Shortcomings of new parametrizations of inflation

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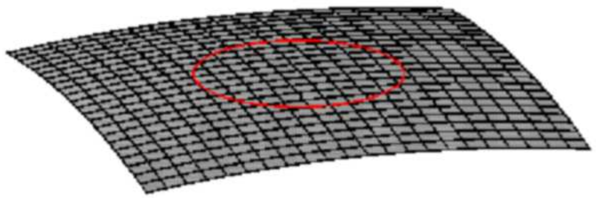
Outline

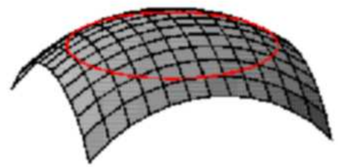
J. Martin, CR and V. Vennin: arXiv:1609.04739

Introduction

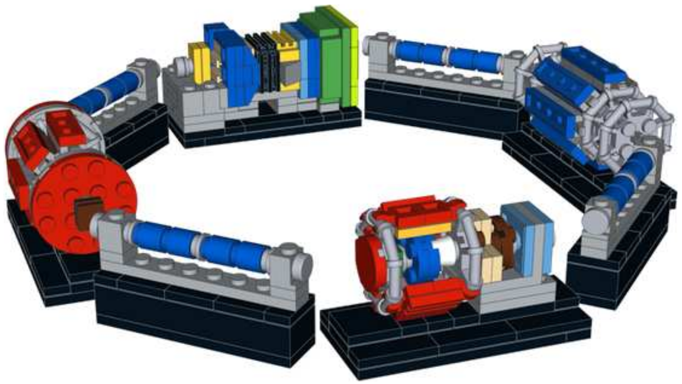
What is primordial inflation?

- A yet to be proven theoretical paradigm describing the early Universe:
  - Our Universe should have undergone a phase a exponentially fast accelerated expansion
  - Length scales $\times e^N$ with $N > 60$ (e-folds)
  - Occured at a redshift: $z_{\rm inf} > 10^{10}$
  - Could have lasted from $10^{-32}\,\mathrm{s}$ to an infinite amount of time





- Energy involved: $10\,\mathrm{MeV} \ll E_{\rm inf} < 10^{16}\,\mathrm{GeV}$
  - $10^{16}\,\mathrm{GeV} = 1000$ billion times the energy of the LHC (7.5 billion €)



Motivations for inflation

- Originally proposed to solve the "monopole" problem [Guth:1981], inflation ends up adressing various issues of the Friedmann-Lemaître cosmology [Linde:1982].
- Unexplanable or inconsistent with the standard Big-Bang model:
  - Flatness of the spatial sections: $\Omega_{\mathrm{K}} = 0.0008 \pm 0.004$
  - Statistical isotropy of the observable Universe (horizon problem)
  - Origin of the CMB anisotropies and large scale structures
  - Gaussianity of the CMB fluctuations: $f_{\mathrm{NL}} = 0.8 \pm 5.0$
  - Adiabaticity of the cosmological perturbations: isocurv. $< 4\%$
  - Almost scale invariance of the primordial perturbations: $n_{\mathrm{S}} = 0.9667 \pm 0.004$
- Within General Relativity (GR) inflation requires "repulsive gravity"
  - Negative pressure
  - Or deviations from GR?

Making observable predictions

Single field inflation

- Dynamics given by ($\kappa^2 = 1/M_{\mathrm{P}}^2$)

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \mathcal{L}(\phi) \right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- Can be used to describe:
  - ✦ Minimally coupled scalar field to General Relativity
  - ✦ Scalar-tensor theory of gravitation in the Einstein frame

    the graviton' scalar partner is also the inflaton (HI, RPI1,. . .)

- Everything can be consistently solved in the slow-roll approximation
  - ✦ Background evolution $\phi(N)$ where $N \equiv \ln a$
  - ✦ Linear perturbations for the field-metric system $\zeta(t, \boldsymbol{x})$, $\delta\phi(t, \boldsymbol{x})$

- Slow-roll = expansion in terms of the Hubble flow functions [Schwarz 01]

$$\epsilon_0 \equiv \frac{H_{\mathrm{ini}}}{H}, \quad \epsilon_{i+1} \equiv \frac{\mathrm{d} \ln |\epsilon_i|}{\mathrm{d}N} \qquad \text{measure deviations from de-Sitter}$$

Decoupling field and space-time evolution

- Friedmann-Lemaître equations in e-fold time (with $M_{\mathrm{P}}^2 = 1$)

$$\left\{\begin{aligned} H^2 &= \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right) \\ \frac{\ddot{a}}{a} &= -\frac{1}{3}\left(\dot{\phi}^2 - V\right)\end{aligned}\right. \Rightarrow \left\{\begin{aligned} H^2 &= \frac{V}{3 - \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2} \\ -\frac{\mathrm{d}\ln H}{\mathrm{d}N} &= \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2\end{aligned}\right. \Leftrightarrow \left\{\begin{aligned} H^2 &= \frac{V}{3-\epsilon_1} \\ \epsilon_1 &= \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2\end{aligned}\right.$$

- Klein-Gordon equation in e-folds: relativistic kinematics with friction

$$\frac{1}{3-\epsilon_1}\frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} + \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi} \quad \Leftrightarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{3-\epsilon_1}{3-\epsilon_1+\frac{\epsilon_2}{2}}\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}$$

- Slow-roll approximation: all $\epsilon_i = \mathcal{O}(\epsilon)$ and $\epsilon_1 < 1$ is the definition of inflation ($\ddot{a} > 0$)
  - ✦ The trajectory can be solved for N

$$N - N_{\mathrm{end}} \simeq \int_{\phi}^{\phi_{\mathrm{end}}} \frac{V(\psi)}{V'(\psi)}\,\mathrm{d}\psi$$

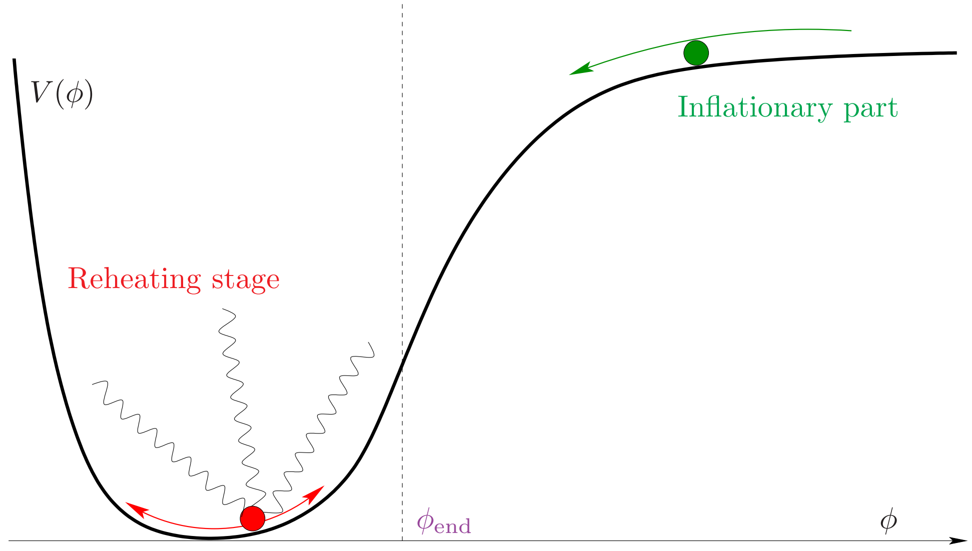
The end of inflation and after

- Accelerated expansion stops for $\epsilon_1 > 1$ ($\ddot{a} < 0$) at $N = N_{\text{end}}$
  - Naturally happens during field evolution (graceful exit) at $\phi = \phi_{\text{end}}$

$$\epsilon_1(\phi_{\text{end}}) = 1$$

  - Or, there is another mechanism ending inflation (tachyonic instability) and ϕ_{end} is a model parameter that has to be specified
- The reheating stage: everything after N_{end} till radiation domination
  - Basic picture $\longrightarrow$
  - But in reality a very complicated process, microphysics dependent
  - Reheating duration is unknown:

$$\Delta N_{\text{reh}} \equiv N_{\text{reh}} - N_{\text{end}}$$



Redshift at which reheating ends

- Denoting $N = N_{\mathrm{reh}}$ the end of reheating = beginning of radiation era
  - ✦ If thermalized, and no extra entropy production: $a_{\mathrm{reh}}^3 s_{\mathrm{reh}} = a_0^3 s_0$

$$\begin{cases} s_{\mathrm{reh}} = q_{\mathrm{reh}} \dfrac{2\pi^2}{45} T_{\mathrm{reh}}^3 \\ \rho_{\mathrm{reh}} = g_{\mathrm{reh}} \dfrac{\pi^2}{30} T_{\mathrm{reh}}^4 \end{cases} \Rightarrow \quad \frac{a_0}{a_{\mathrm{reh}}} = \left(\frac{q_{\mathrm{reh}}^{1/3} g_0^{1/4}}{q_0^{1/3} g_{\mathrm{reh}}^{1/4}} \right) \frac{\rho_{\mathrm{reh}}^{1/4}}{\rho_\gamma^{1/4}}$$

$$\text{or} \quad 1 + z_{\mathrm{reh}} = \left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma} \right)^{1/4}$$

- Depends on ρ_{reh} and $\tilde{\rho}_\gamma \equiv \mathcal{Q}_{\mathrm{reh}} \rho_\gamma$
  - ✦ Energy density of radiation today: $\rho_\gamma = 3 \dfrac{H_0^2}{M_{\mathrm{P}}^2} \Omega_{\mathrm{rad}}$
  - ✦ Change in the number of entropy and energy relativistic degrees of freedom (small effect compared to $\rho_{\mathrm{reh}}/\rho_\gamma$)

$$\mathcal{Q}_{\mathrm{reh}} \equiv \frac{g_{\mathrm{reh}}}{g_0} \left(\frac{q_0}{q_{\mathrm{reh}}} \right)^{1/4}$$

Redshift at which inflation ends

- Depends on the redshift of reheating

$$1+z_{\mathrm{end}}=\frac{a_0}{a_{\mathrm{end}}}=\frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}(1+z_{\mathrm{reh}})=\frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}\left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma}\right)^{1/4}=\frac{1}{R_{\mathrm{rad}}}\left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

  - ✦ The reheating parameter $R_{\mathrm{rad}}\equiv\frac{a_{\mathrm{end}}}{a_{\mathrm{reh}}}\left(\frac{\rho_{\mathrm{end}}}{\rho_{\mathrm{reh}}}\right)^{1/4}$
  - ✦ Encodes any observable deviations from a radiation-like or instantaneous reheating $R_{\mathrm{rad}}=1$

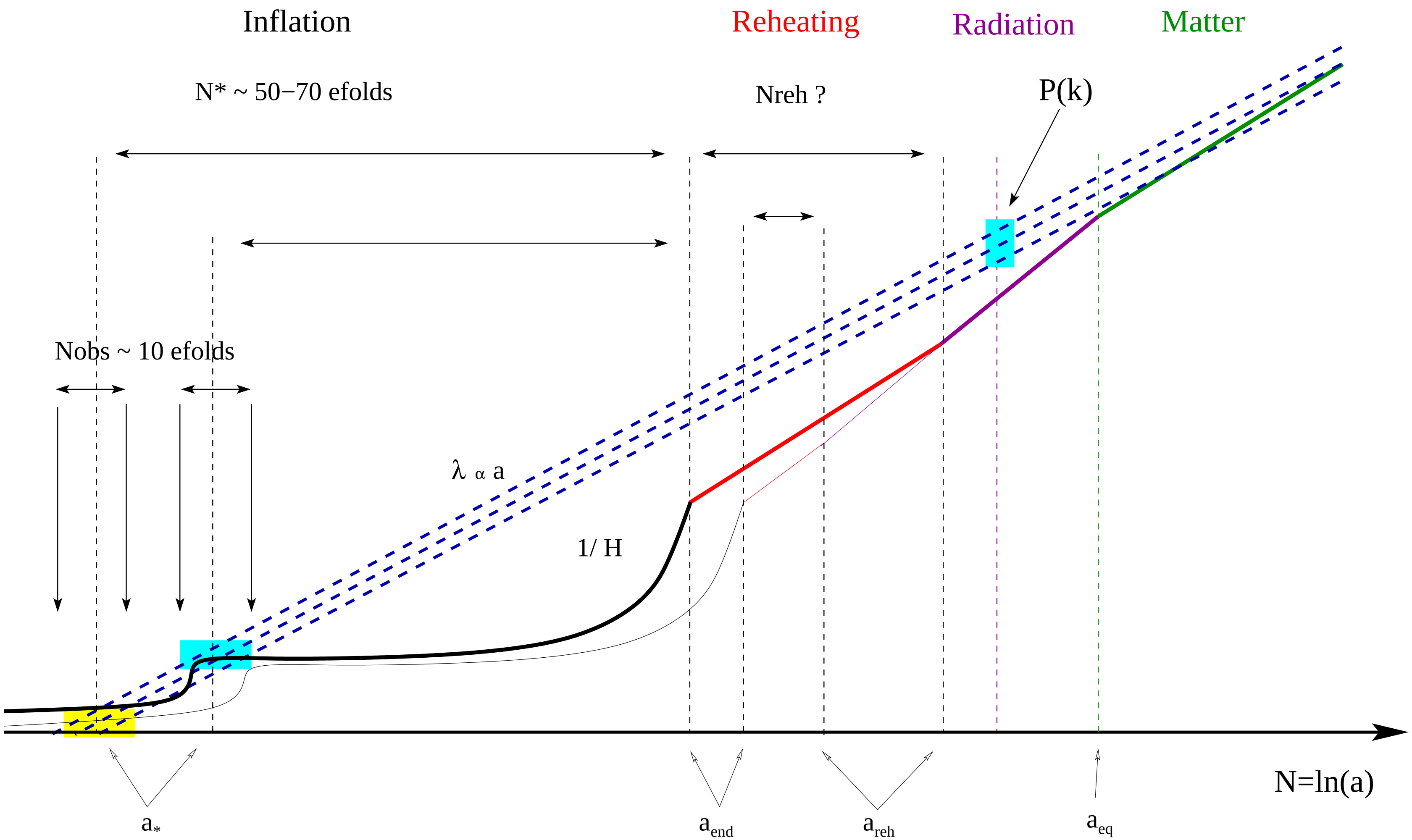
- R_{rad} can be expressed in terms of $(\rho_{\mathrm{reh}},\overline{w}_{\mathrm{reh}})$ or $(\Delta N_{\mathrm{reh}},\overline{w}_{\mathrm{reh}})$

$$\ln R_{\mathrm{rad}}=\frac{\Delta N_{\mathrm{reh}}}{4}(3\overline{w}_{\mathrm{reh}}-1)=\frac{1-3\overline{w}_{\mathrm{reh}}}{12(1+\overline{w}_{\mathrm{reh}})}\ln\left(\frac{\rho_{\mathrm{reh}}}{\rho_{\mathrm{end}}}\right)$$

where $\overline{w}_{\mathrm{reh}}\equiv\frac{1}{\Delta N_{\mathrm{reh}}}\int_{N_{\mathrm{end}}}^{N_{\mathrm{reh}}}\frac{P(N)}{\rho(N)}\mathrm{d}N$

- A fixed inflationary parameters, z_{end} can still be affected by R_{rad}

Reheating effects on inflationary observables



- Model testing: reheating effects must be included!

Inflationary perturbations in slow-roll

- Equations of motion for the linear perturbations

$$\left.\begin{aligned} \mu_{\mathrm{T}} &\equiv ah \\ \mu_{\mathrm{S}} &\equiv a\sqrt{2}\phi_{,N}\zeta \end{aligned}\right\} \Rightarrow \mu_{\mathrm{TS}}'' + \left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]\mu_{\mathrm{TS}} = 0$$

- Can be consistently solved using slow-roll and pivot expansion [Stewart:1993, Gong:2001, Schwarz:2001, Leach:2002, Martin:2002, Habib:2002, Casadio:2005, Lorenz:2008, Martin:2013, Beltran:2013]

$$\begin{aligned} \mathcal{P}_\zeta = \frac{H_*^2}{8\pi^2 M_{\mathrm{P}}^2\epsilon_{1*}} \Bigg\{ & 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^2}{2} - 3 + 2C + 2C^2\right)\epsilon_{1*}^2 + \left(\frac{7\pi^2}{12} - 6 - C + C^2\right)\epsilon_{1*}\epsilon_{2*} \\ & + \left(\frac{\pi^2}{8} - 1 + \frac{C^2}{2}\right)\epsilon_{2*}^2 + \left(\frac{\pi^2}{24} - \frac{C^2}{2}\right)\epsilon_{2*}\epsilon_{3*} \\ & + \left[-2\epsilon_{1*} - \epsilon_{2*} + (2+4C)\epsilon_{1*}^2 + (-1+2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^2 - C\epsilon_{2*}\epsilon_{3*}\right]\ln\left(\frac{k}{k_*}\right) \\ & + \left[2\epsilon_{1*}^2 + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^2 - \frac{1}{2}\epsilon_{2*}\epsilon_{3*}\right]\ln^2\left(\frac{k}{k_*}\right)\Bigg\}, \end{aligned}$$

$$\begin{aligned} \mathcal{P}_h = \frac{2H_*^2}{\pi^2 M_{\mathrm{P}}^2}\Bigg\{ & 1 - 2(1+C)\epsilon_{1*} + \left[-3 + \frac{\pi^2}{2} + 2C + 2C^2\right]\epsilon_{1*}^2 + \left[-2 + \frac{\pi^2}{12} - 2C - C^2\right]\epsilon_{1*}\epsilon_{2*} \\ & + \left[-2\epsilon_{1*} + (2+4C)\epsilon_{1*}^2 + (-2-2C)\epsilon_{1*}\epsilon_{2*}\right]\ln\left(\frac{k}{k_*}\right) + \left(2\epsilon_{1*}^2 - \epsilon_{1*}\epsilon_{1*}\right)\ln^2\left(\frac{k}{k_*}\right)\Bigg\} \end{aligned}$$

- Notice that: $H_* \equiv H(\Delta N_*)$ and $\epsilon_{i*} \equiv \epsilon_i(\Delta N_*)$ with $k_*\eta(\Delta N_*) = -1$

The power law parameters

- From the observable point of view, one defines spectral index, running, tensor-to-scalar ratio, . . .

$$n_{\mathrm{S}} - 1 \equiv \left.\frac{\mathrm{d}\ln \mathcal{P}_\zeta}{\mathrm{d}\ln k}\right|_{k_*}, \qquad \alpha_{\mathrm{S}} \equiv \left.\frac{\mathrm{d}^2 \ln \mathcal{P}_\zeta}{\mathrm{d}(\ln k)^2}\right|_{k_*}, \qquad r \equiv \left.\frac{\mathcal{P}_h}{\mathcal{P}_h}\right|_{k_*}$$

- They are read-off from the previous slow-roll expression

$$\begin{aligned} n_{\mathrm{S}} &= 1 - 2\epsilon_{1*} - \epsilon_{2*} - (3+2C)\epsilon_{1*}\epsilon_{2*} - 2\epsilon_{1*}^2 - C\epsilon_{2*}\epsilon_{3*} + \mathcal{O}\left(\epsilon^3\right) \\ \alpha_{\mathrm{S}} &= -2\epsilon_{1*}\epsilon_{2*} - \epsilon_{2*}\epsilon_{3*} + \mathcal{O}\left(\epsilon^3\right) \\ r &= 16\epsilon_{1*}\left(1 + C\epsilon_{2*}\right) + \mathcal{O}\left(\epsilon^3\right) \end{aligned}$$

- One has to know the functions $\epsilon_i(\Delta N_*)$ and the value of ΔN_* to make predictions

Solving for the time of pivot crossing

- To make inflationary predictions, one has to solve $k_*\eta_* = -1$

$$\frac{k_*}{a_0} = \frac{a(N_*)}{a_0}H_* = e^{N_*-N_{\mathrm{end}}}\frac{a_{\mathrm{end}}}{a_0}H_* = \frac{e^{\Delta N_*}H_*}{1+z_{\mathrm{end}}} = e^{\Delta N_*}R_{\mathrm{rad}}\left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{-\frac{1}{4}}H_*$$

- Defining $N_0 \equiv \ln\left(\frac{k_*}{a_0}\frac{1}{\tilde{\rho}_\gamma^{1/4}}\right)$ (number of e-folds of deceleration)
  - ✦ This is a non-trivial integral equation that depends on: model + how inflation ends + reheating + data

$$-\left[\int_{\phi_{\mathrm{end}}}^{\phi_*}\frac{V(\psi)}{V'(\psi)}\mathrm{d}\psi\right] = \ln R_{\mathrm{rad}} - N_0 + \frac{1}{4}\ln(8\pi^2 P_*) - \frac{1}{4}\ln\left\{\frac{9}{\epsilon_1(\phi_*)[3-\epsilon_1(\phi_{\mathrm{end}})]}\frac{V(\phi_{\mathrm{end}})}{V(\phi_*)}\right\}$$

  - ✦ Result: one gets ϕ_*, or equivalently ΔN_*, as a function of inflationary model parameters and R_{rad}

Hubble-flow functions from the potential

- One would prefer a "slow-roll" hierarchy based on $V(\phi)$ only

$$\epsilon_{v_0}(\phi) \equiv \sqrt{\frac{3}{V(\phi)}}\,, \qquad \epsilon_{v_{i+1}}(\phi) \equiv \frac{\mathrm{d}\ln \epsilon_{v_i}(\phi)}{\mathrm{d}\tilde{N}} \quad \text{with} \quad \frac{\mathrm{d}}{\mathrm{d}\tilde{N}} \equiv -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}$$

- Can be mapped with the Hubble flow hierarchy

$$\epsilon_{v_0} = \frac{\epsilon_0}{\sqrt{1-\epsilon_1/3}}\,, \qquad \epsilon_{v_1} = \epsilon_1\left(1 + \frac{\epsilon_2/6}{1-\epsilon_1/3}\right)^2$$

$$\epsilon_{v_2} = \epsilon_2\left[1 + \frac{\epsilon_2/6+\epsilon_3/3}{1-\epsilon_1/3} + \frac{\epsilon_1\epsilon_2^2}{(3-\epsilon_1)^2}\right], \qquad \epsilon_{v_3} = \cdots$$

- Inversion can only be made perturbatively

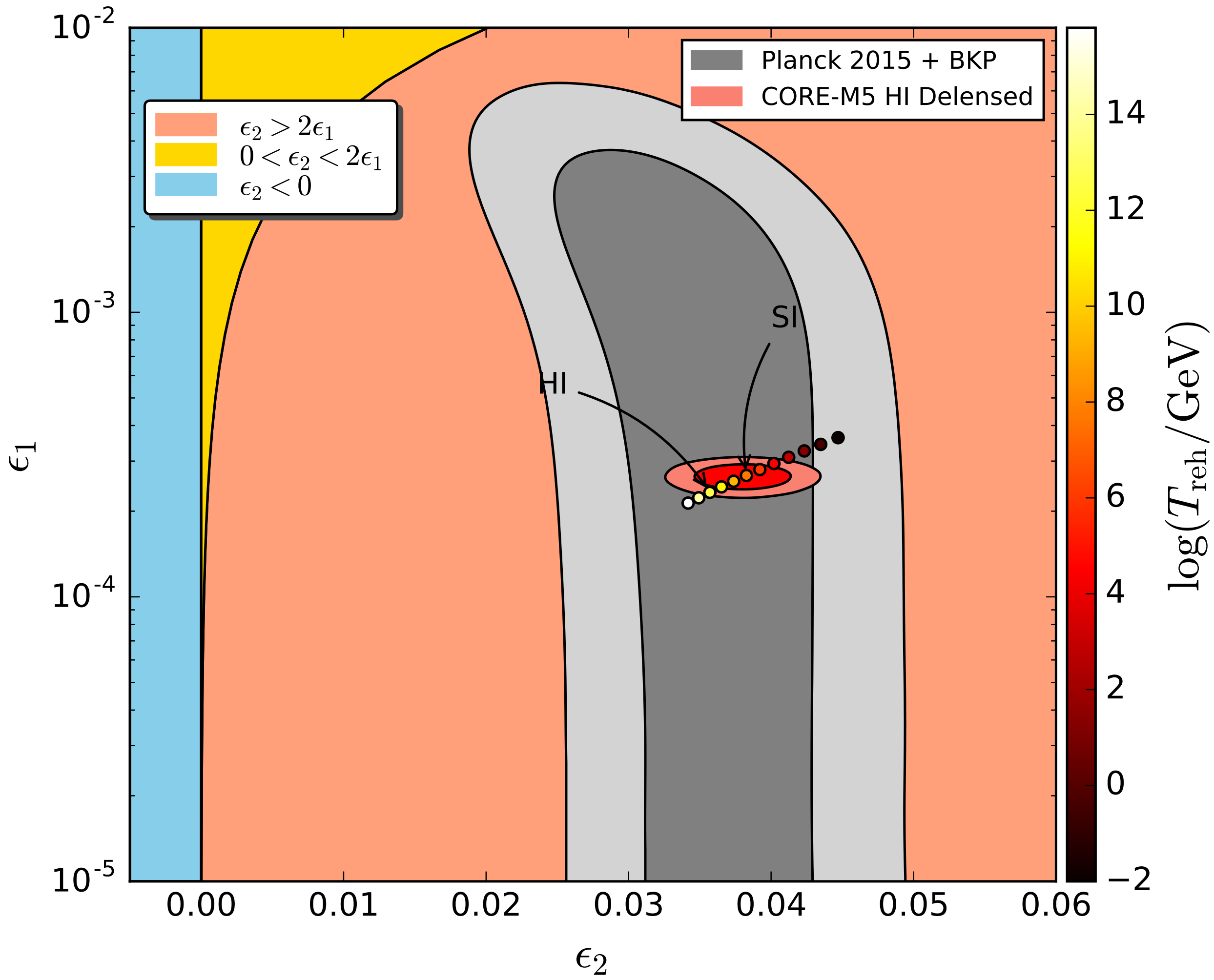
$$\epsilon_1 = \epsilon_{v_1} - \frac{1}{3}\epsilon_{v_1}\epsilon_{v_2} - \frac{1}{9}\epsilon_{v_1}^2\epsilon_{v_2} + \frac{5}{36}\epsilon_{v_1}\epsilon_{v_2}^2 + \frac{1}{9}\epsilon_{v_1}\epsilon_{v_2}\epsilon_{v_3} + \mathcal{O}(\epsilon^4)$$

$$\epsilon_2 = \epsilon_{v_2} - \frac{1}{6}\epsilon_{v_2}^2 - \frac{1}{3}\epsilon_{v_2}\epsilon_{v_3} - \frac{1}{6}\epsilon_{v_1}\epsilon_{v_2}^2 + \frac{1}{18}\epsilon_{v_2}^3 - \frac{1}{9}\epsilon_{v_1}\epsilon_{v_2}\epsilon_{v_3} + \frac{5}{18}\epsilon_{v_2}^2\epsilon_{v_3} + \frac{1}{9}\epsilon_{v_2}\epsilon_{v_3}^2 + \frac{1}{9}\epsilon_{v_2}\epsilon_{v_3}\epsilon_{v_4} + \mathcal{O}(\epsilon^4)$$

Example with Higgs and Starobinski inflation

- Same potential but not the same reheating

$$V(\phi) \propto \left(1 - e^{-\sqrt{2/3}\,\phi/M_{\mathrm{P}}}\right)^2$$



The new parametrizations

Universality classes

- Because hundred of inflationary models have been proposed since the 80s $\Rightarrow$ some desire to avoid specifying a potential, its parameters, the reheating, ...
- Are there "universality classes" favoured by Planck?
  - Proposals of Refs. [arXiv:0706.2215, arXiv:1309.1285, arXiv:1412.0678]: the large ΔN_* limit is somehow universal

$$\epsilon_{1*} = \frac{\beta}{(\Delta N_*)^{\alpha}} + \cdots$$

  - Order one: $\epsilon_{1*} \propto 1/\Delta N_*$ (currently under pressure), motivates to search of next order $\epsilon_{1*} \propto 1/\Delta N_*^2$ (typical of Starobinski inflation)
- Universality classes would avoid specifying a model (bottom to top approach)
  - Only two parameters to fit: α and the order β
  - Effective approach as in Particle Physics [arXiv:1407.0820]
- Unfortunately...

Not universal

- One of the most favoured models by Planck is Khäler Moduli Inflation
  - Two parameters $\bar{\alpha}$ and $\bar{\beta}$

$$V(\phi) \propto 1 - \bar{\alpha}\left(\frac{\phi}{M_{\mathrm{P}}}\right)^{4/3} \exp\left[-\bar{\beta}\left(\frac{\phi}{M_{\mathrm{P}}}\right)^{4/3}\right]$$

  - Slow-roll parameter in the large ΔN_* limit

$$\epsilon_{v_1 *} = \frac{\ln^{5/2}\left(16\bar{\alpha}\sqrt{\frac{9\bar{\beta}^{1/2}}{8}}\Delta N_*\right)}{324\bar{\beta}^{3/2}\Delta {N_*}^2} + \mathcal{O}\left(\frac{1}{\Delta {N_*}^3}\right)$$

- Many models are not in $1/(\Delta N_*)^\alpha$!
- Proposal of [1402.2059]: there are more than one "Universality Classes"
  - Perturbative (the original one), "logarithmic" (see above) and "non-perturbative" (exponentials in ϵ_{v_1})
- Unfortunately...

Inflation of the number of classes

- The simplest inflationary model at next-to-leading order (SI):

$$\epsilon_{v_1*} = \frac{3}{4\Delta N_*^2} - \frac{9}{8\Delta N_*^3}\left[\frac{2}{\sqrt{3}} - \ln\left(1+\frac{2}{\sqrt{3}}\right) + \ln\left(\frac{4}{3}\Delta N_*\right)\right] + \mathcal{O}\left(\frac{1}{\Delta N_*{}^4}\right)$$

  - ✦ SI belongs to the “perturbative class” at leading order but becomes “logarithmic” at next-to-leading order!

- Other big troubles: $1/\Delta N_*$ expansion may not make sense!
  - ✦ Quadratic small field model: $V(\phi) \propto 1 - (\phi/\mu)^2$

$$\epsilon_{v_1*} = \frac{M_{\mathrm{P}}^4}{\mu^4}\left(\sqrt{1+2\frac{\mu^2}{M_{\mathrm{P}}^2}} - 1\right)^2 e^{-\frac{M_{\mathrm{P}}^2}{\mu^2}\left(4\Delta N_* + 1 + \frac{\mu^2}{M_{\mathrm{P}}^2} - \sqrt{1+2\frac{\mu^2}{M_{\mathrm{P}}^2}}\right)} + \mathcal{O}(f_*)$$

$$\epsilon_{v_2*} = 4\frac{M_{\mathrm{P}}^2}{\mu^2} + \mathcal{O}(f_*) \quad \text{where} \quad f_* \equiv e^{-4\frac{M_{\mathrm{P}}^2}{\mu^2}\Delta N_*}$$

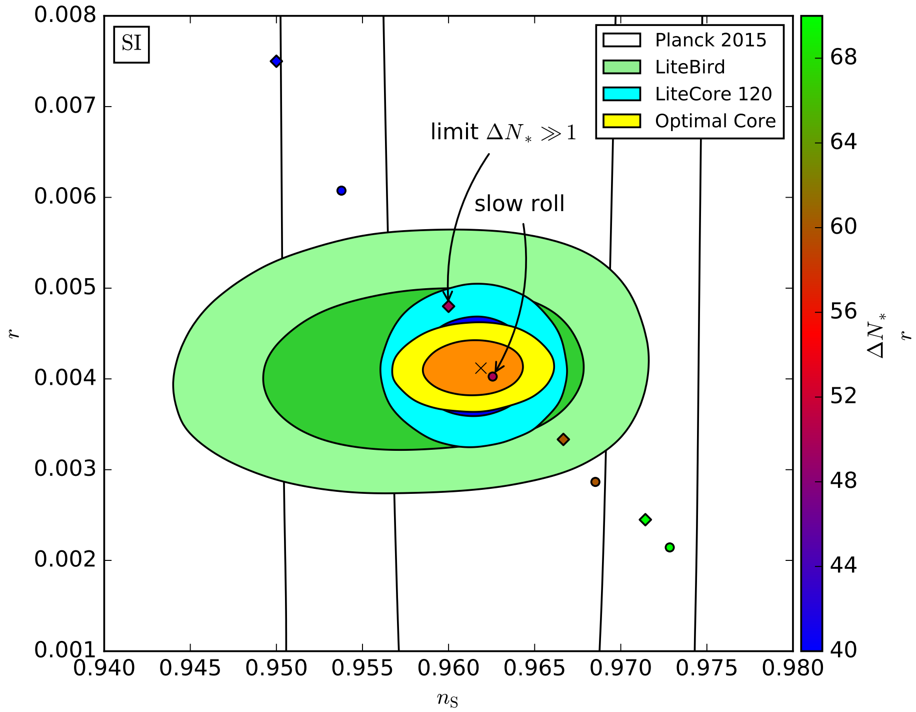
  - ✦ Expansion makes sense for $\Delta N_* M_{\mathrm{P}}^2/\mu^2 \gg 1$ in which $\mu < M_{\mathrm{P}}$ breaks slow-roll!

Insufficiently accurate

- The large ΔN_* limit (when it exists) leads to inaccurate predictions

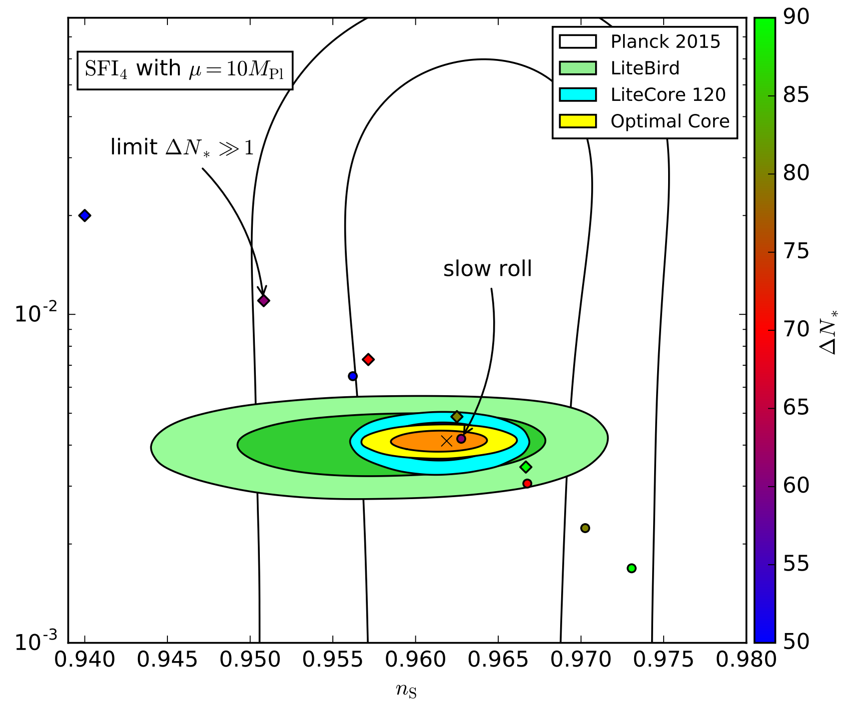
Starobinski Inflation

$$V(\phi) \propto \left(1 - e^{-\sqrt{2/3}\,\phi/M_\mathrm{P}}\right)^2$$



Quartic Small Field Inflation

$$V(\phi) \propto 1 - (\phi/\mu)^4$$



- ΔN_* without a potential is unpredictable: an additional model parameter?

Equation-of-state inflation

- Instead of $V(\phi)$, one fixes $w(\Delta N_*) \equiv P(\Delta N_*)/\rho(\Delta N_*)$
  - Hydrodynamical approached proposed by Mukhanov [arXiv:1303.3925].
  - It is not an expansion $\Rightarrow$ does not suffer from the previous inconsistencies
- At the background level, ends up being equivalent to a scalar field
  - Hydrodynamical Friedmann-Lemaître equations

$$H^2 = \frac{\rho(N)}{3M_{\mathrm{P}}^2}\,, \qquad \frac{\mathrm{d}H}{\mathrm{d}N} = -\frac{3}{2}\left[1+w(N)\right]H(N)$$

  - By comparison with the ones coming from a scalar field, one gets:

$$\epsilon_1(N) = \frac{3}{2}[1+w(N)], \qquad \phi(N) = \phi_0 \pm \sqrt{3}\,M_{\mathrm{P}} \int_{N_0}^{N} \sqrt{1+w(n)}\,\mathrm{d}n$$

$$V(N) = V_0 \exp\left\{-3\int_{N_0}^{N}\left[1+w(n)\right]\mathrm{d}n\right\}$$

Example for the perturbative class

- Assuming inflation is driven by:

$$w(\Delta N_*) + 1 = \frac{\beta}{(c + \Delta N_*)^\alpha}$$

  - ✦ End of inflation at $w(\Delta N_* = 0) = -1/3 \Rightarrow c = (3\beta/2)^{1/\alpha}$
  - ✦ Solving for $\phi(N)$, $V(N)$ and $V[N(\phi)]$:

$$V(\phi) \propto \left[1 - \frac{\beta}{2\left(1 + \frac{2-\alpha}{2\sqrt{3\beta}}\frac{\phi}{M_\mathrm{P}}\right)^{\frac{2\alpha}{2-\alpha}}}\right] \times \exp\left\{\frac{3\beta}{1-\alpha}\left[\left(1 + \frac{2-\alpha}{2\sqrt{3\beta}}\frac{\phi}{M_\mathrm{P}}\right)^{\frac{2(1-\alpha)}{2-\alpha}} - 1\right]\right\}$$

- Ends up being a particular inflationary potential!

Hydrodynamical cosmological perturbations

- For the Bardeen potential

$$\Phi_{\mathrm{B}}'' + 3\mathcal{H}\left(1 + c_{\mathrm{S}}^2\right)\Phi_{\mathrm{B}}' + \left[2\mathcal{H}' + \mathcal{H}^2\left(1 + 3c_{\mathrm{S}}^2\right)\right]\Phi_{\mathrm{B}} + c_{\mathrm{S}}^2 k^2 \Phi_{\mathrm{B}} = \frac{a^2}{2M_{\mathrm{P}}^2}\delta P_{\mathrm{nad}}$$

$$\delta P_{\mathrm{nad}} \equiv \delta P - c_{\mathrm{S}}^2 \delta\rho$$

- For a fluid, $c_{\mathrm{s}}^2(\Delta N_*)$ and $\delta P_{\mathrm{nad}}(\Delta N_*)$ should also be specified
- Cosmological perturbations during inflation would evolve as in scalar field inflation provided
  - ✦ $c_{\mathrm{s}}^2 = 1 - \dfrac{4}{9[1 - w(N)^2]}\left\{3 + 3w(N) - \dfrac{\mathrm{d}\ln[1 - w(N)]}{\mathrm{d}N}\right\}$
  - ✦ $\delta P_{\mathrm{nad}} = -2M_{\mathrm{P}}^2\left(1 - c_{\mathrm{S}}^2\right)\dfrac{k^2}{a^2}\Phi_{\mathrm{B}}$
  - ✦ This is implicitely assumed when one uses the standard expressions for the power spectra
  - ✦ How to justify these relations if the gravitating fluid is not a scalar field?

Conclusion

- New parametrizations of inflation fail in
  - Being universal: number of classes blow-up at higher orders
  - Being predictive: ΔN_* becomes an arbitrary parameter
  - Being accurate: already obsoleted by the Planck satellite accuracy
  - Being useful?
- Equation-of-state inflation is
  - Consistent
  - Equivalent to scalar field inflation (or incomplete)
  - A new way to construct exact solutions
- How to be model independent?
  - Use slow-roll. . .
- Why being model independant?
  - Planck 2015 has already ruled-out 30% of all inflationary models
  - Theoreticians should do their job: making observable predictions