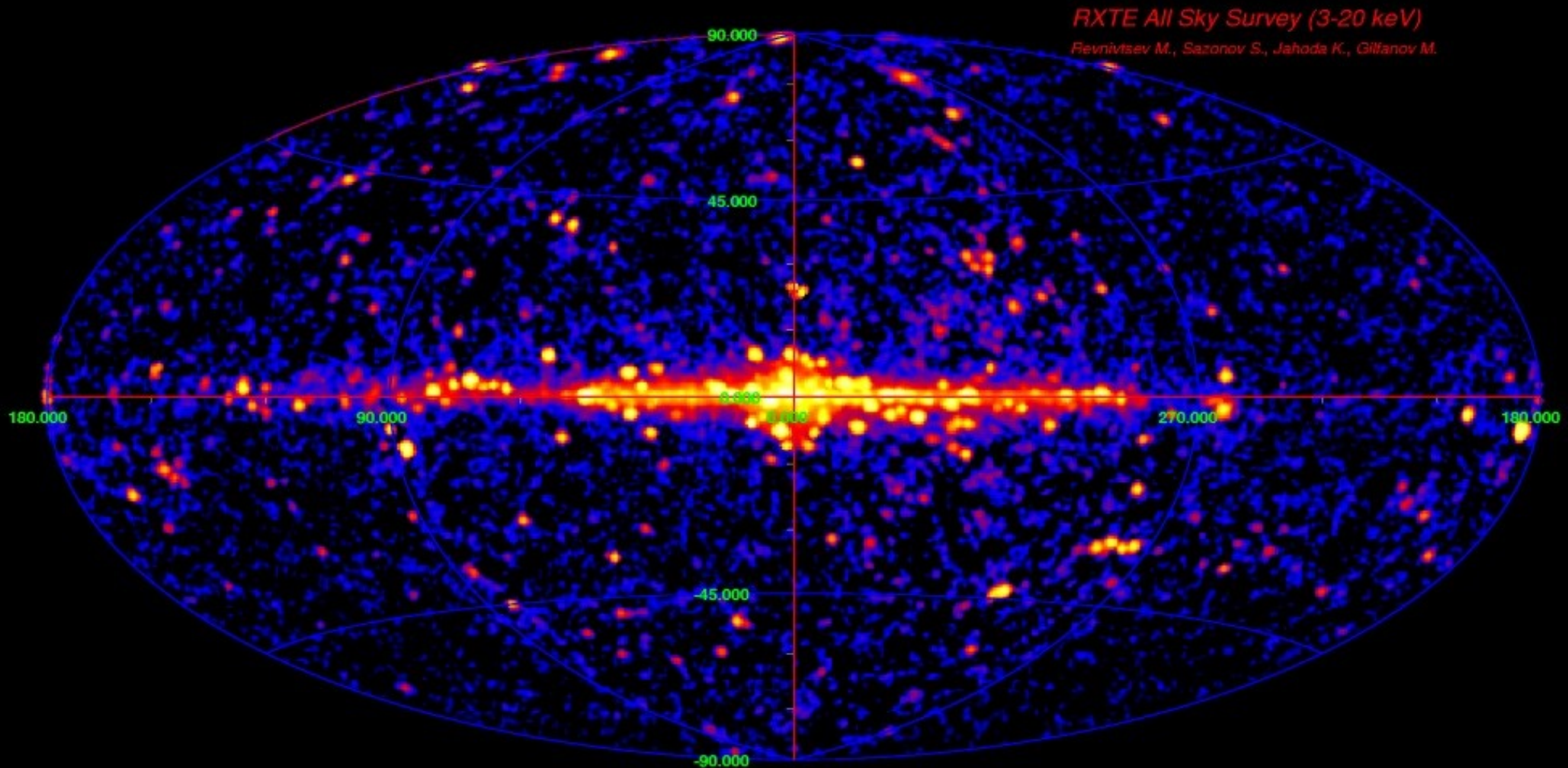
4. **LESS LUMINOUS**
than
standard disc model

ION TORUS

5. Summary



1. Introduction: The X-ray sky



X-rays are only produced in the most energetic events.

Many sources are so bright that they require accretion of matter onto a compact object

1. Introduction: Compact sources

Stellar evolution: As stars age they reach a point where they can no longer support their self-gravity by burning => collapse => compact object

New equilibrium stages (listed for increasing progenitor mass):

→ **White Dwarf:** e- degenerate, non-classical Fermi pressure (Chandrasekhar limit !)

→ **Neutron Star:** nuclei degenerate, repulsive nuclear forces (TOV limit !)

→ **Quark Star / Strange Star / Boson Star / ?:** quark degeneracy (hypothetical !)

1. Compact sources: Rotating black holes

Rotating black holes are creatures conjured by Albert Einstein's General Theory of Relativity.

Stationary black holes are completely described by 2 parameters

- * the **MASS**
- * the **SPIN**

They are found, one reckons, where **GRAVITY** and **ROTATION** coalesce in a confined space.

Where these 2 primordial properties of the universe meet - extreme conditions (curvature) arise.

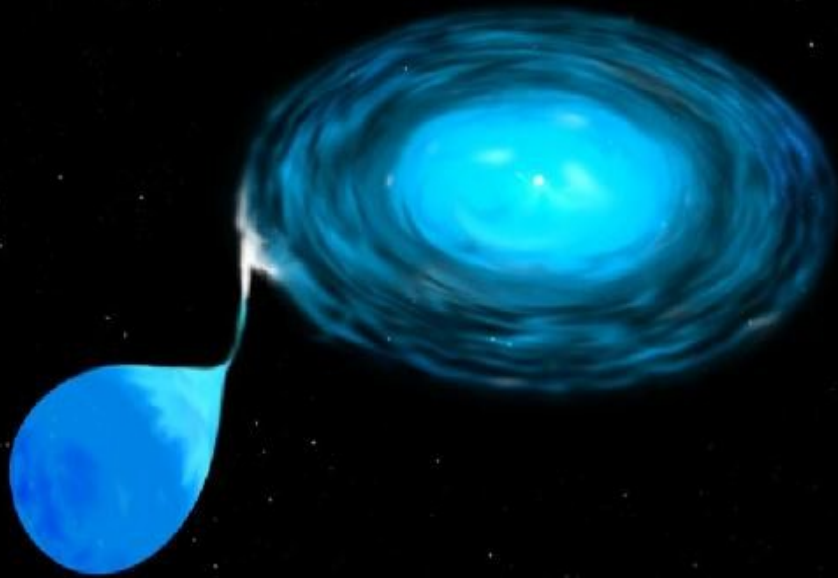


They are beasts akin to the smile of the Cheshire cat.

They are enormous stars that have winked out but are still there.

(C. Sagan, 1973)

1. Compact sources: X-ray binaries



Low-mass XRB:

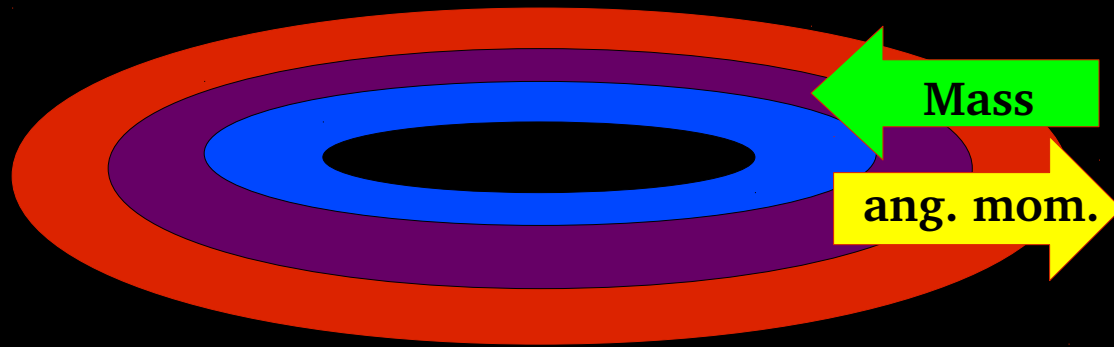
Roche lobe overflow



High-mass XRB:

Wind accretion

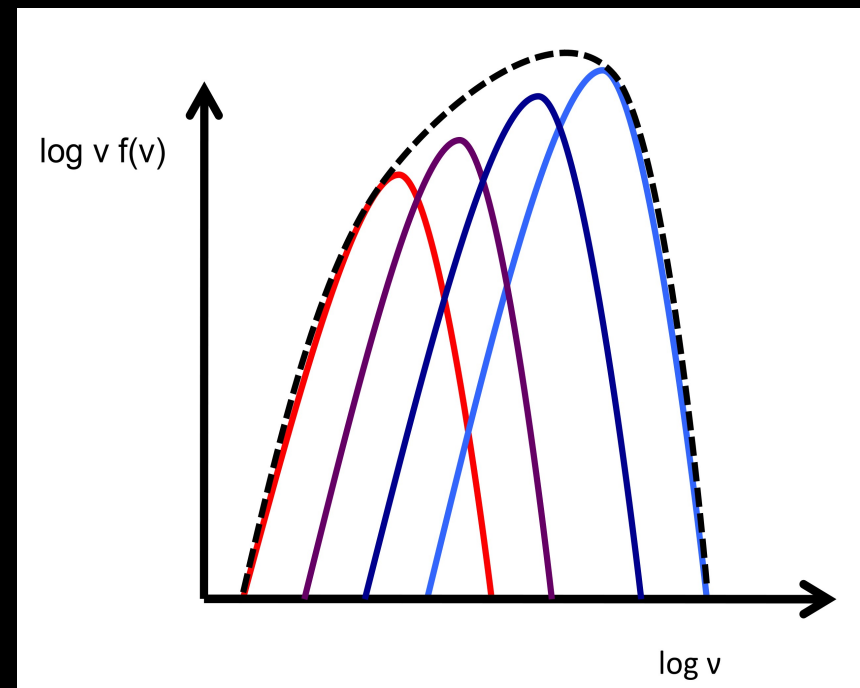
2. Accretion discs: Basics



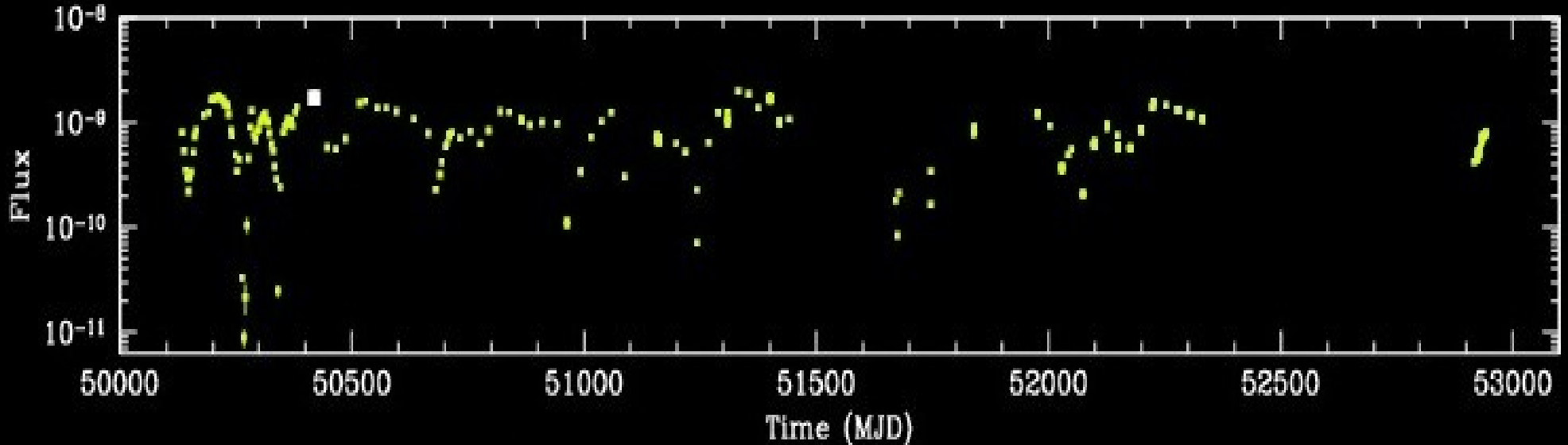
The piling of matter induces all kinds of stresses that trigger energy release via radiation, so that a part of the initial angular momentum is lost and matter can make way by moving slightly inward.

A multicolour disc blackbody, produced by blackbody emission over multiple disc annuli.

The hottest blackbody component comes from the annulus closest to the black hole.

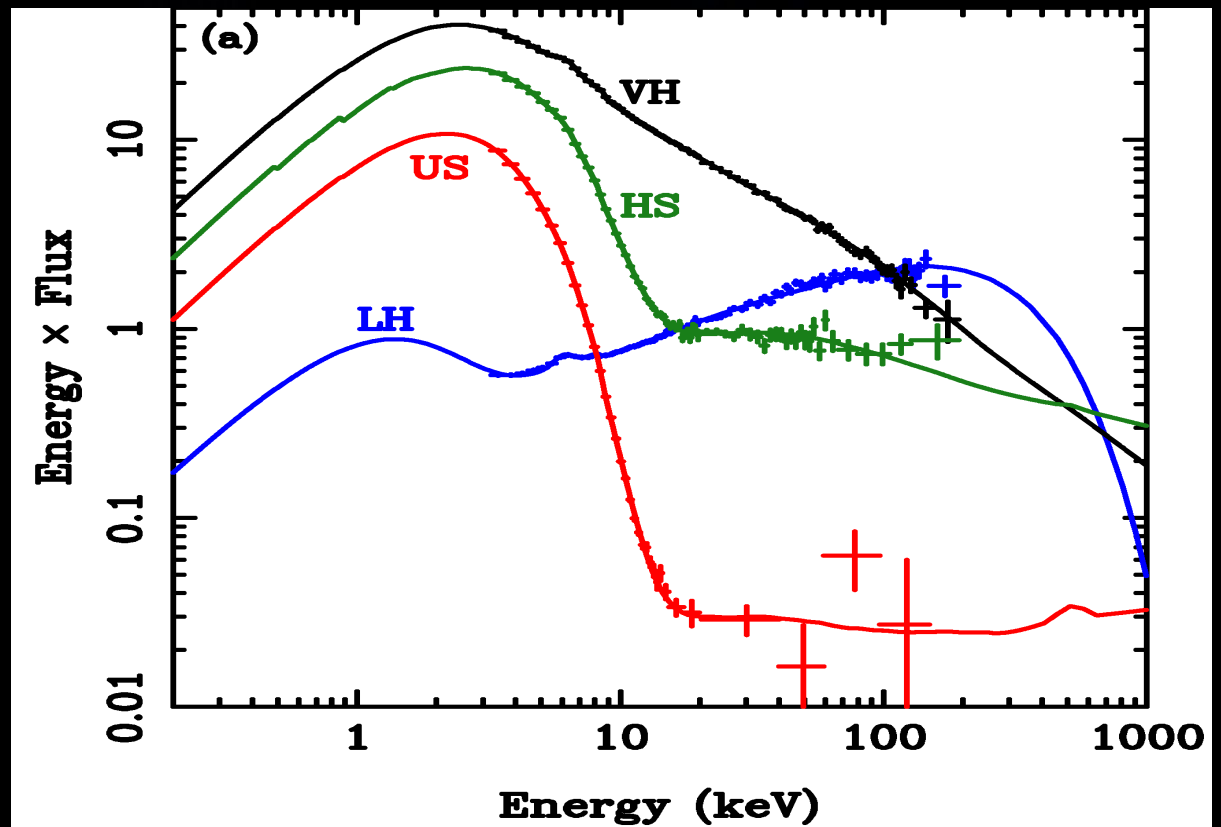


2. Accretion discs: Observational evidence



The spectral appearance changes as a function of luminosity.

Evidence of disc in the **Ultra-Soft** (thermal) state...



2. Accretion discs: Simple model

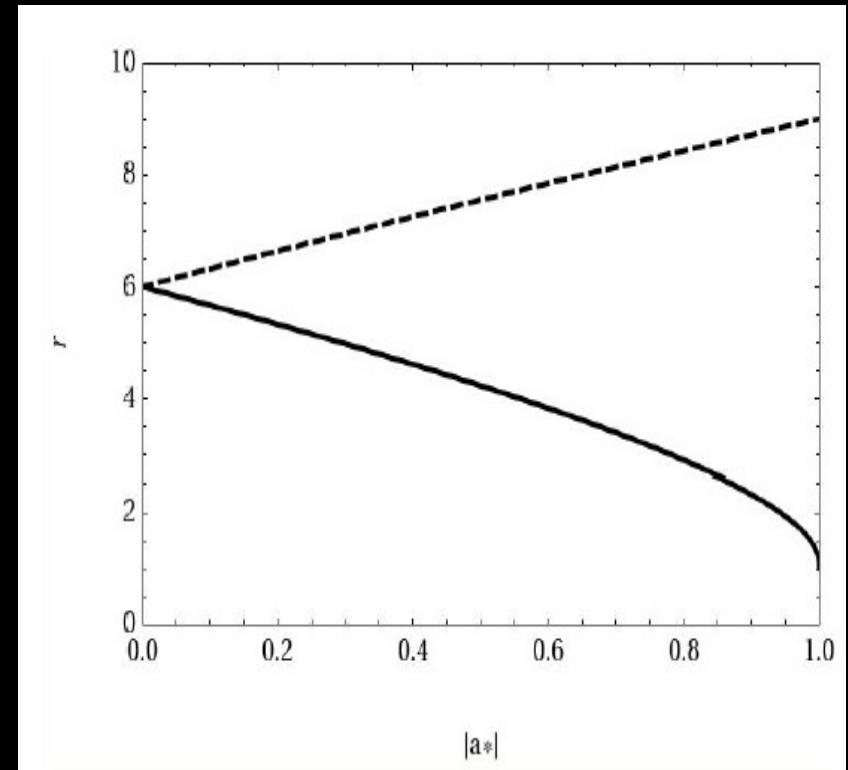
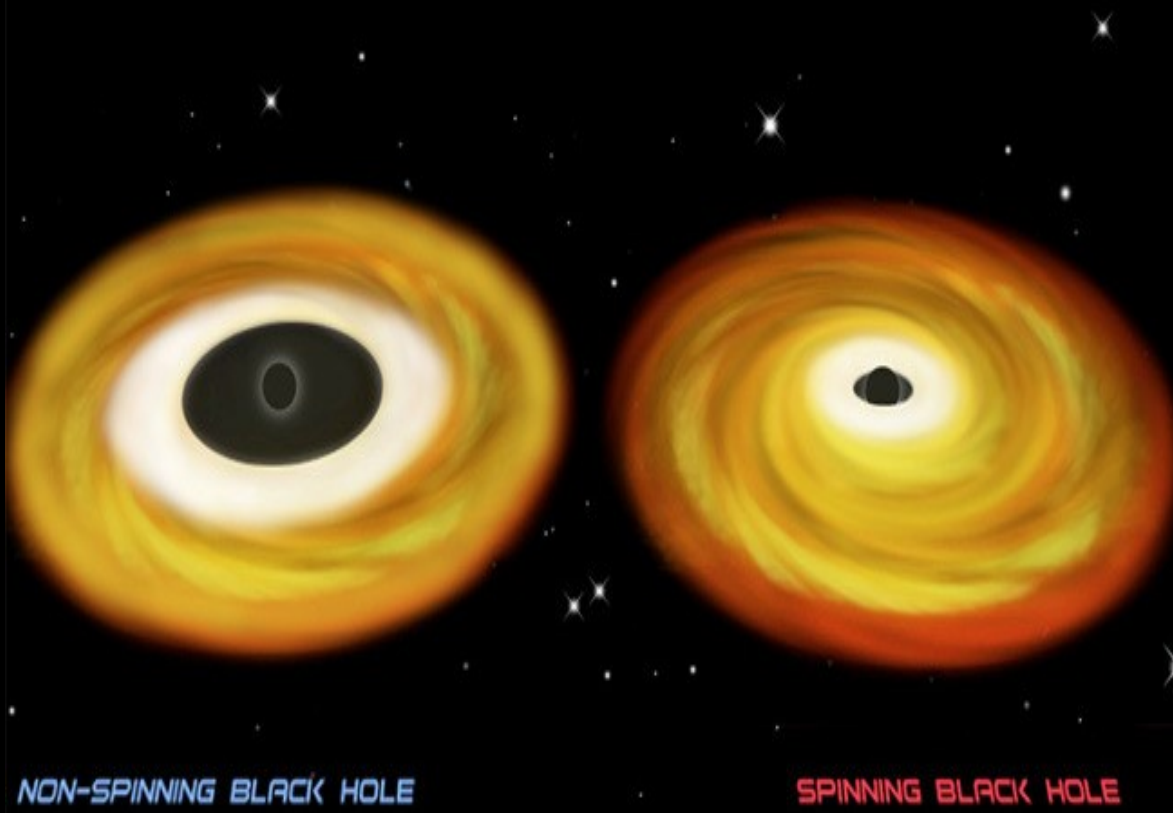
$$F(R) = \frac{3GM\dot{M}}{8\pi R^3} \left(1 - \sqrt{\frac{R_{\text{in}}}{R}}\right)$$

$$T(R) = T_0 \left(\frac{R_{\text{in}}}{R}\right)^{3/4} \left(1 - \sqrt{\frac{R_{\text{in}}}{R}}\right)^{1/4}$$

$$T_0 = \left(\frac{3GM\dot{M}}{8\pi\sigma R_{\text{in}}^3}\right)^{1/4}, \quad L_{\text{disk}} = \frac{GM\dot{M}}{2R_{\text{in}}}$$

$$R_{\text{in}} = \sqrt{\frac{3}{4\pi\sigma} \frac{L_{\text{disk}}^{1/2}}{T_0^2}}$$

2. Accretion discs: BH properties



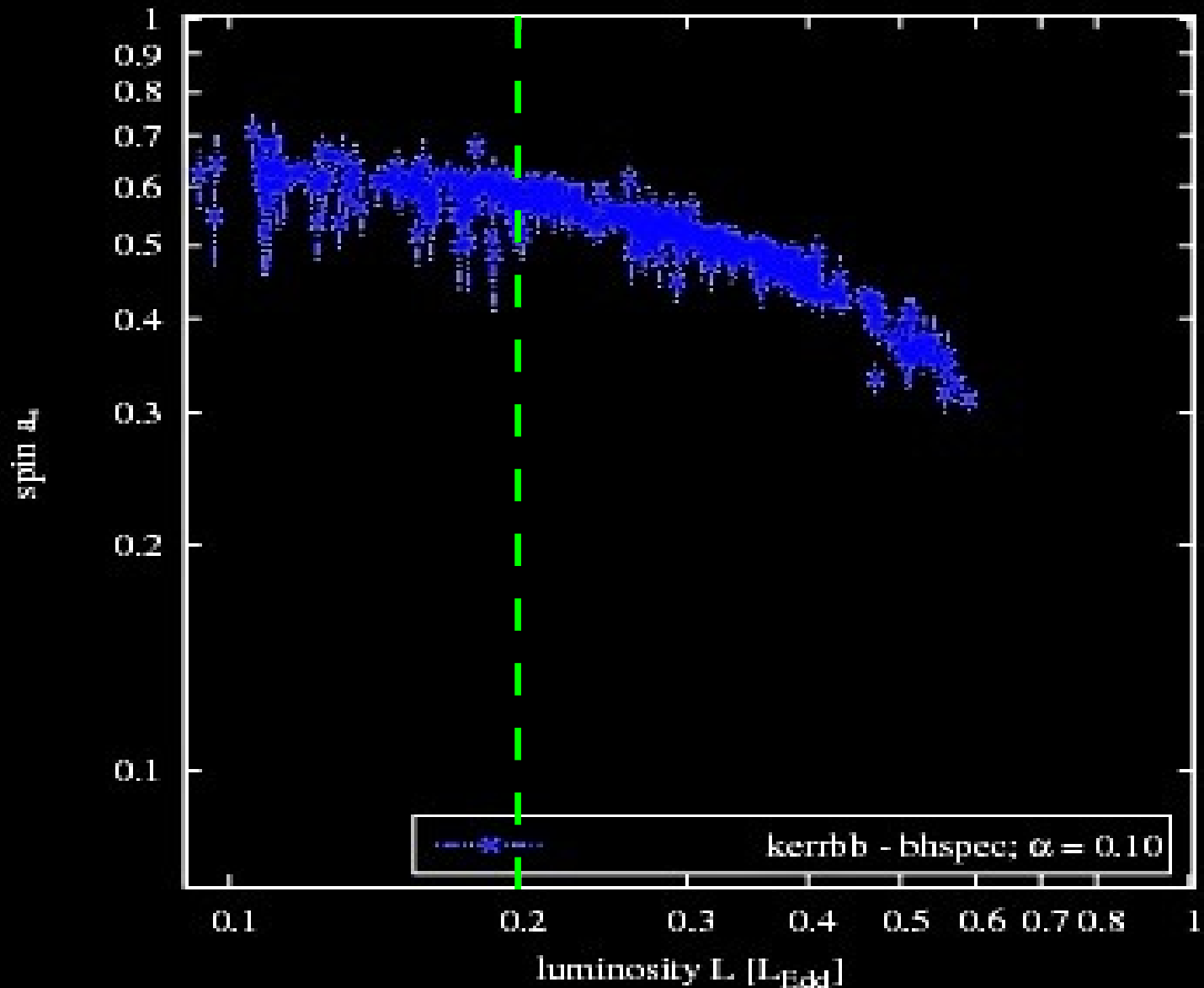
$$L_{\star} = 4\pi D^2 F_{obs} = 4\pi R_{\star}^2 \sigma T^4$$

* M, D, i from ground-based observations

* $R_{in} = R_{isco}$

$$\rightarrow a^* = Jc/GM^2$$

2. Troubles with high L spectra



The spin is not constant for all luminosity? → Go back to the drawing board!

Supplement: The Eddington limit

The theoretical limit at which the force generated by radiation pressure of a light-emitting body equals its gravitational attraction. A star emitting radiation at greater than the Eddington limit would break up.

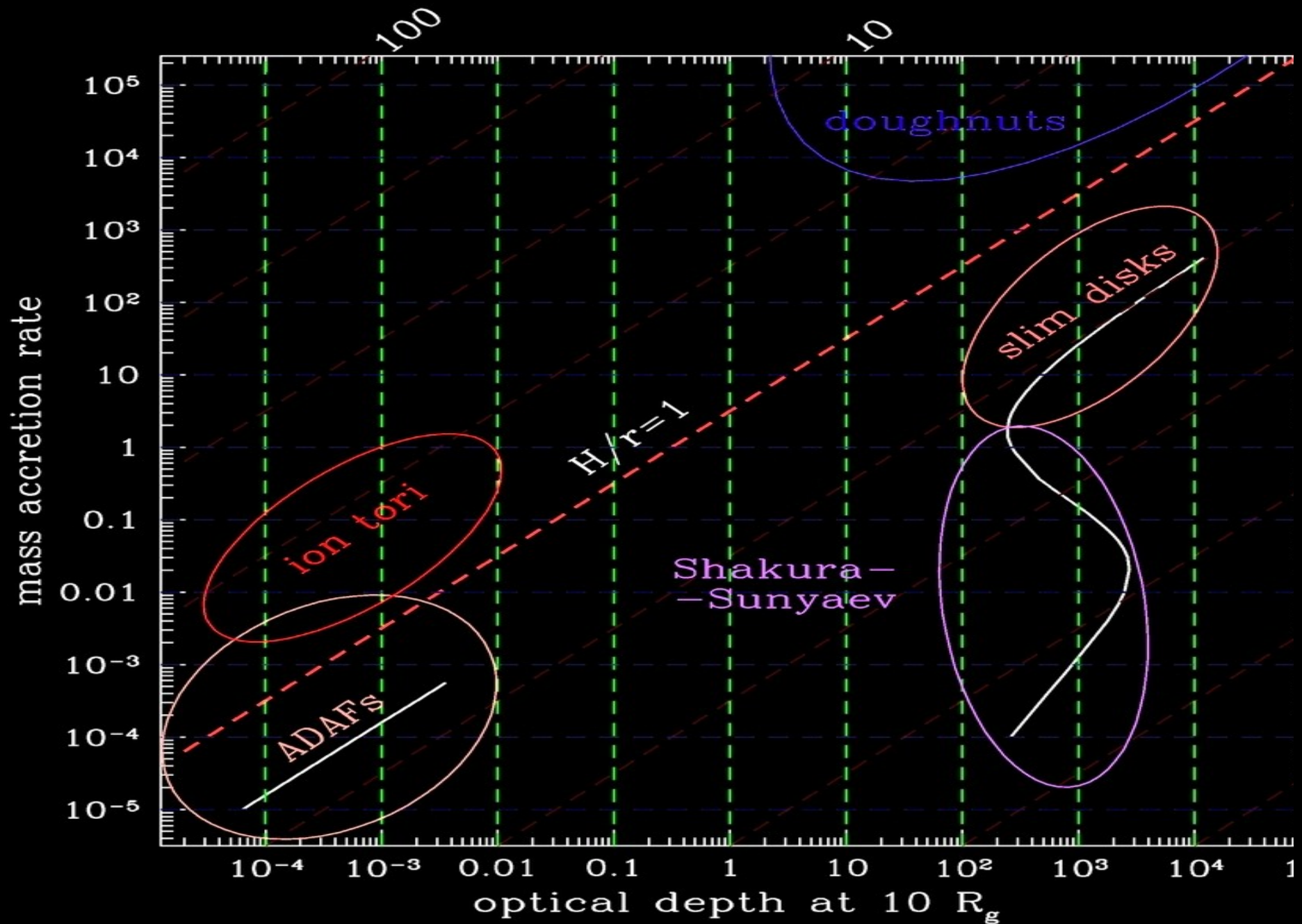
$$\frac{L\sigma_T}{4\pi r^2 c} = \frac{GMm_p}{r^2}$$

$$\Rightarrow L_{Edd} = \frac{4\pi cGMm_p}{\sigma_T} = 1.2 \times 10^{38} \left(\frac{M}{M_{sun}} \right) \text{ erg/s}$$

The Eddington mass accretion rate:

$$L_{Edd} = \eta c^2 \dot{M}_{Edd}$$

Supplement: Modelling accretion discs



(Abramowicz & Straub, Scholarpedia, 2012)

3. Luminous accretion: Slim disc equations

1) mass conservation

V , gas radial velocity

$$\dot{M} = -2\pi\Sigma\Delta^{1/2} \frac{V}{\sqrt{1-V^2}}$$

2) radial momentum conservation

$$\frac{V}{1-V^2} \frac{dV}{dr} = -\frac{MA}{r^2\Delta\Omega_K^+\Omega_K^-} \frac{(\Omega - \Omega_K^+)(\Omega - \Omega_K^-)}{1 - \Omega_K^2 R}$$

3) angular momentum conservation

$P=2H_p$, vertically integr. pressure

$\alpha=0.1$

$$\frac{\dot{M}}{2\pi} (\mathcal{L} - \mathcal{L}_{in}) = \frac{\sqrt{A\Delta}\gamma}{r} \alpha P$$

4) energy conservation

$$F^+ - F^- = -\frac{\alpha P A \gamma^2}{r^3} \frac{d\Omega}{dr} - \frac{32\sigma T^4}{3\kappa\Sigma}$$

5) vertical equilibrium

ϵ , conserved energy

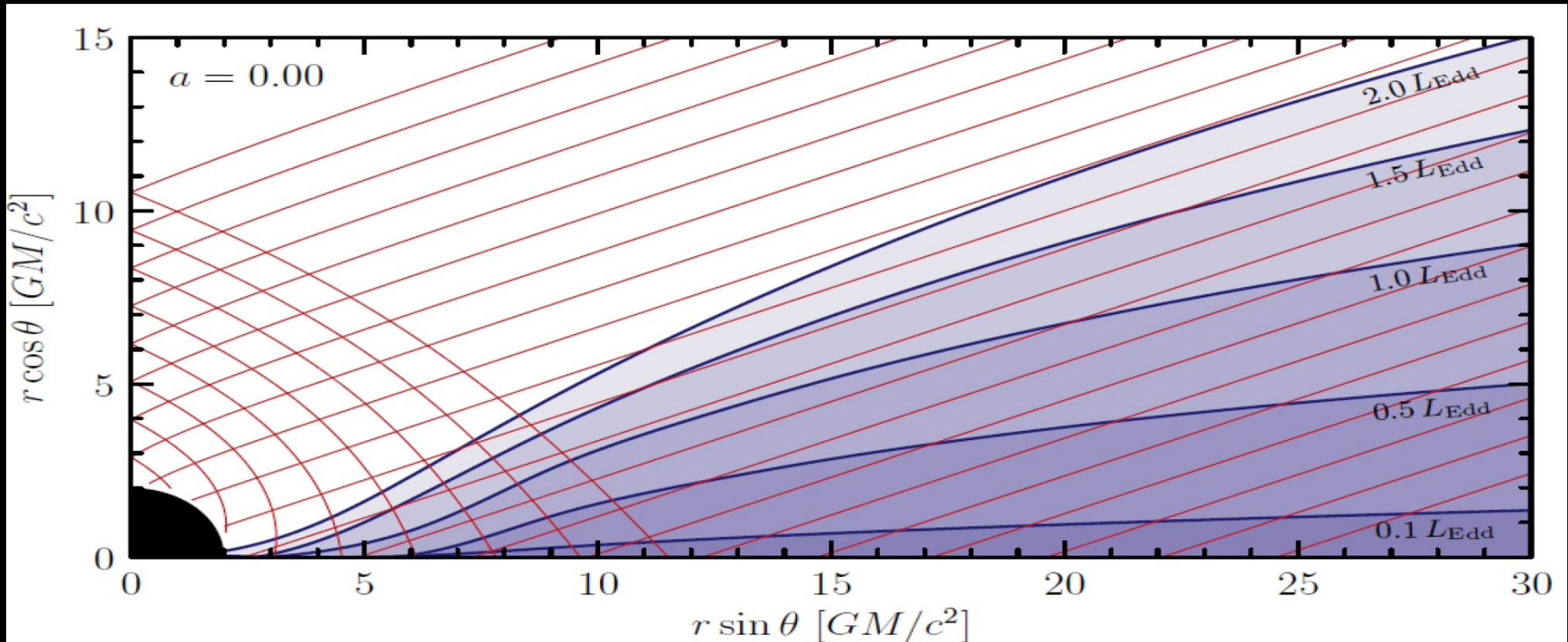
$$\frac{P}{\Sigma H^2} = \frac{\mathcal{L}^2 - a^2(\epsilon^2 - 1)}{2r^4}$$

3. Luminous accretion: Advection

Slim discs are cooled by advection.

Advection sweeps some of the emitted energy along with the flow because photons are trapped in the optically thick disc.

The photons can be released again at lower radii as the material accelerates towards the black hole



(A. Sadowski)

3. Luminous accretion: What is different?

THIN discs

Keplerian rotation

inner disk terminates at the ISCO

all locally released energy (photons) is radiated away

Shakura & Sunyaev (1973)

SLIM discs

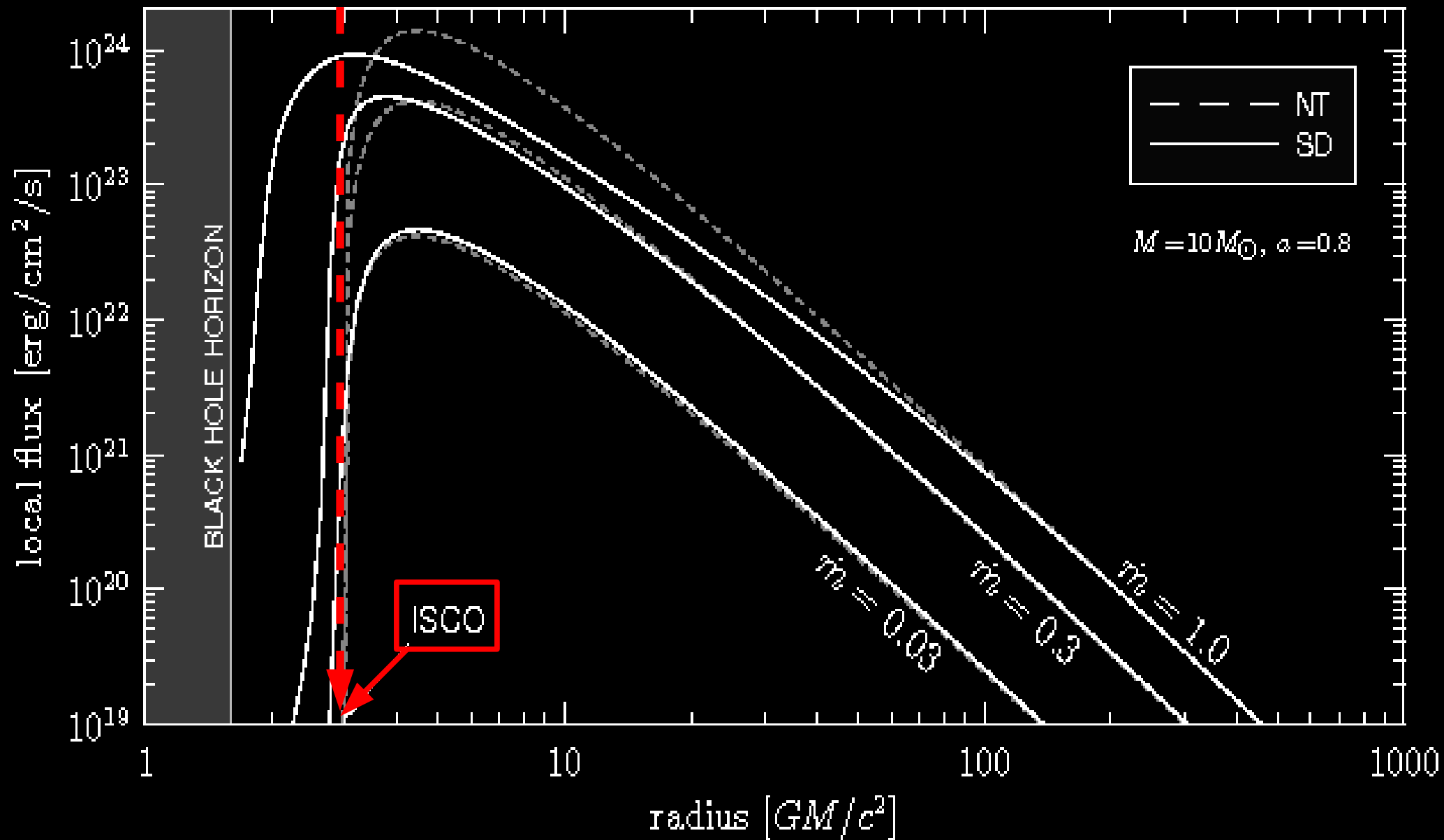
super-Keplerian rotation in the innermost part of the disk, otherwise sub-Keplerian

inner disk edge can be closer to the BH than the ISCO (for higher accretion rates)

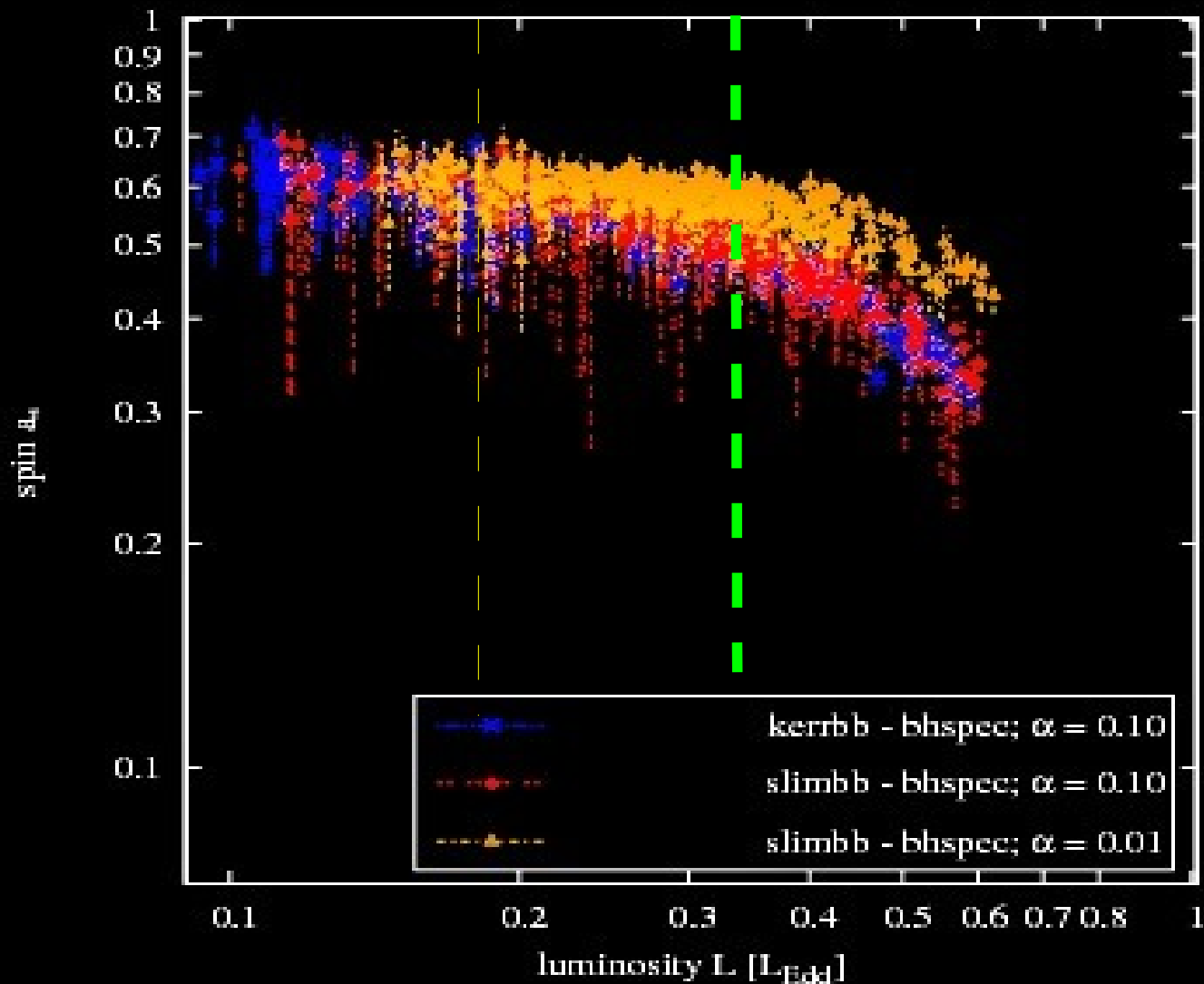
a part of the dissipated energy is captured in the flow and advected inwards

Abramowicz et al. (1988)

3. Luminous accretion: Flux

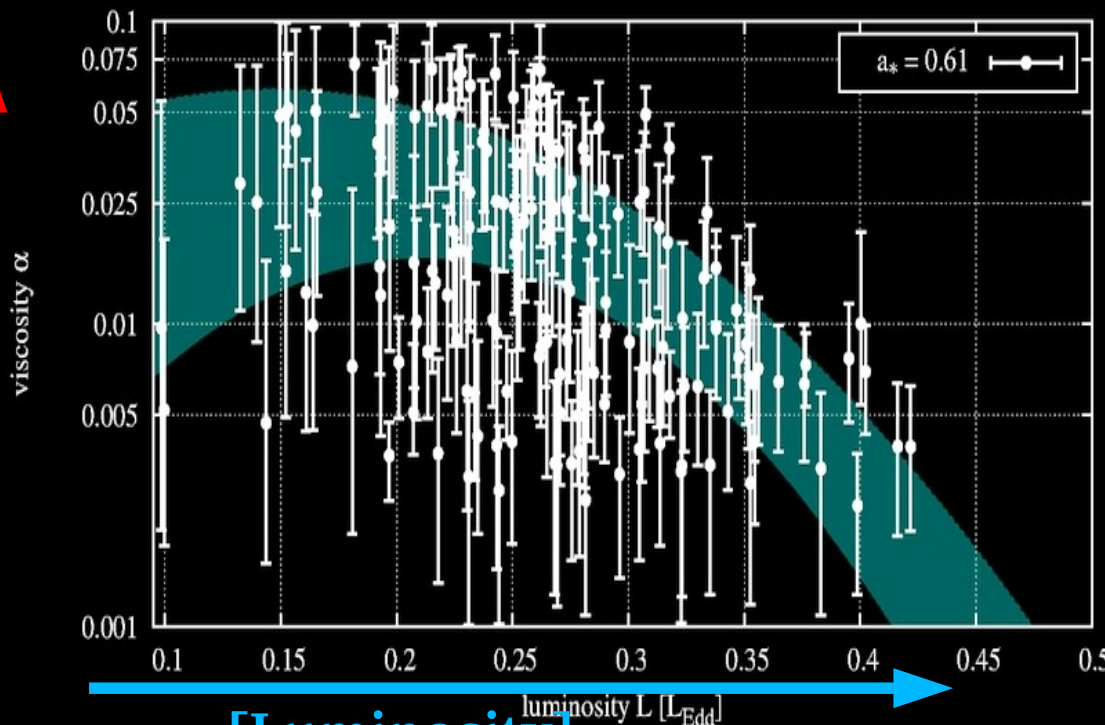


3. Luminous accretion: Advection



The spin is STILL not constant for all luminosity? → Go back to the drawing board!

3. Luminous accretion: Viscosity

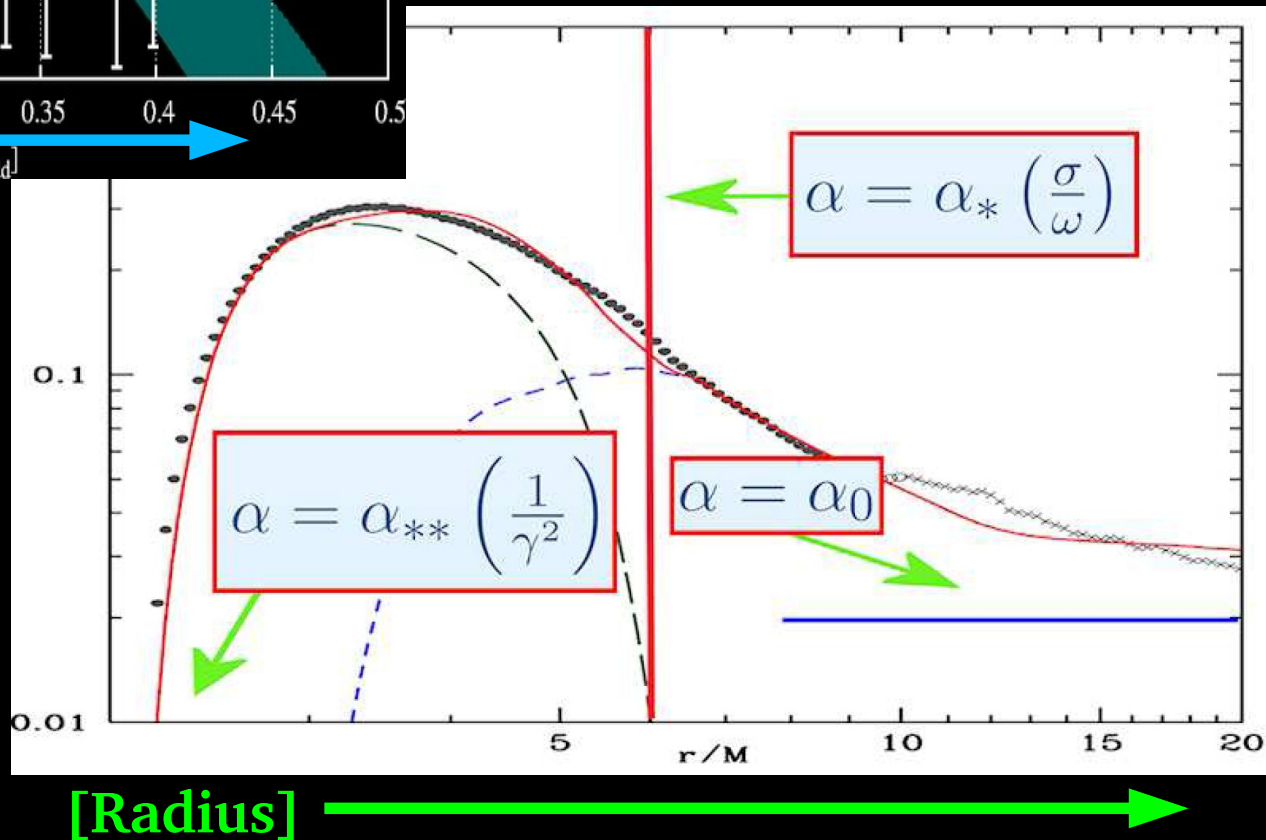


(Penna et al., 2013)

[Luminosity]

α form data

α form GRMHD simulations



[Radius]

3. Luminous accretion: ULX

Great Nebula in Andromeda (M 31 - NGC 224)

Type: **Galaxy**
Magnitude: 3.4
RA/DE (J2000): 0h42m42.0s/+41°16'00.0"
RA/DE (of date): 0h43m25s/+41°20'16"
Hour angle/DE: 6h37m14s/+41°20'16" (geometric)
Hour angle/DE: 6h37m10s/+41°21'21" (apparent)
Az/Alt: +300°46'53"/+30°24'31" (geometric)
Az/Alt: +300°46'53"/+30°25'52" (apparent)
Size: +2°58'00"



3. Luminous accretion: ULX

CXOM31 J004253.1+411422 (M31 ULX-1
hereafter)

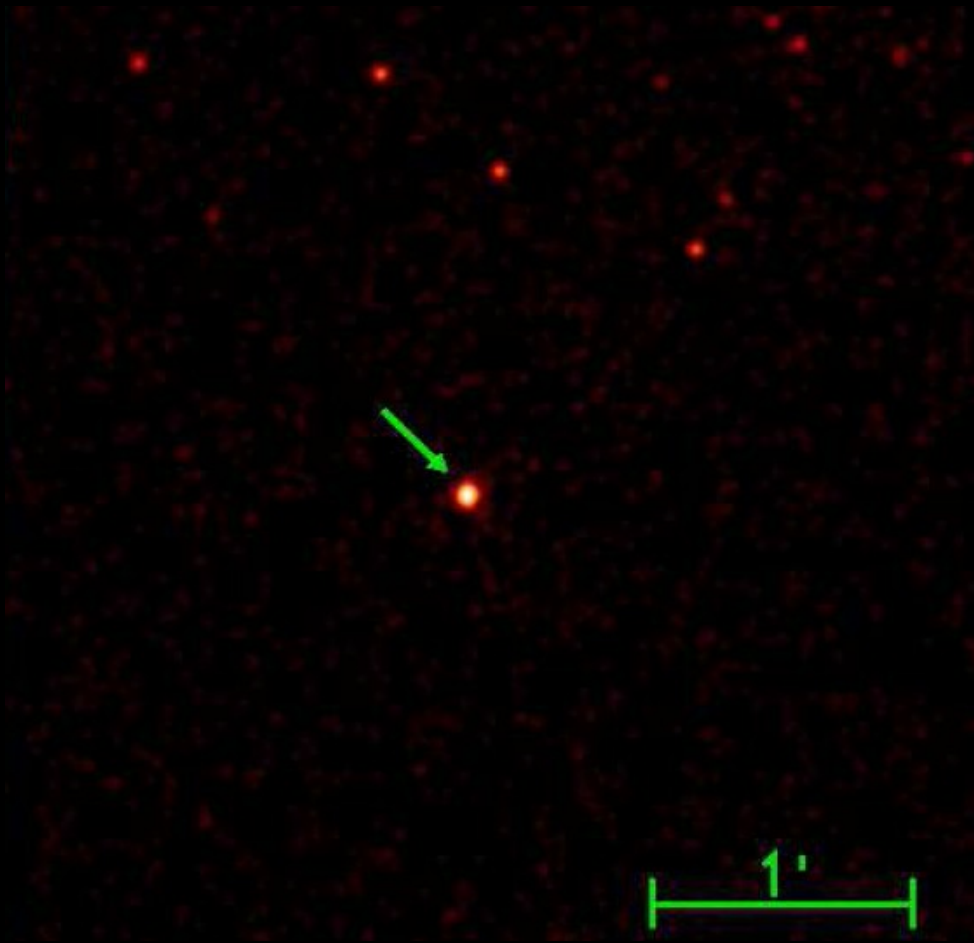
1. $L > 10^{39}$ erg/s
(roughly L_{Edd} of a BH of $20 M_{\text{sun}}$)

2. Distinct from centre of galaxy

3. Not coincident with background AGN or
QSOs

4. Often found in star-forming regions
(Gao et al. 2003)

The fact that ULXs have Eddington luminosities larger than that of stellar mass objects implies that they may be different from normal X-ray binaries. - There are several models for ULXs, and it is likely that different models apply for different sources.



(Chandra: M31 ULX-1)

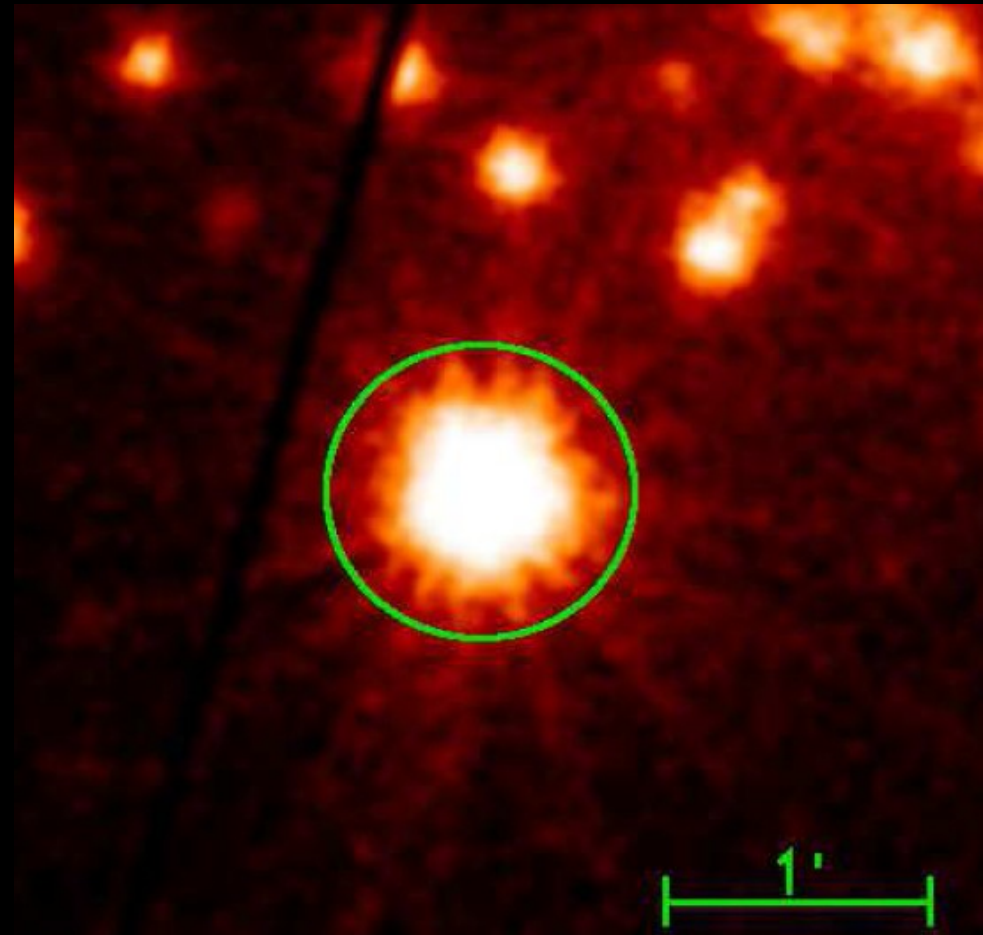
3. Luminous accretion: ULX

Solution 1: Super-Eddington accretion onto stellar mass BHs

=> This would mean that Eddington accretion can be readily observed

Solution 2: Sub-Eddington accretion onto 'Intermediate mass BHs'

=> Need new physics for their formation (potentially seen already in GCs: Maccarone & Servillat 2008)



(XMMN: M31

ULX 1)

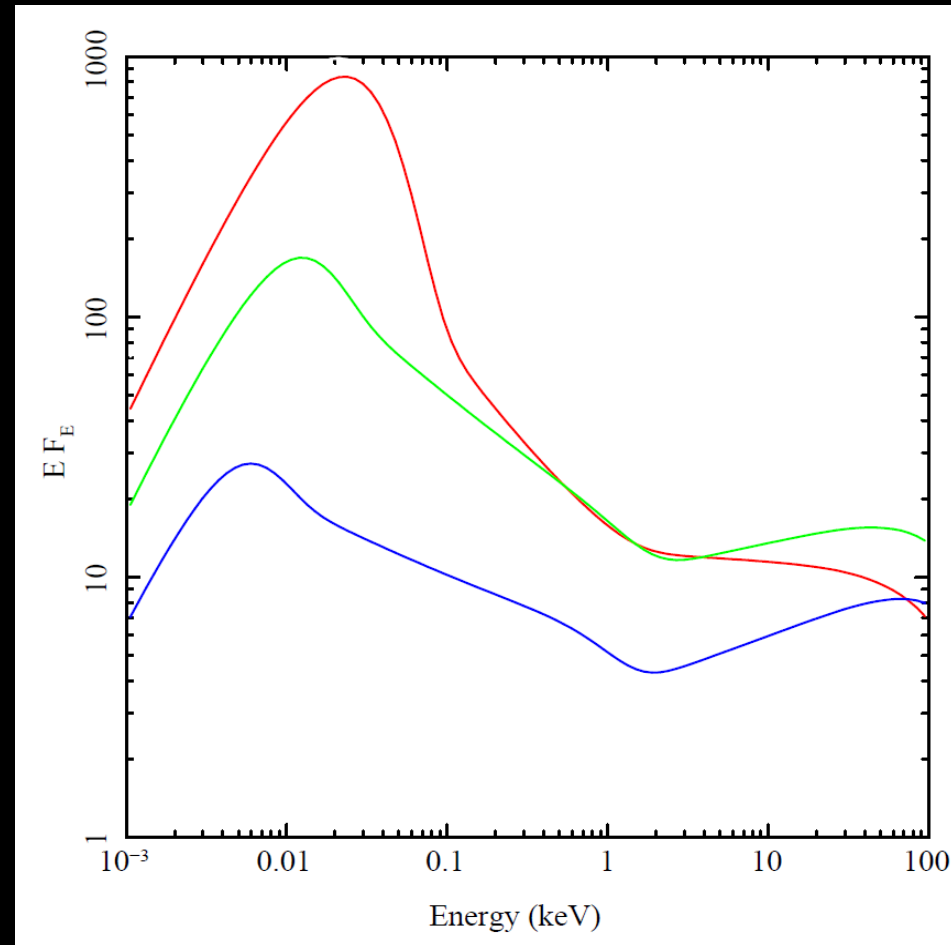
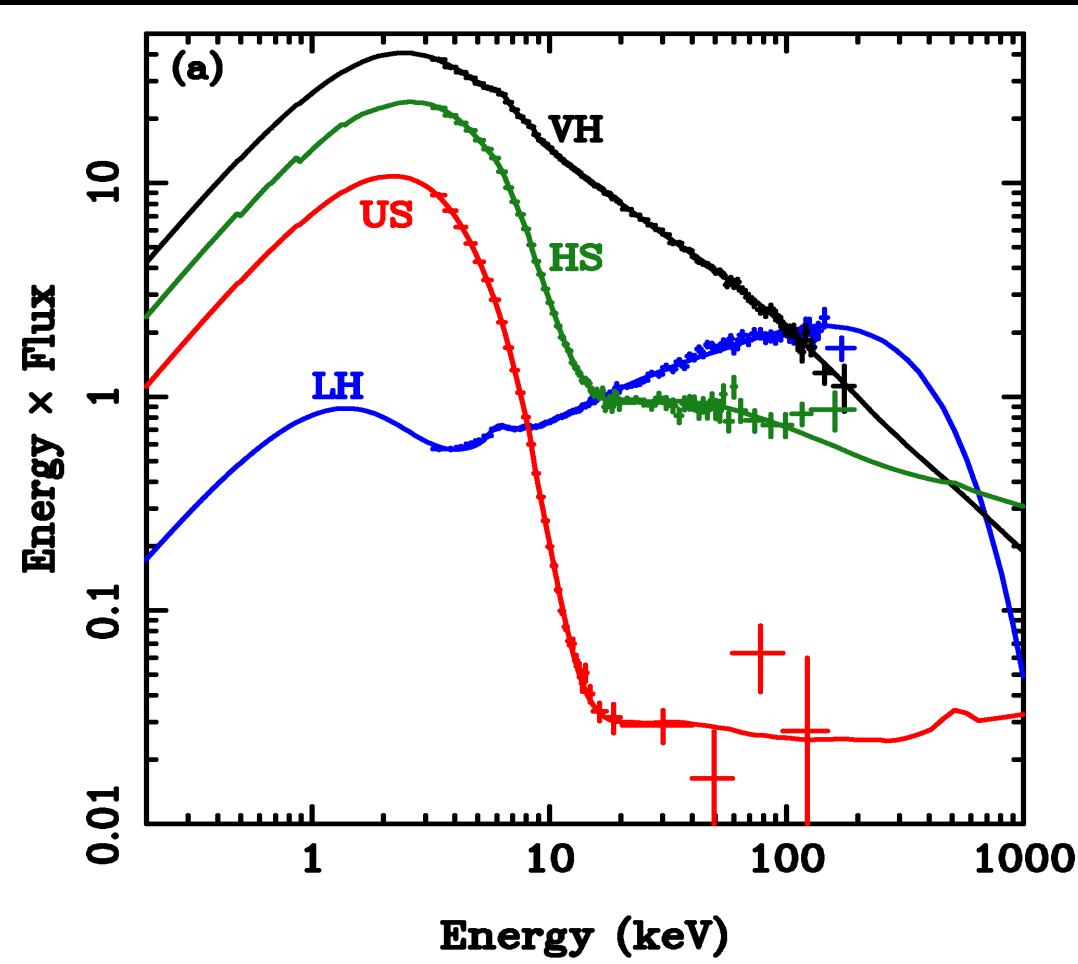
3. Luminous accretion: ULX

How can we distinguish between the two solutions?

Axiom of accretion physics: **Timing and spectral properties scale with mass**

* Timing: Features of PDS shifted in frequency due to mass

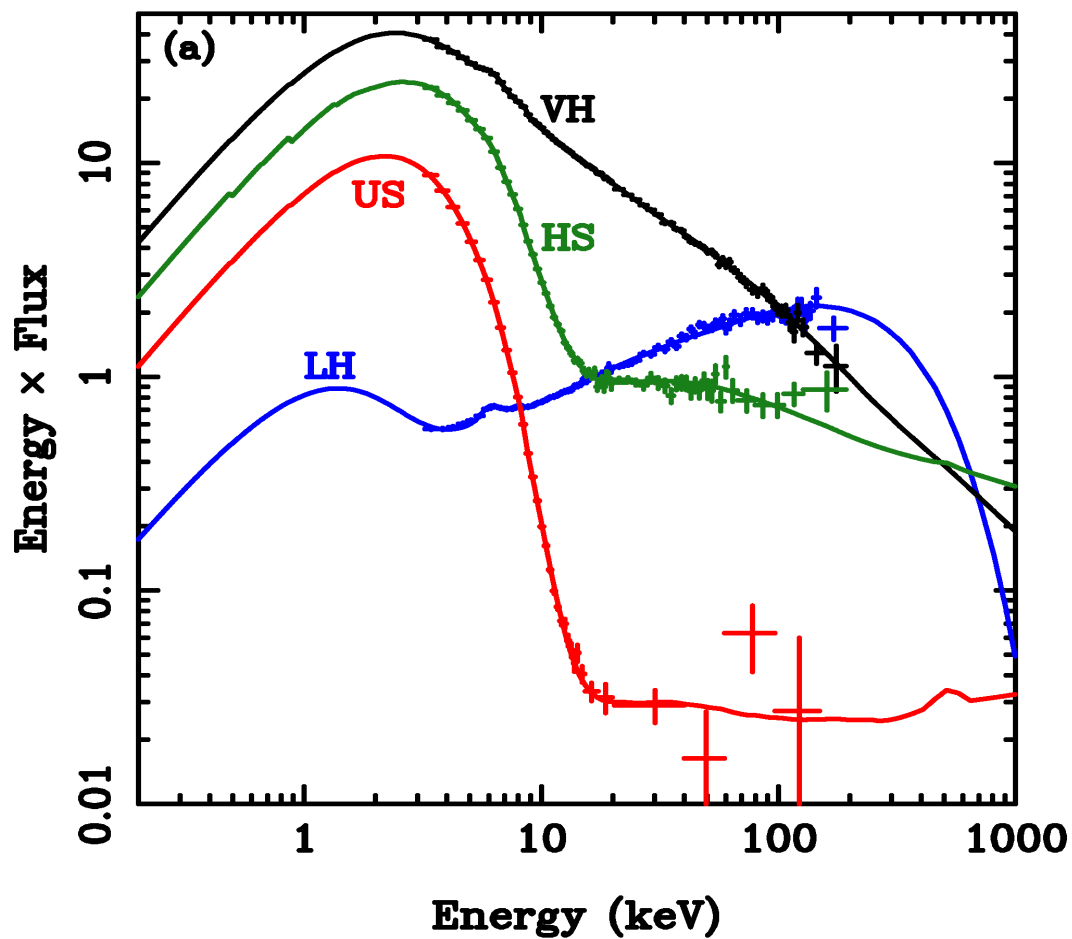
* Spectra:



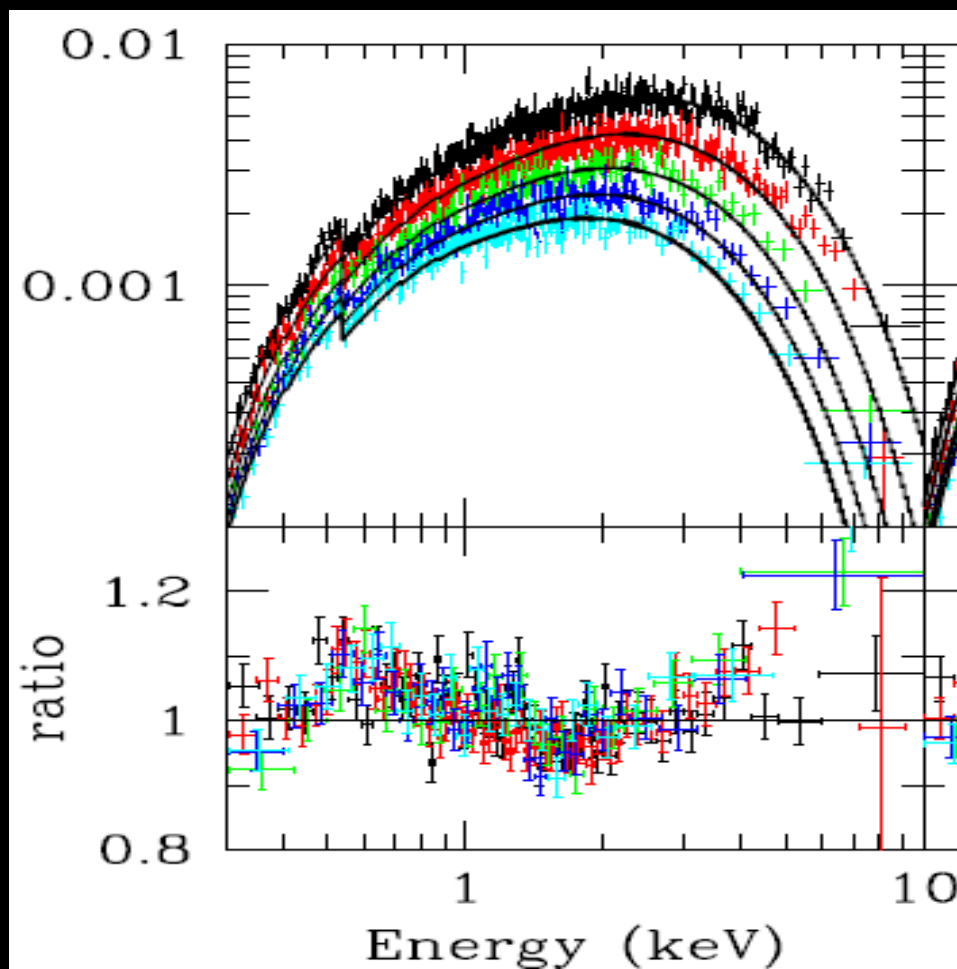
(XRB spectra: Done et al., 2007)

(AGN spectra: Done et al., 2011)

3. Luminous accretion: ULX

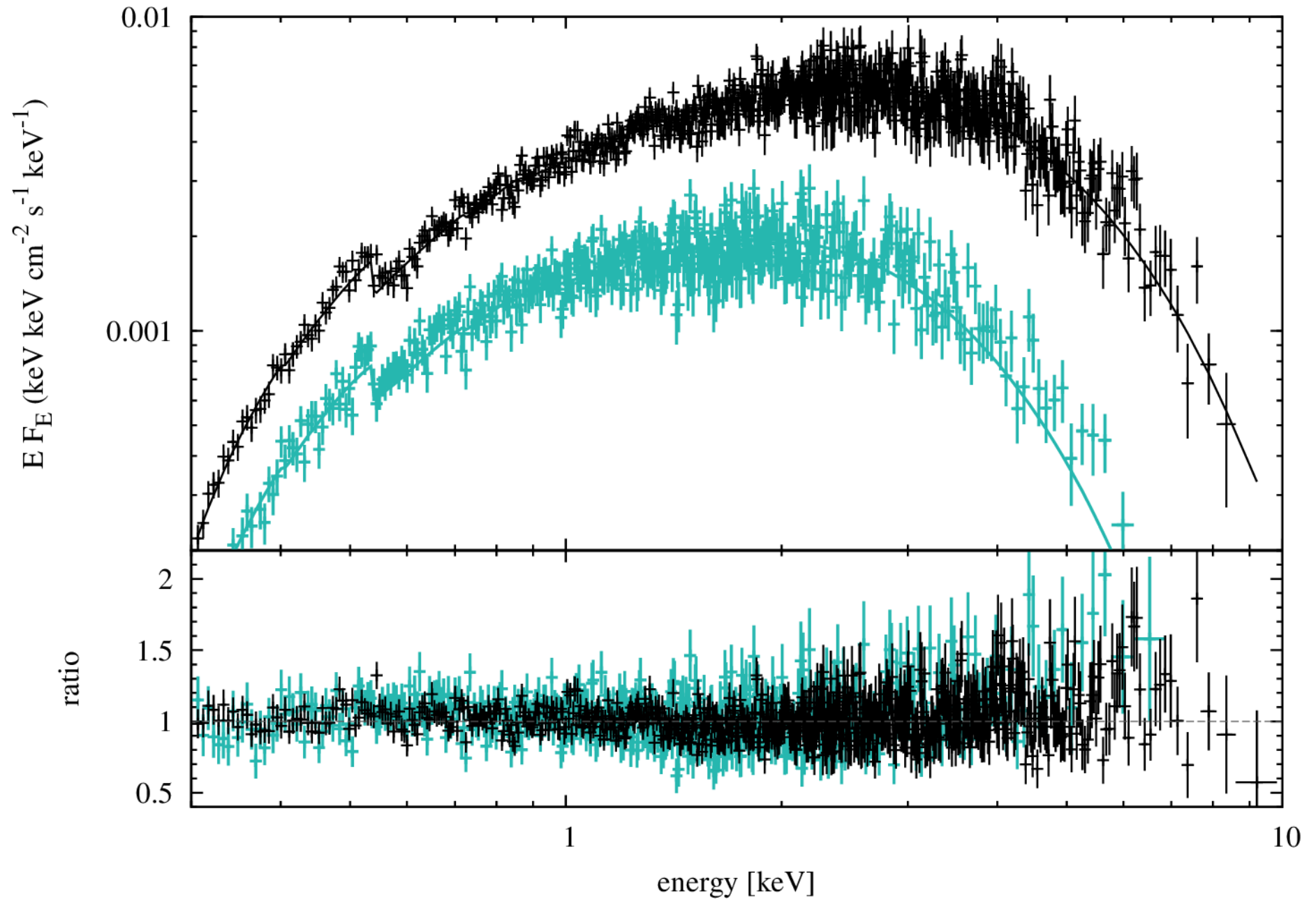


(XRB spectra: Done et al., 2007)



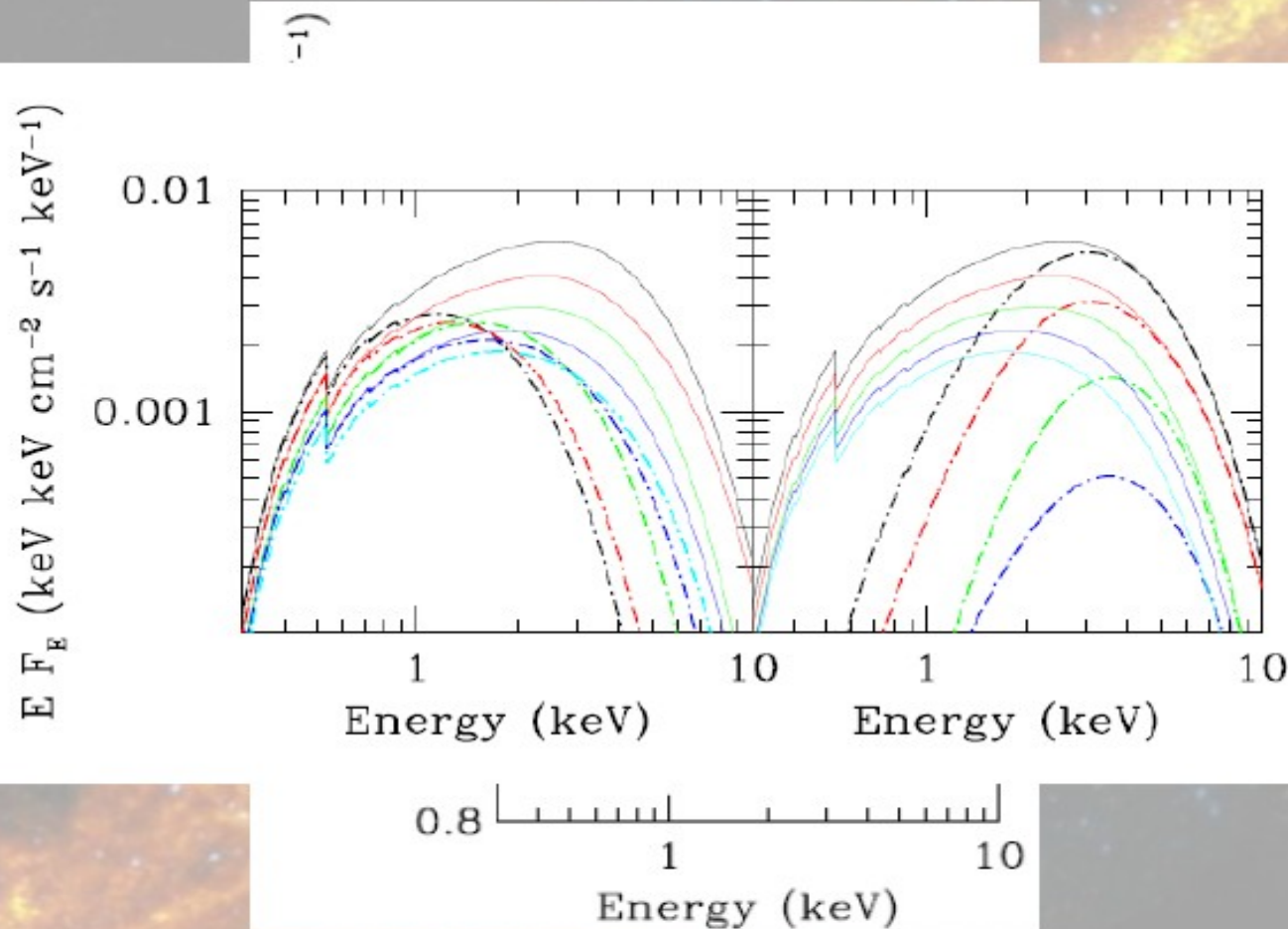
(ULX-1 spectra: Middleton et al., 2012)

3. Luminous accretion: Interpreting data



M31 ULX-1: Middleton et al. 2012

ULX model – better, can incorporate intrinsic absorption

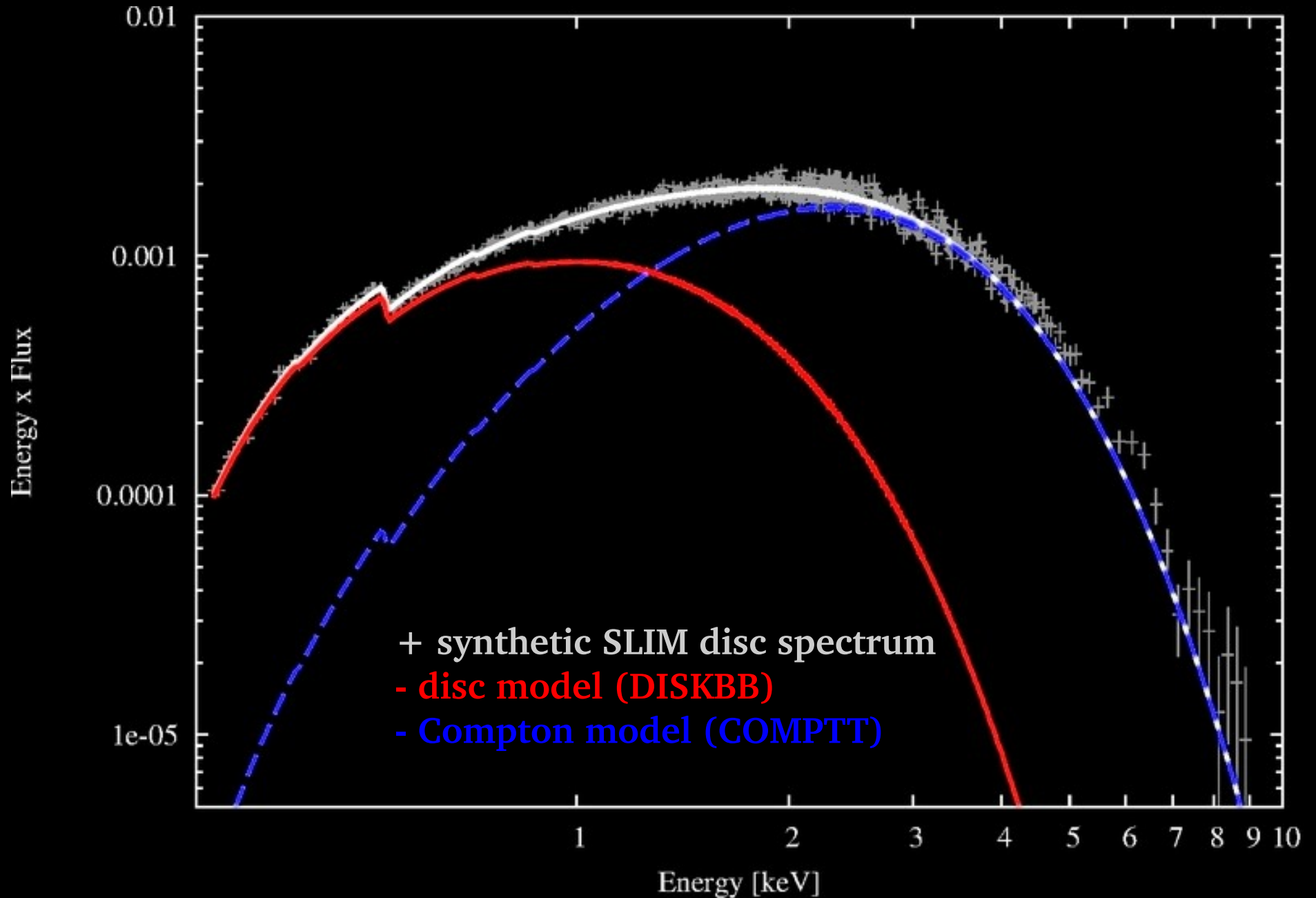


With decreasing luminosity the disc gets hotter and more advection dominated, the 'wind' component gets hotter and less important

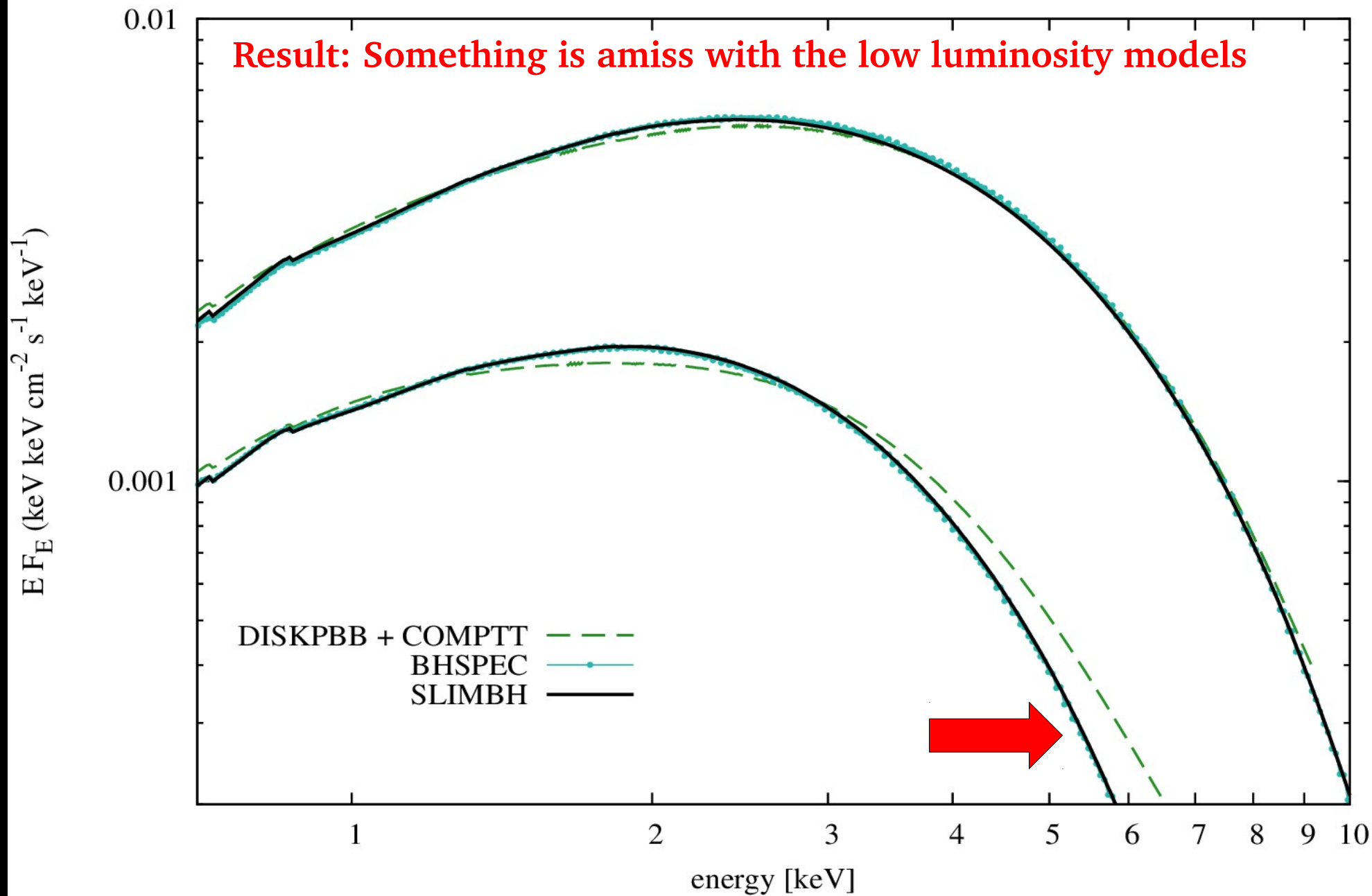
Best fit with 2 model components.

BUT: more parameters = more degrees of freedom! → can fit ANYTHING

3. Phenomenological vs physical models



3. Luminous accretion: ULX



3. Luminous accretion: Conclusion

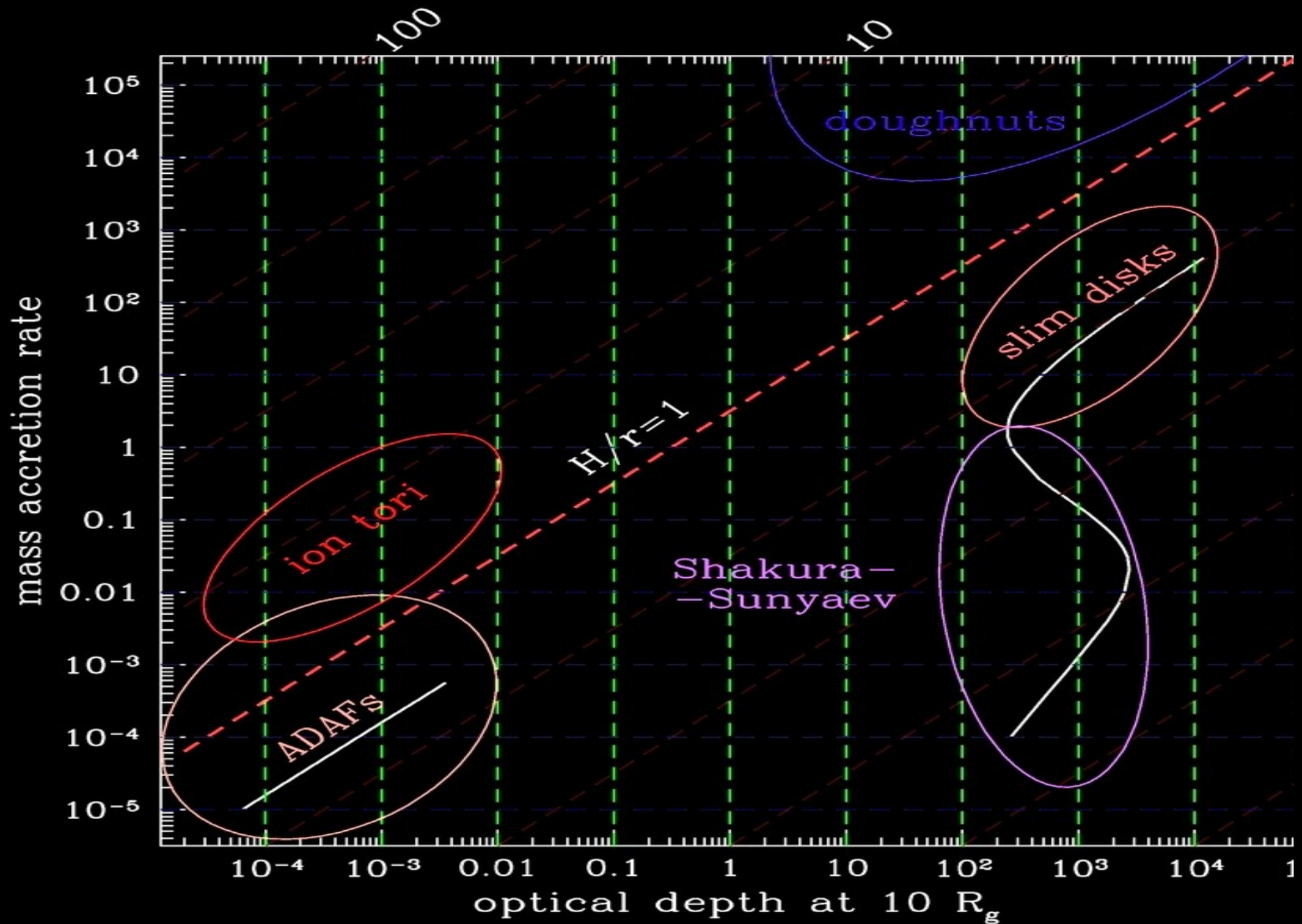
It has long been assumed that in the trans-Eddington luminosity regime two effects could become increasingly important for accretion disc models,

- (i) advection (Abramowicz et al. 1988; Mineshige et al. 2000) and/or
- (ii) outflows (Shakura & Sunyaev 1973; Poutanen et al. 2007; Ohsuga et al. 2009).

- Advection makes very little difference to spectra below the Eddington luminosity.
- Model/Disc comparison show exactly opposite behaviour with respect to luminosity than expected from either winds, advection, bulk turbulence and/or inhomogeneities in the disc.
- Instead **variable alpha** and most likely a **missing physical process(es)** ...

Solution? → Find a better viscosity prescription & include e.g. magnetic fields

Supplement: Modelling accretion discs



(Abramowicz & Straub, Scholarpedia, 2012)

4. Journey into the heart of the galaxy



A few 100 pc - optical

4. Journey into the heart of the galaxy



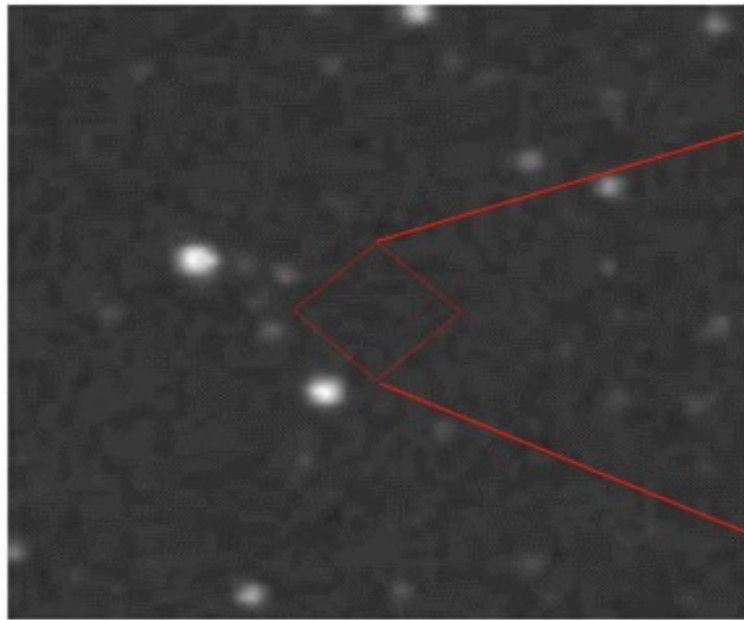
A few 100 pc - infrared

4. Journey into the heart of the galaxy

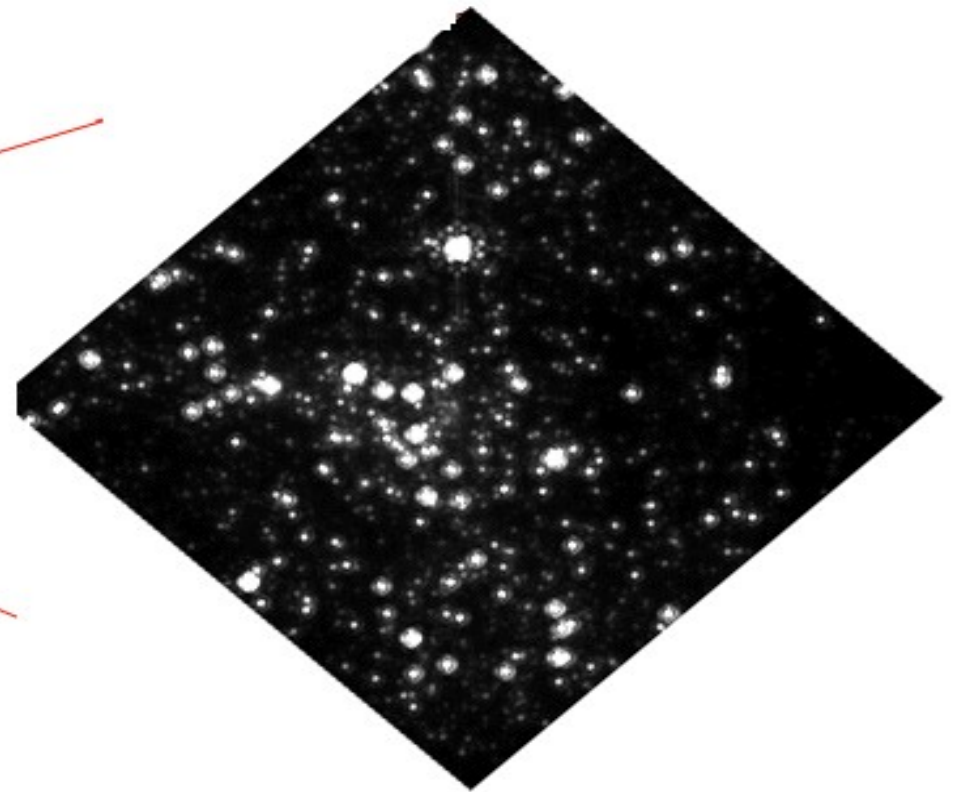


150 pc = 1° (Chandra)

4. Journey into the heart of the galaxy



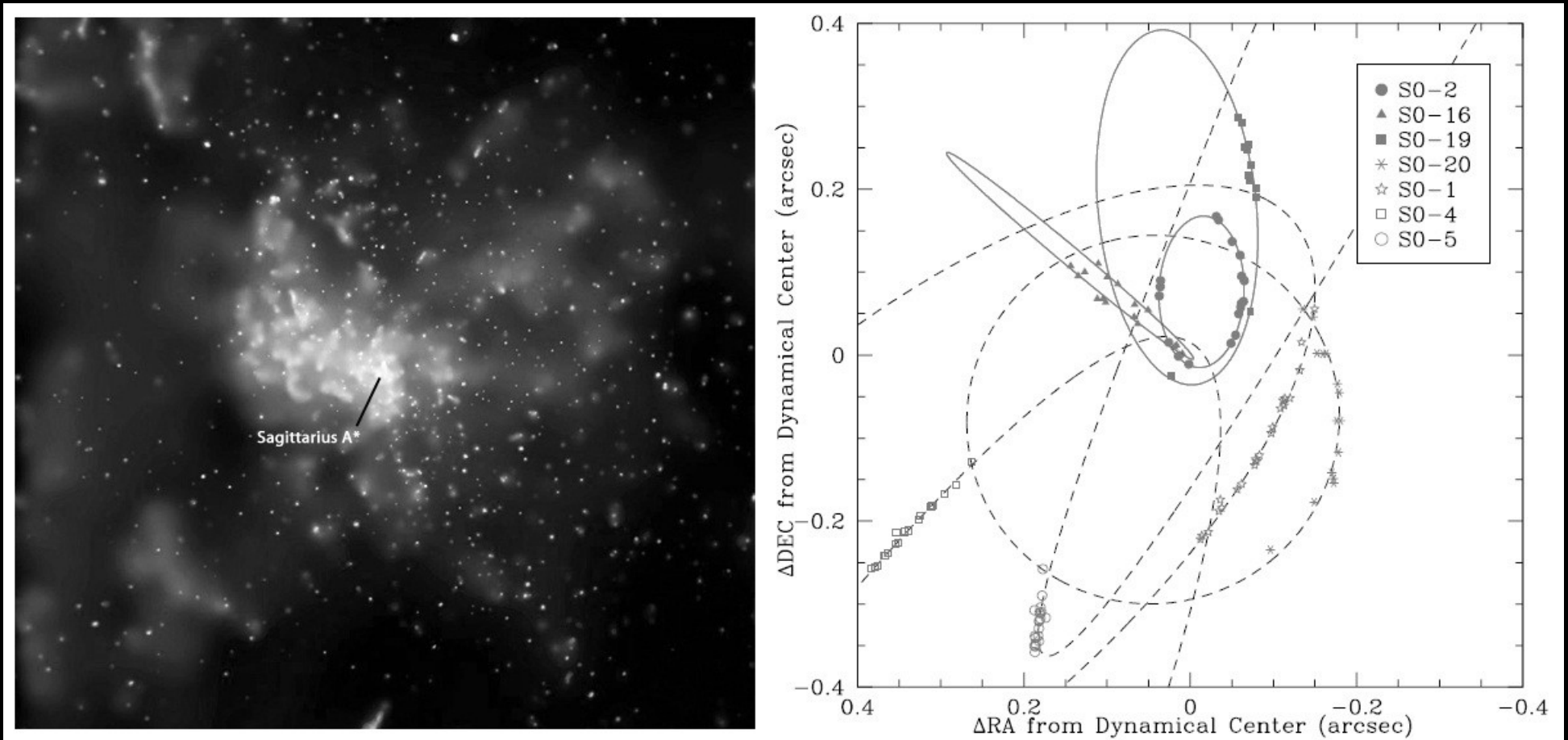
Visible Light



Infrared Light

few 10' (Chandra)

4. Journey into the heart of the galaxy



1" (VLT)

→ to resolve the horizon of Sgr A* we need **MICRO** ARCSECONDS

4. Journey into the heart of the galaxy



(EHT, 2020)

4. Geometrically thick disc: Doughnut family

construction of a geometrically THICK disc model: The Polish Doughnut.

* Euler equation of a perfect fluid
[A := -u_t]

$$\frac{\nabla_{\mu} p}{p + \rho} = -\nabla_{\mu} \ln A + \frac{l \nabla_{\mu} \Omega}{1 - \Omega l}.$$

* Von Zeipel condition
→ simplest solution: $l = \text{const.}$

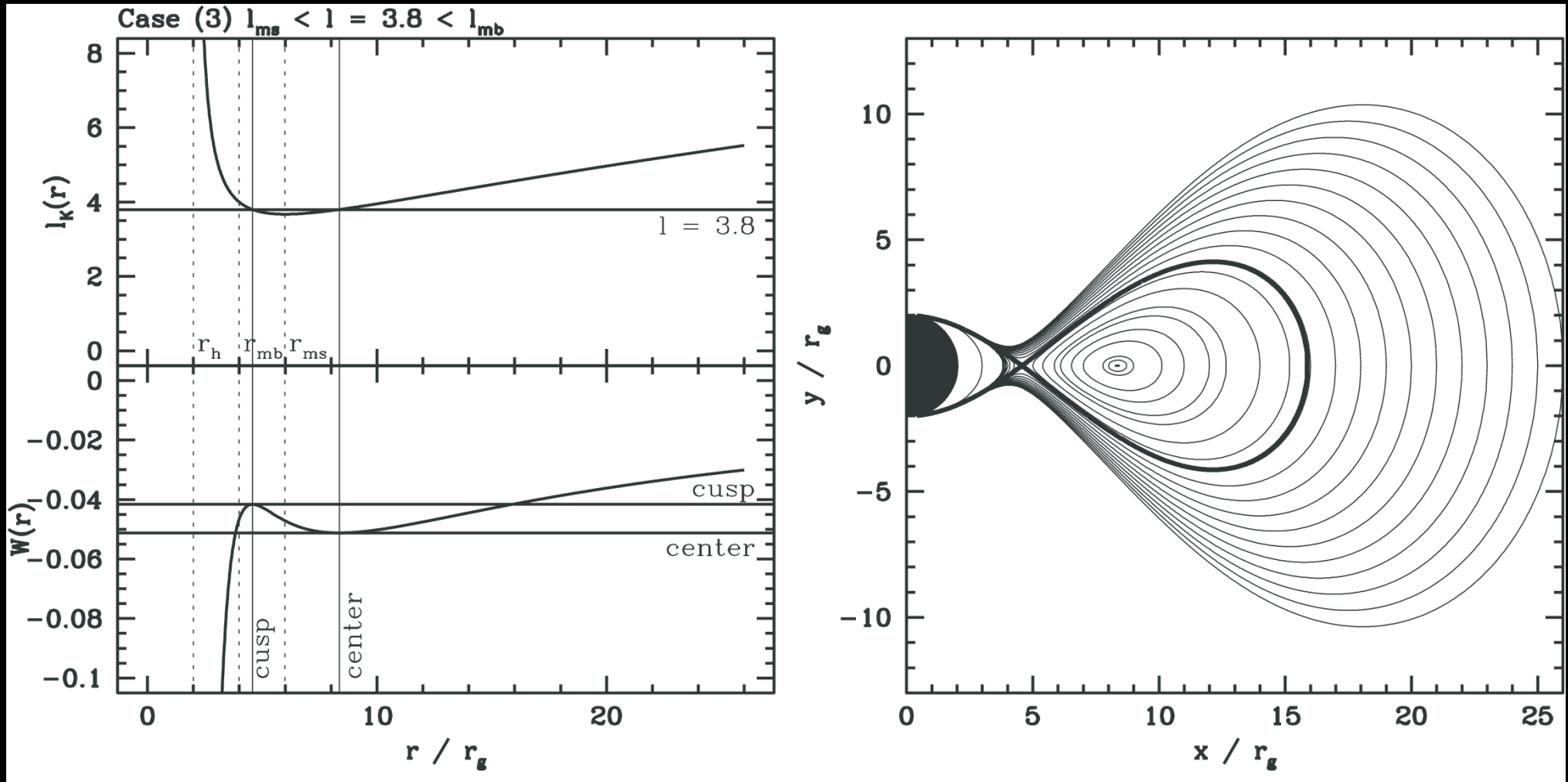
$$p = p(\rho) \iff l = l(\Omega)$$

→ define potential function:

$$\nabla_{\mu} \mathcal{W} \equiv -\nabla_{\mu} \ln A + \frac{l \nabla_{\mu} \Omega}{1 - \Omega l}.$$

→ find equipotential surfaces

4. Geometrically thick disc: Doughnut family

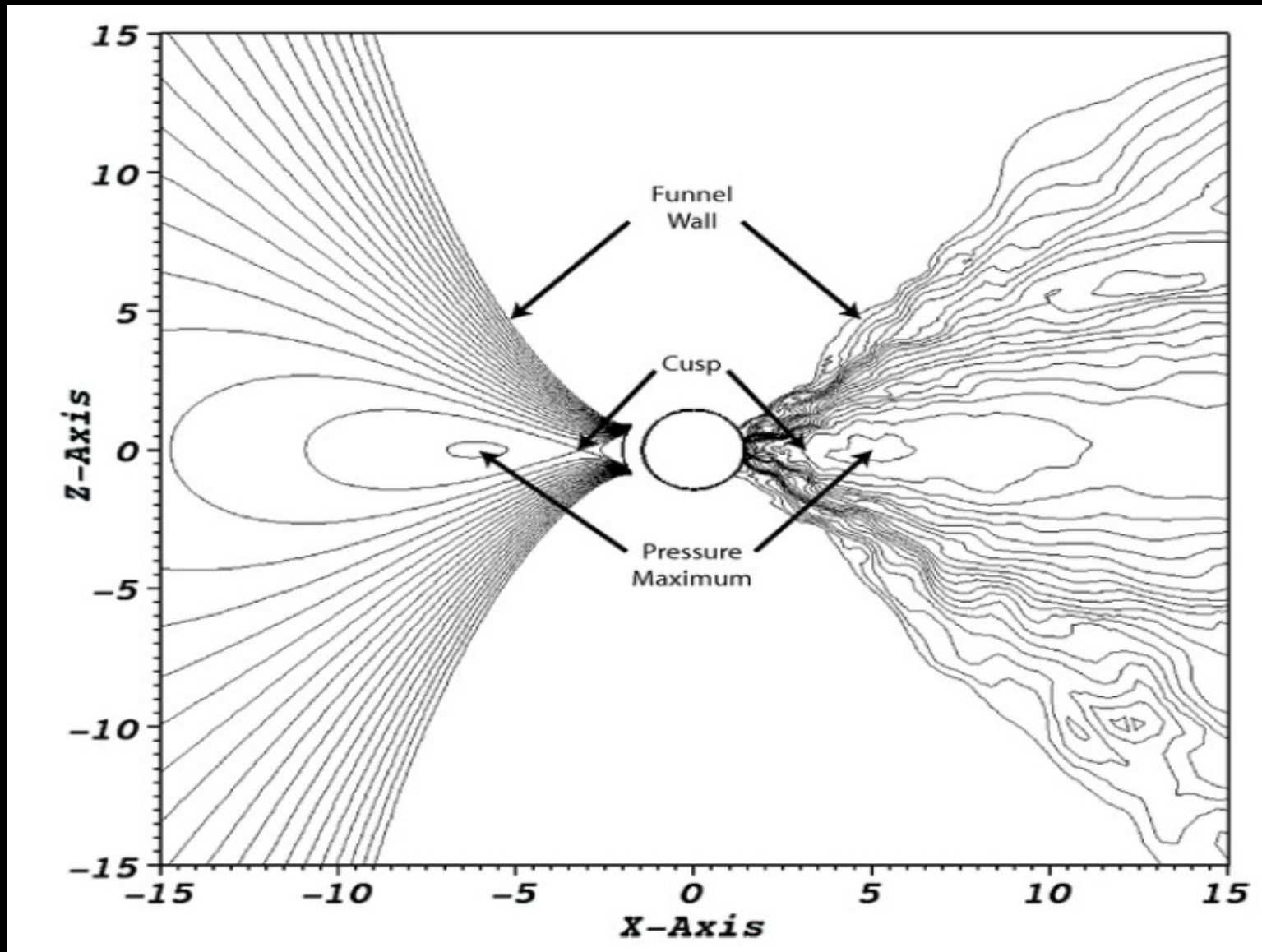


(Abramowicz & Fragile, Living Reviews, 2013)

→ torus surface is the self-crossing equipotential surface

→ depending on choice of angular momentum ($l=\text{const.}$) we get different size tori

4. Torus model vs MHD simulations



(Qian et al., 2009)

- Analytical structure, few simple parameters
- Reproduces basic features of sophisticated GRMHD simulations

4. Optically thin disc: Ion torus family

* ION TORI are optically thin and advection dominated like ADAFs.

→ magnetic pressure

$$p_{tot} = p_{gas} + p_{mag}$$

with

$$p_{mag} = \beta p_{tot}$$

p_{gas} = perfect gas law
for electrons and ions

→ EOS for total gas pressure: polytropic

$$p_{tot} = \kappa (\rho c^2)^{1+1/n}$$

$$p_{mag} = \beta * p_{tot}$$

→ express all thermodynamic quantities in terms of the potential function W

→ Radiative processes: Bremsstrahlung & synchrotron emission, their inverse Compton scattering

→ Emission coefficient given by the radiative power of the above radiative processes (as in ADAFs, e.g., Narayan & Yi, 1995)

$$dE = j_{\nu} dV d\Omega dt d\nu$$

→

$$j_{\nu} = j_{\nu}^{br} + j_{\nu}^{sy} + j_{\nu}^{br,c} + j_{\nu}^{sy,c}$$

4. Optically thin disc: Radiative transfer

The transfer equation for radiation in the emitter's frame is (RL79)

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Intensity in the emitter's frame

To solve the equation one requires the emission and absorption coefficients

$$j_\nu = j_\nu^{br} + j_\nu^{sy} + j_\nu^{br,c} + j_\nu^{sy,c}$$
$$\alpha_\nu = \frac{j_\nu}{B_\nu} \text{ (negligible at ion torus temperatures)}$$

The transfer equation is **integrated by a ray-tracing code** to get the monochromatic intensity of the emitted photons (expressed in the emitter's frame).

4. Ray-tracing basics: Geodesic equation

Ray-tracing means integrating the null geodesics of photons from a distant observer's screen to the source.

A geodesic is given by the **geodesic equation**:

$$\ddot{x}^\alpha + \Gamma_{\mu\nu}^\alpha \dot{x}^\mu \dot{x}^\nu = 0$$

(derivative with respect to affine parameter λ) where

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2} g^{\alpha\sigma} (\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$$

are the **Christoffel symbols**.

4. Ray-tracing basics: Constants of motion

There are 3 obvious constants of (geodesic) motion:

$$m^2 = -p \cdot p$$

the photon's **mass** (scalar prod. between 4-vectors)

$$E = -p_t$$

the photon's **energy**

$$L = p_\phi$$

the photon's ϕ -component of the **angular mom.**

where $p^\mu = \dot{x}^\mu$ the photon's 4-momentum (E, L seen at infinity).

And there is the 4th constant: **Carter's Constant** (in Kerr)

$$Q = p_\theta^2 + \cos^2 \theta \left[a^2 (m^2 - p_t^2) + \frac{p_\phi^2}{\sin^2 \theta} \right]$$

These 4 constants allow us to rewrite the geodesic equation (in terms of the 4 constants) and get a system of 4 differential equations...

4. Ray-tracing basics

$$\Sigma \frac{dr}{d\lambda} = \pm \sqrt{R}$$

$$\Sigma \frac{d\theta}{d\lambda} = \pm \sqrt{\Theta}$$

$$\Sigma \frac{d\phi}{d\lambda} = -\left(aE - \frac{L}{\sin^2 \theta}\right) + \frac{a}{\Delta} P$$

$$\Sigma \frac{dt}{d\lambda} = -a(aE \sin^2 \theta - L) + \frac{r^2 + a^2}{\Delta} P$$

Ray-tracing codes integrate the above system over λ in order to determine the photon trajectory from its initial position, velocity (x_0^μ, p_0^μ) .

→ All ray-tracing codes follow these steps (formulations may vary, though).

4. Radiative transfer equation

To get the (monochromatic) intensity in the observer's frame

we use the frame invariant $\mathcal{I} = I_\nu / \nu^3$

and define $g \equiv \frac{\nu^{obs}}{\nu^{em}} = \frac{p^{obs} \cdot u^{obs}}{p^{em} \cdot u^{em}}$

where

p^{obs}, p^{em} is the photon's tangent vector to the geodesic in each frame and

u^{obs}, u^{em} is the respective 4-velocity.

The intensity measured by the observer is then

$$I_\nu^{obs} = g^3 I_\nu$$

4. Radiative transfer equation

A **map** of specific intensity in the observer's frame is called an **image**.

The observer's screen has N pixels. Each pixel is associated to one direction of incidence of the photons.

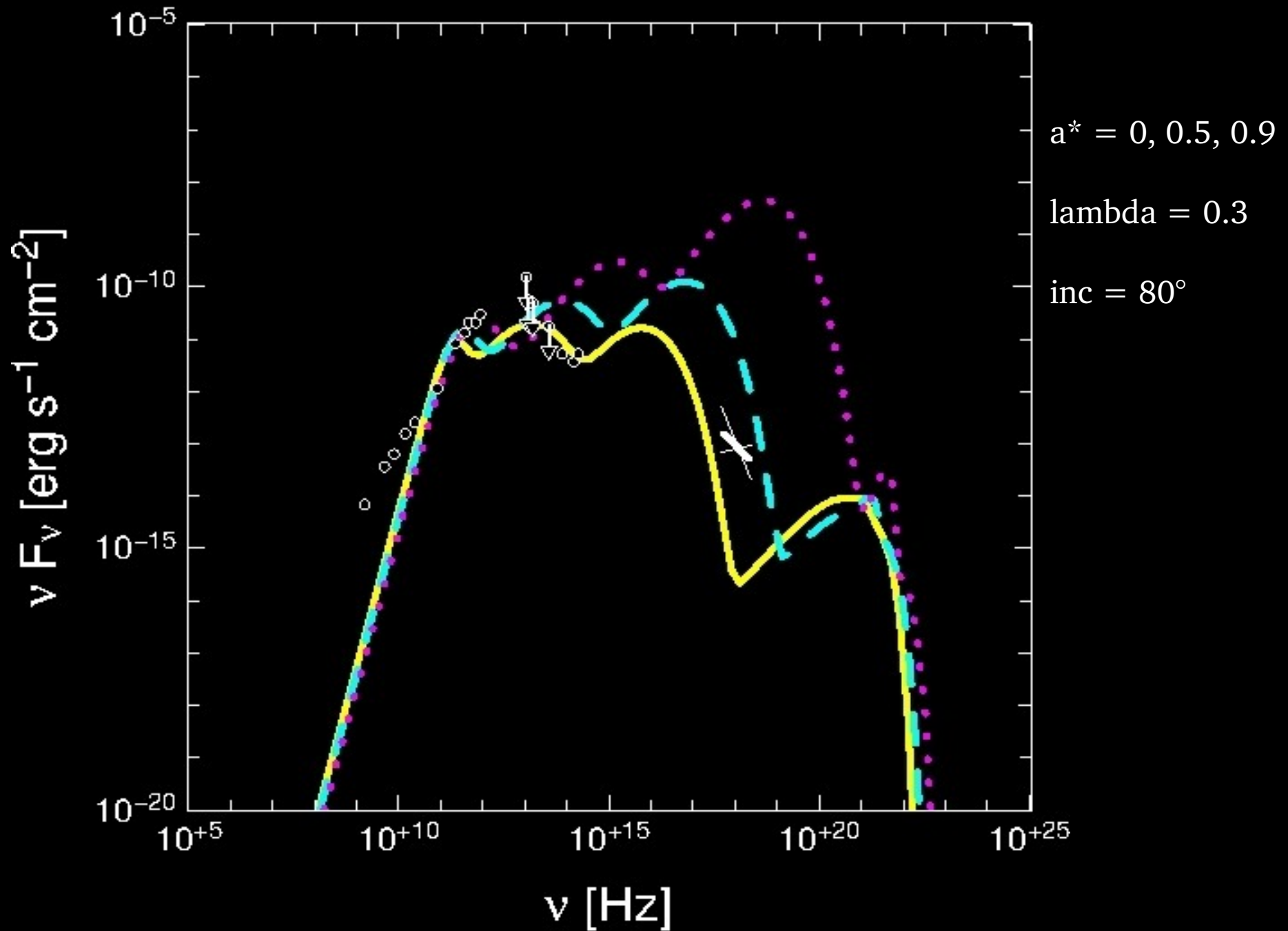
The **flux** is given by the integral over solid angle of the specific intensity. Computing the flux for a range of frequencies gives the **spectrum**.

$$dF_{\nu_{obs}} = I_{\nu}^{obs} \cdot \cos \theta \delta\Omega$$

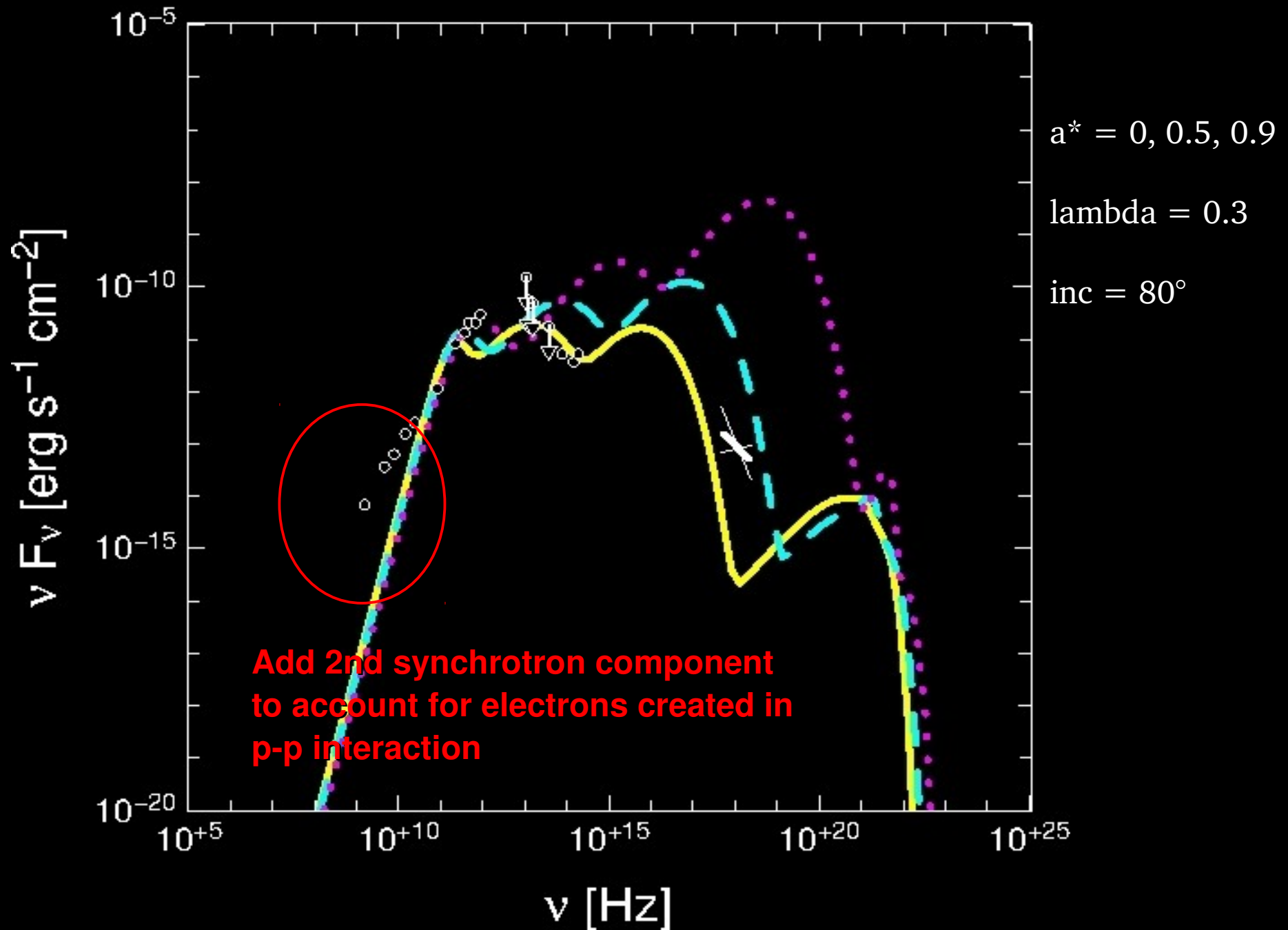
GYOTO code: A pixel on the screen corresponds to a direction on the sky, just like a pixel on a camera detector. **Flux is given by the sum over all pixels.**

(F. Vincent et al., 2011)

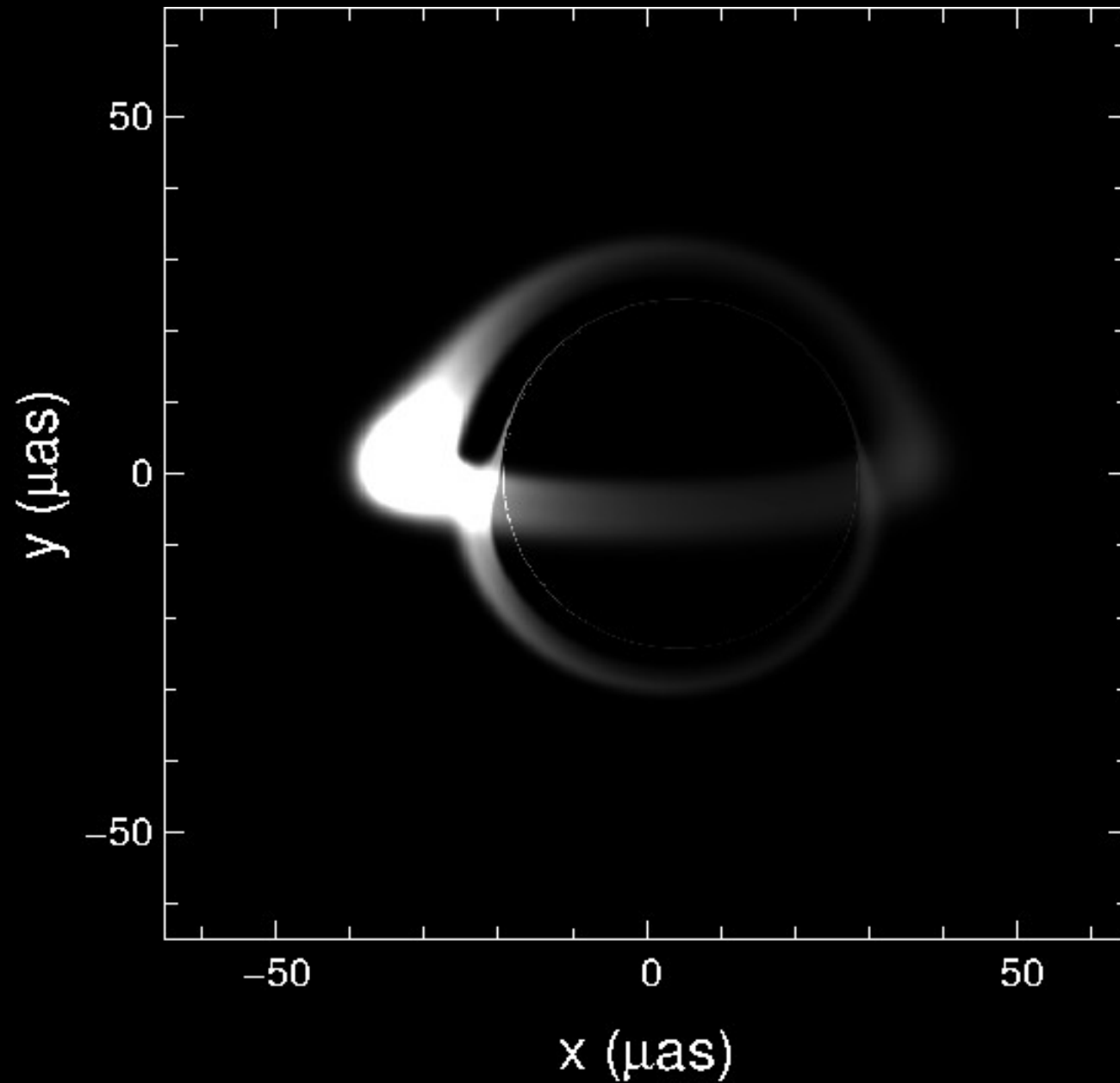
4. Dim accretion: Spectra



4. Dim accretion: Spectra



4. Dim accretion: Images

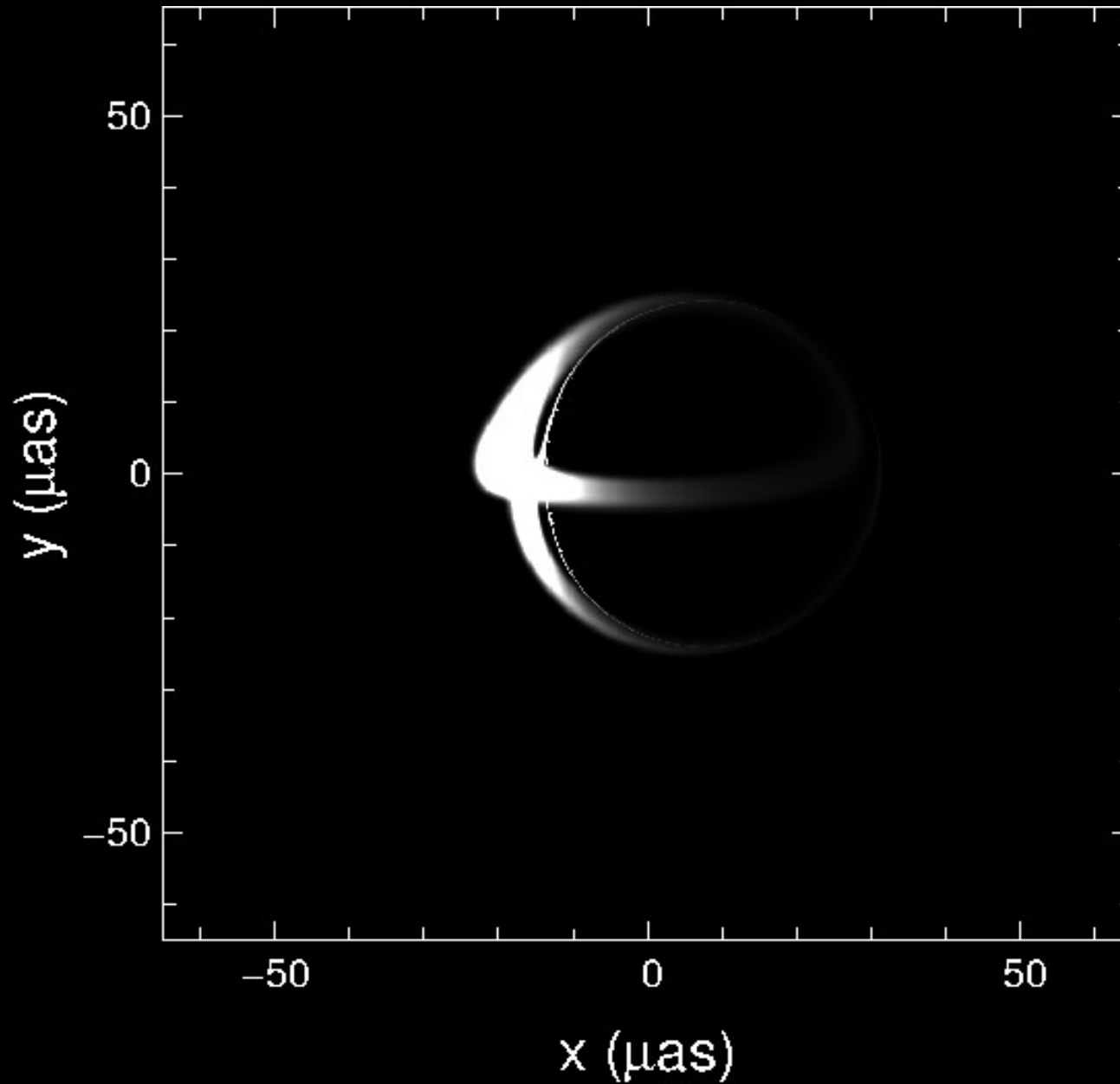


$$a^* = 0.5$$

$$\text{lambda} = 0.3$$

$$\text{inc} = 80^\circ$$

4. Dim accretion: Images

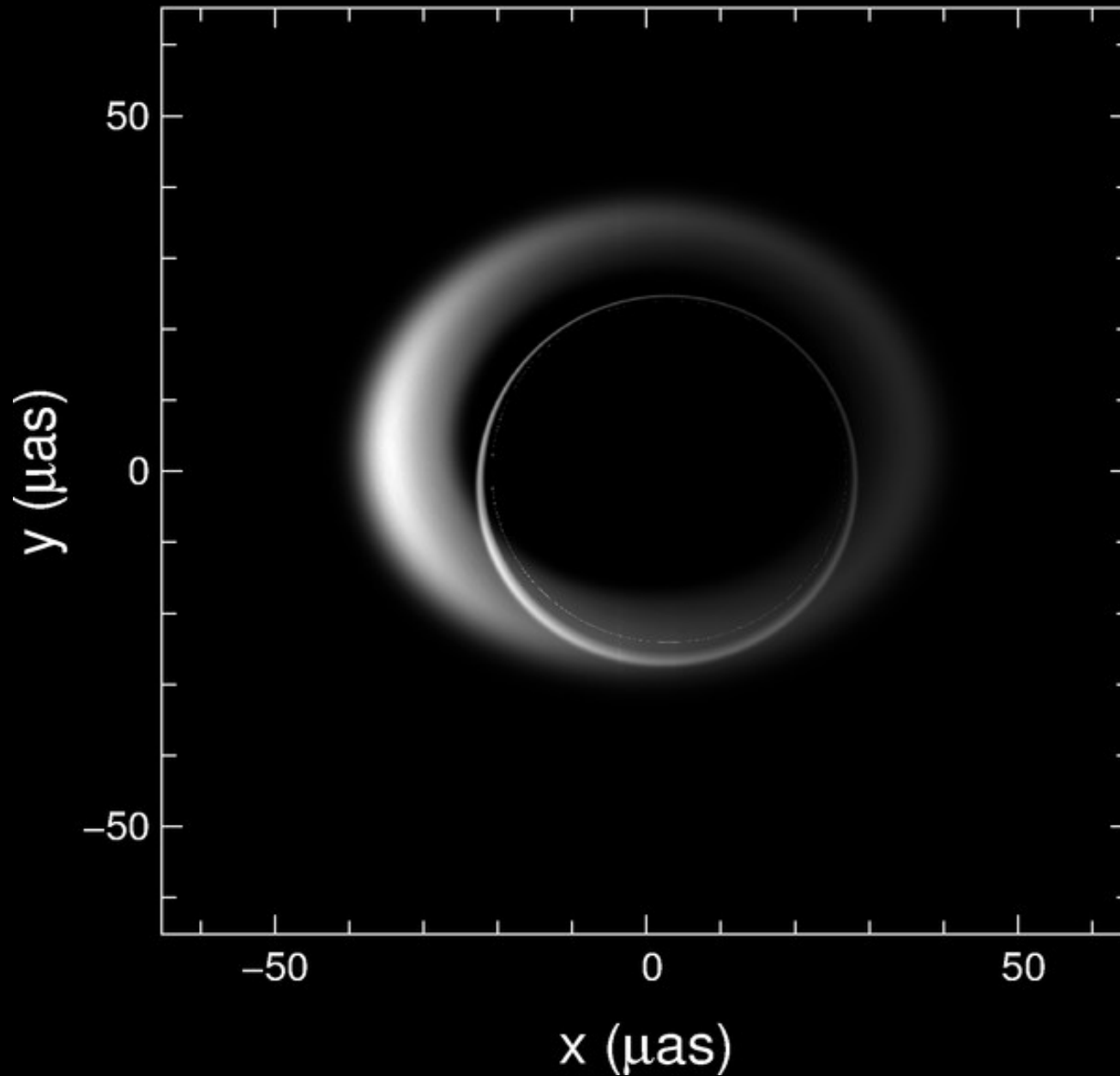


$$a^* = 0.9$$

$$\text{lambda} = 0.3$$

$$\text{inc} = 80^\circ$$

4. Dim accretion: Images

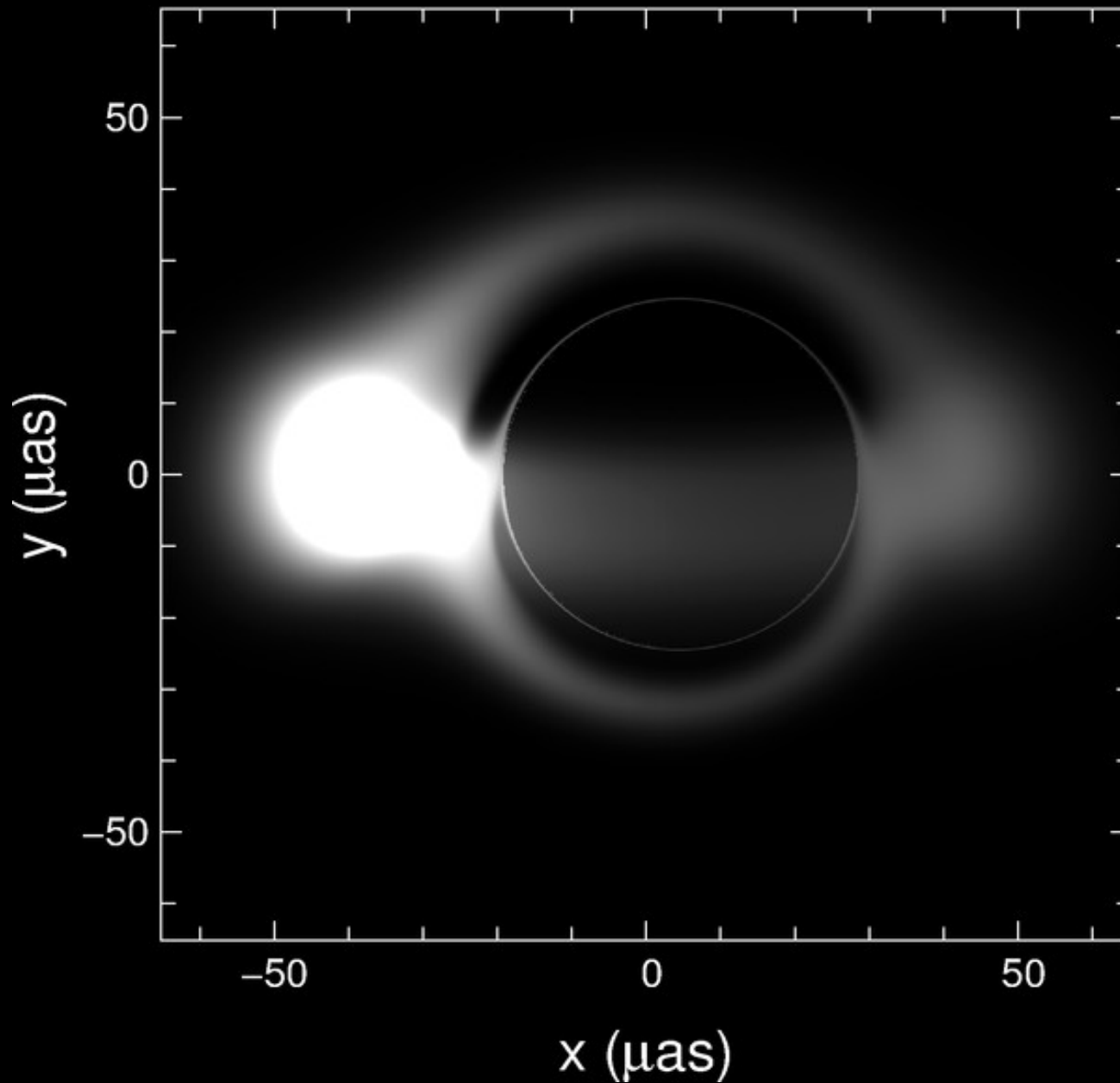


$$a^* = 0.5$$

$$\text{lambda} = 0.3$$

$$\text{inc} = 40^\circ$$

4. Dim accretion: Images

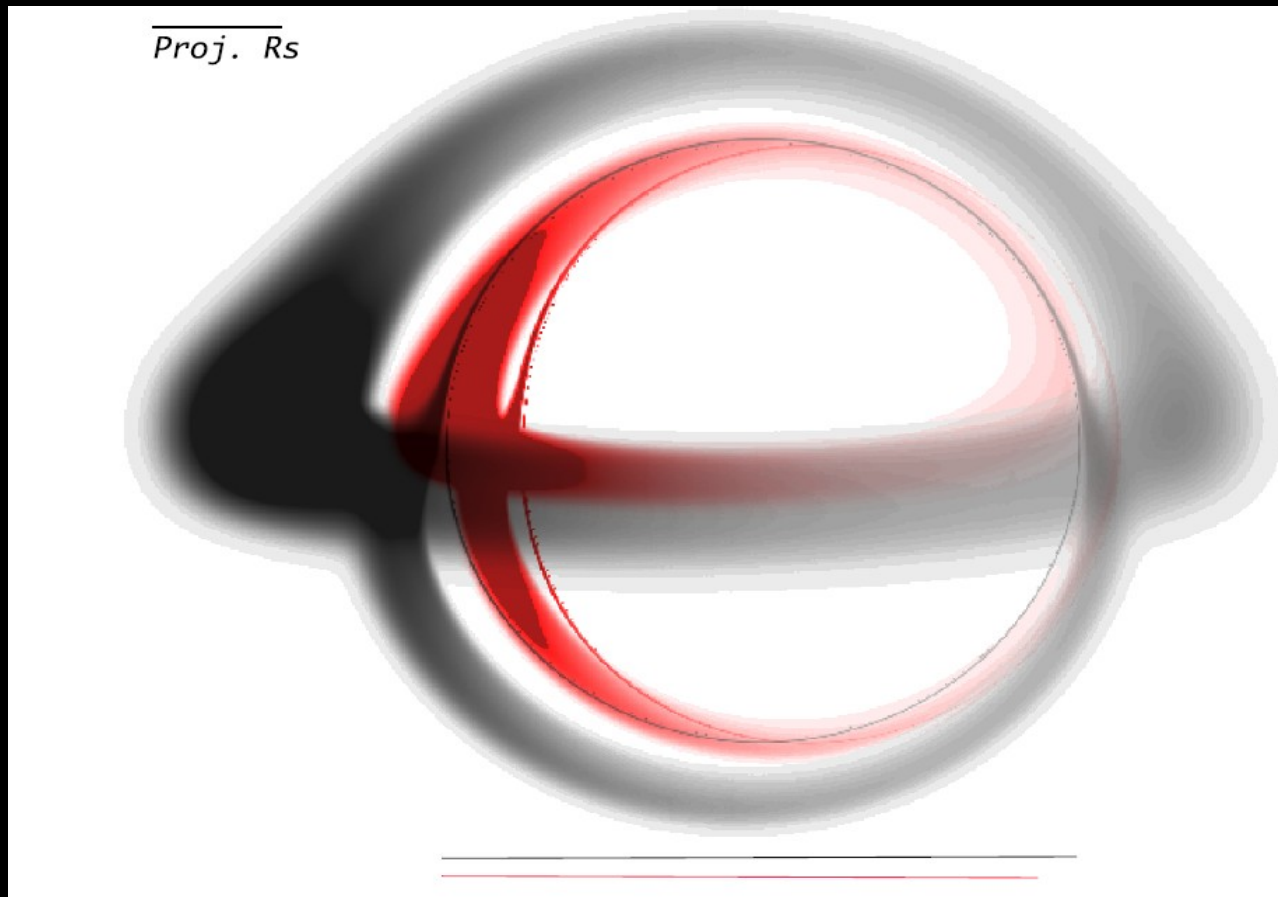


$$a^* = 0.5$$

$$\text{lambda} = 0.7$$

$$\text{inc} = 80^\circ$$

4. Dim accretion: Images



$$A^* = 0.5, 0.9$$

$$\text{lambda} = 0.3$$

$$\text{inc} = 80^\circ$$

Spin constraint ?

Future : measuring the black hole's silhouette

Using VLBI measurements : Doeleman et al.,
Broderick et al., Dexter et al. ...

4. New telescopes

* GRAVITY beam combiner resolution: 10 – 100 μas (ca 2014)

* Event Horizon Telescope resolution: 1 – 10 μas (ca 2020)

Content

Standard **THIN DISC** model of accretion

LESS LUMINOUS
than
standard disc model

ION TORUS

MORE LUMINOUS
than
standard disc model

SLIM DISC

The End

