Relativistic effects on redshift-space distortions at quasi-linear scales

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Collaborators

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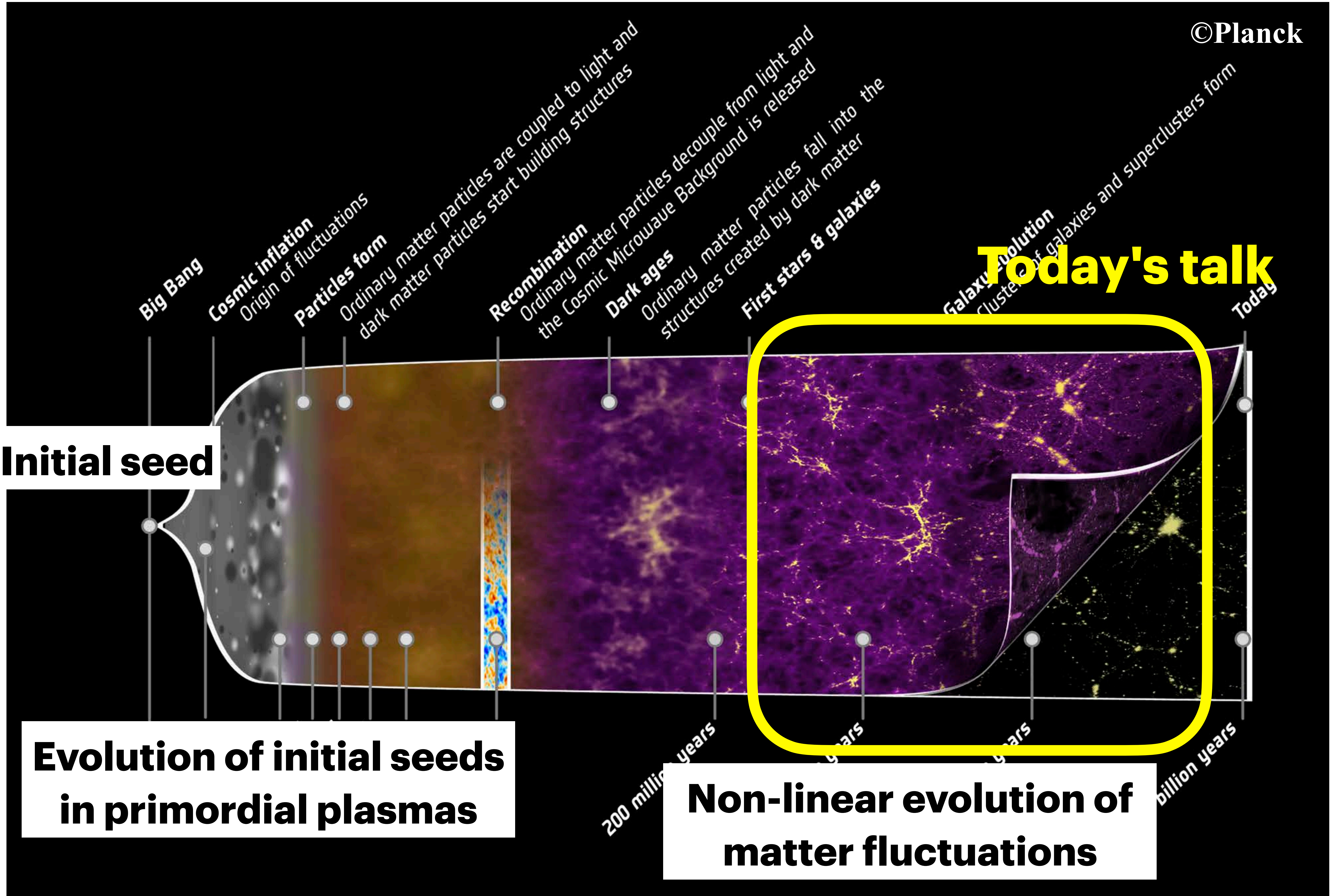
LUTH seminar

Relativistic effects on redshift-space distortions at quasi-linear scales

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, **S.Saga** [1803.04294]
A.Taruya, **S.Saga**, M-A.Breton, Y.Rasera, T.Fujita [1908.03854]
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]
S.Saga et al. in prep.

1.1 History of the universe



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Today's talk

Initial seed

Evolution of initial seeds in primordial plasmas

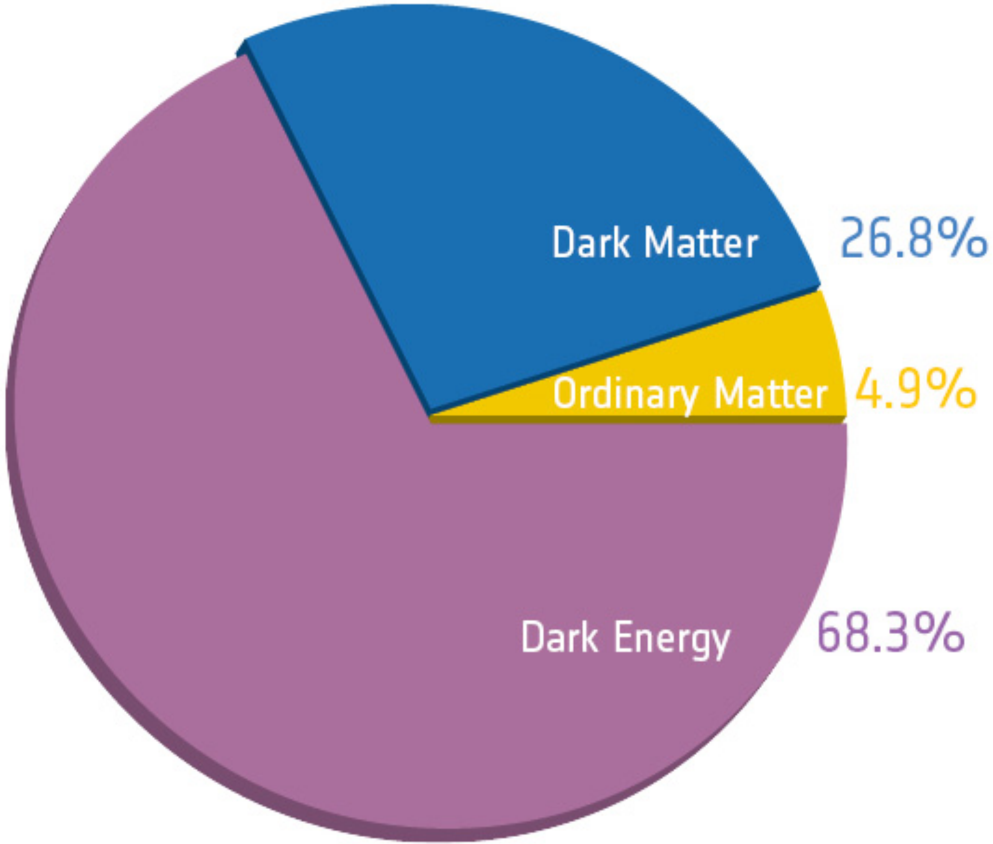
Non-linear evolution of matter fluctuations

1.2 Large-scale structure

Large-scale structure of the universe

the inhomogeneous distribution of galaxies and/or matters on very large scales
(on scales much larger than individual galaxies)

dominated by the **gravity** of **cold dark matters** (CDM) seeded during **inflation**



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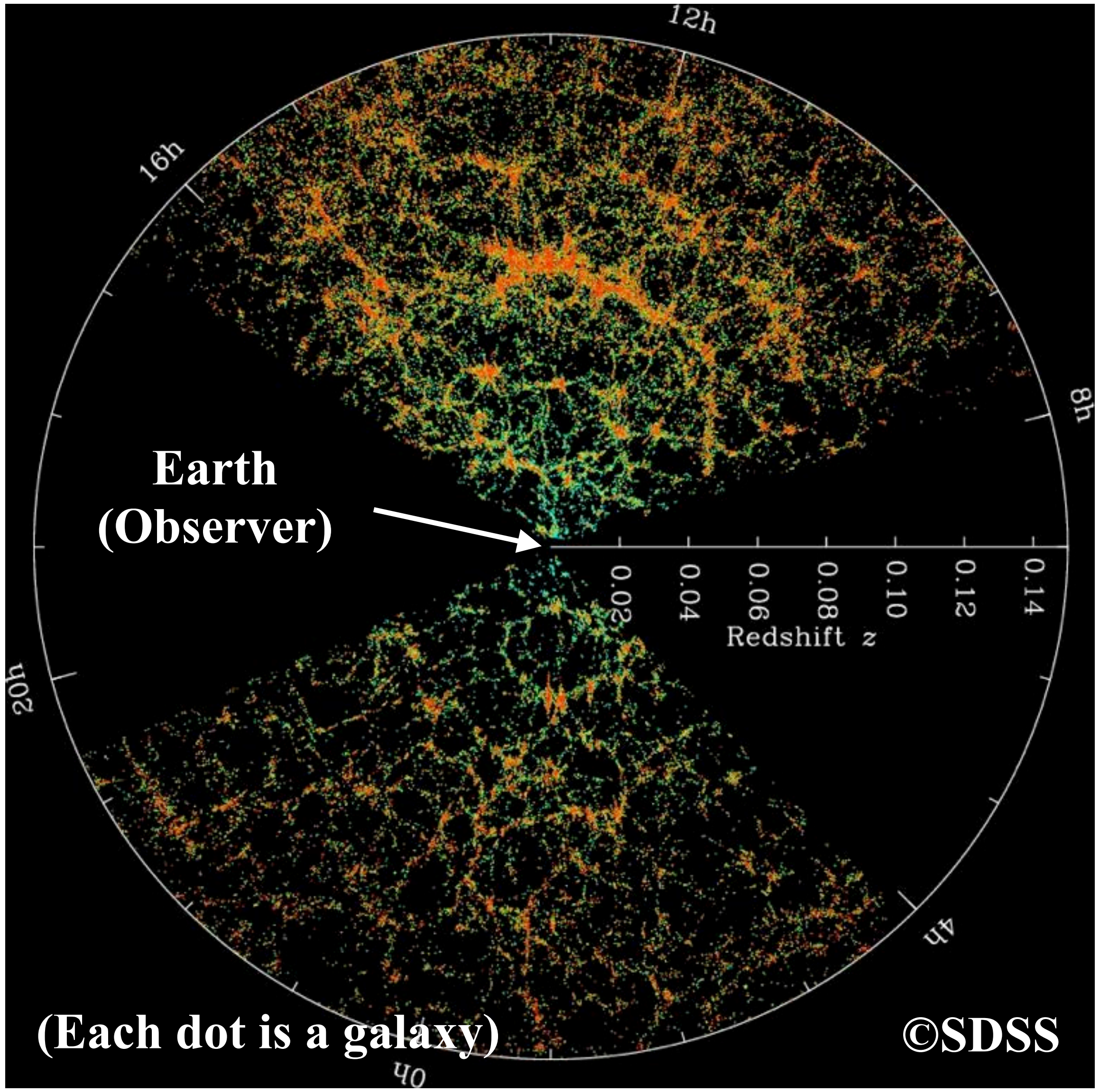

(Each dot is a galaxy)

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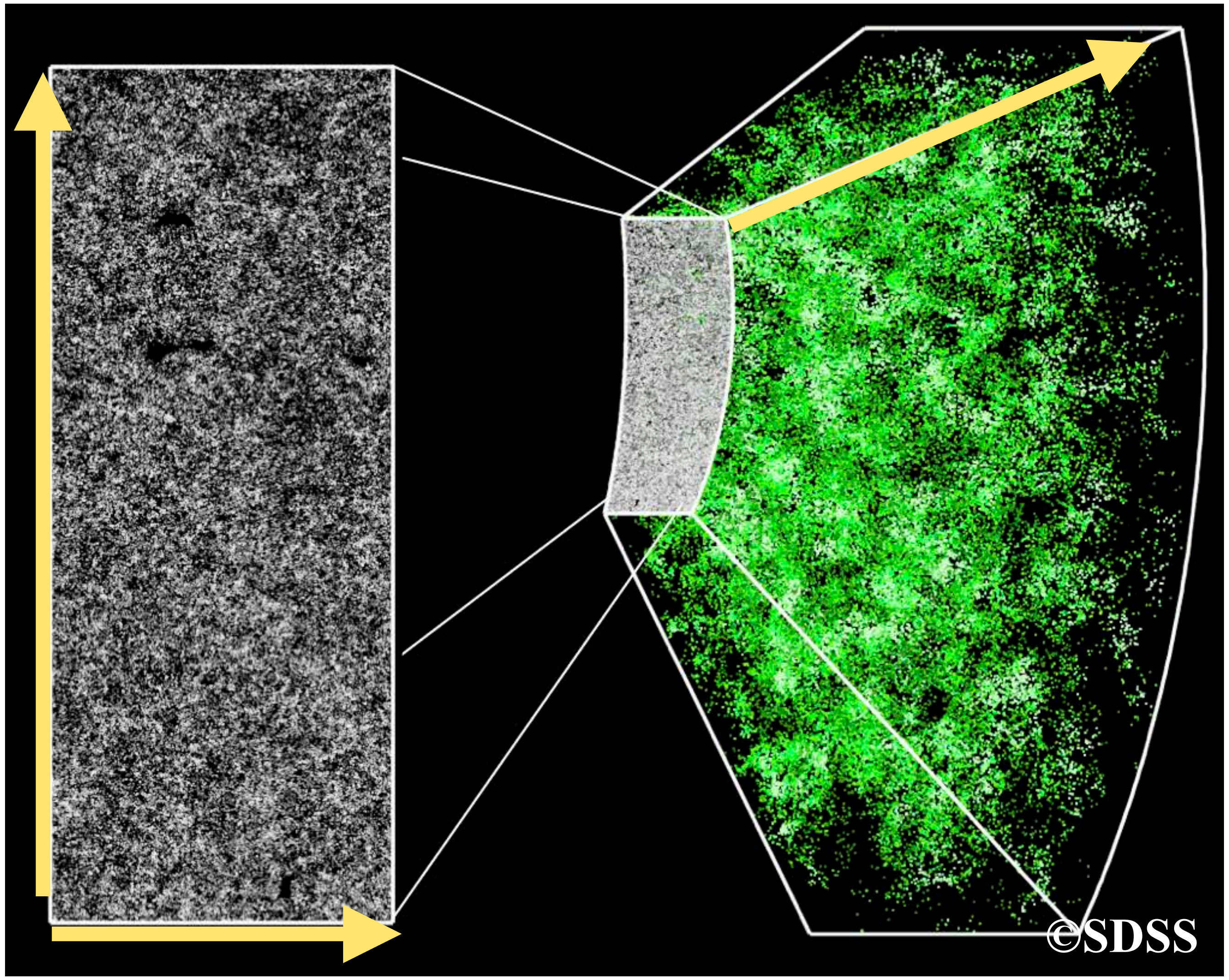
Observing large-scale structure would shed light on the dark components, initial condition, and nature of gravity.

1.3 Galaxy redshift surveys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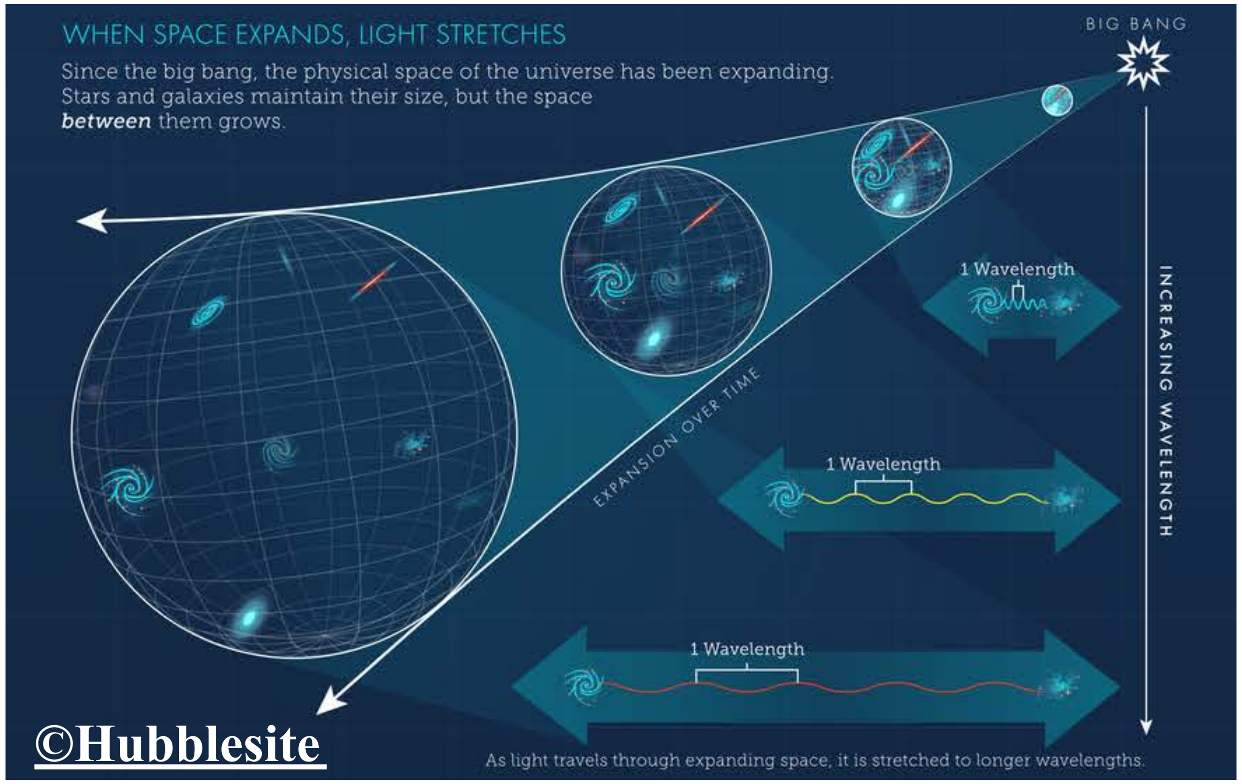
mapping the universe by measuring

redshift: $z = \dfrac{\lambda_{\rm obs} - \lambda_{em}}{\lambda_{\rm em}}$

angular position: (θ, ϕ)


WHEN SPACE EXPANDS, LIGHT STRETCHES

Since the big bang, the physical space of the universe has been expanding. Stars and galaxies maintain their size, but the space *between* them grows.

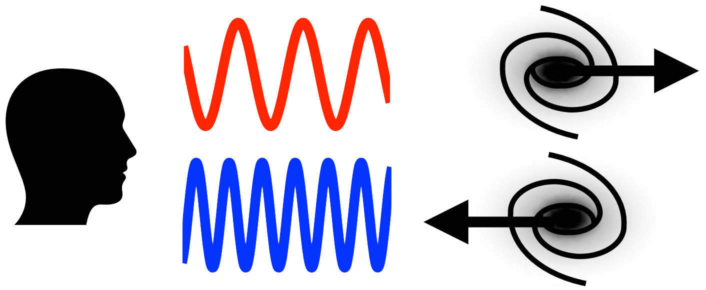
As light travels through expanding space, it is stretched to longer wavelengths.

©Hubblesite

Observed redshift

Cosmological redshift (Hubble flow)

+ **Doppler effect (peculiar velocity)**



Observed position (inferred from redshift) ≠ Actual position

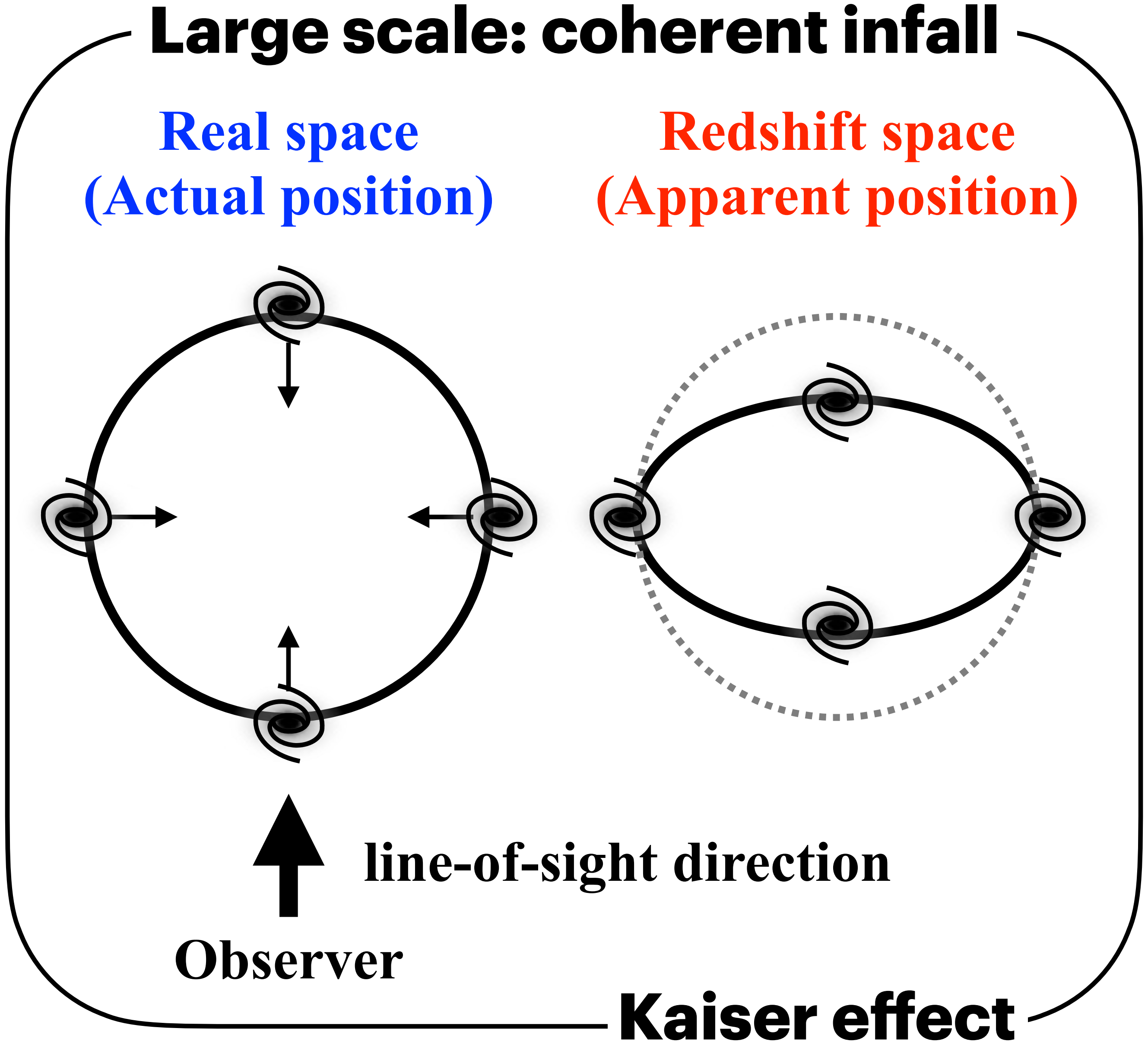
1.4 Redshift space distortions (RSD)

Observed galaxy distribution appears distorted

↓

Redshift space distortions (RSD)

Primary source: Doppler effect induced by peculiar velocity



Large scale: coherent infall

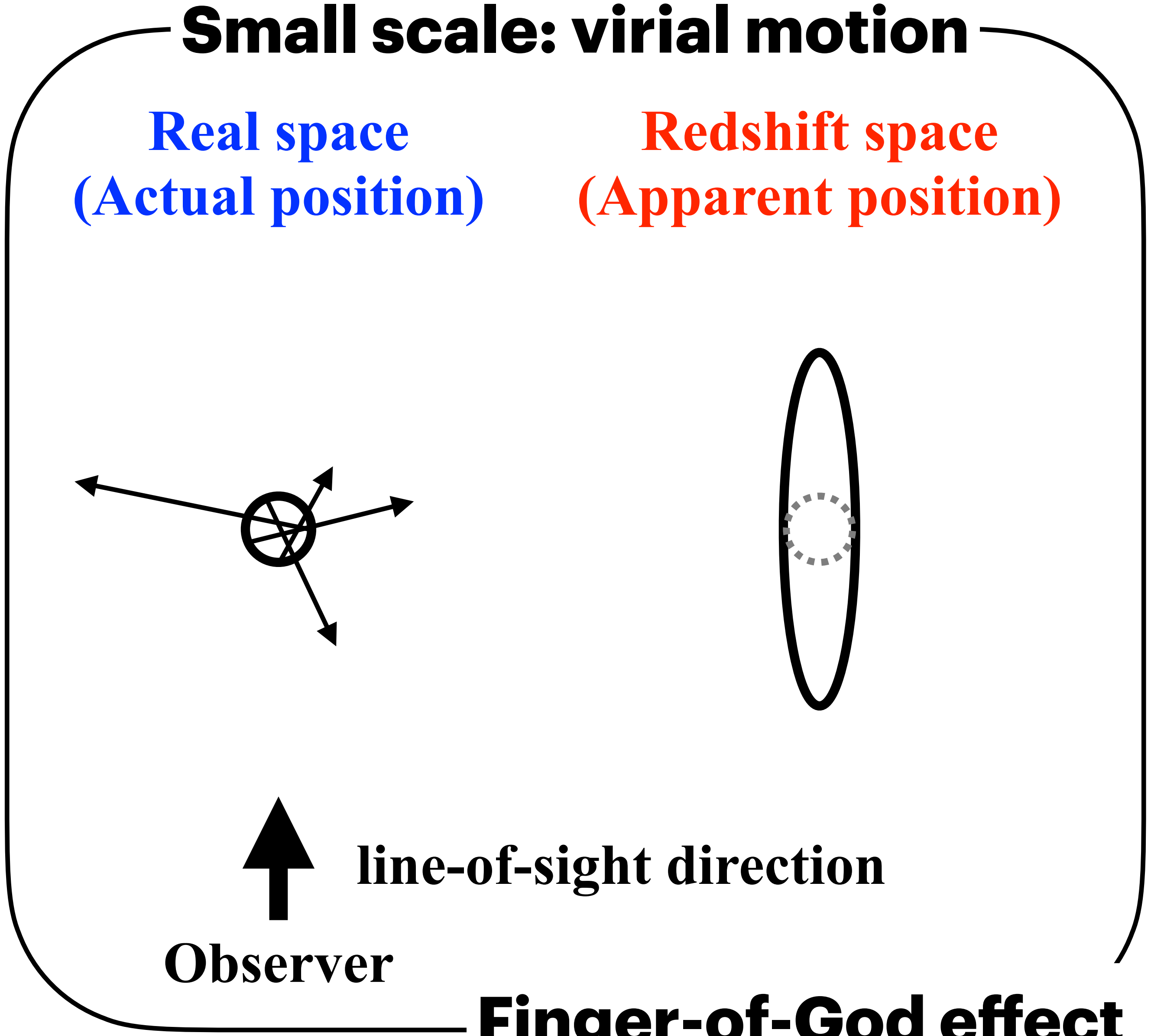
Real space (Actual position)

Redshift space (Apparent position)

line-of-sight direction

Observer

Kaiser effect



Small scale: virial motion

Real space (Actual position)

Redshift space (Apparent position)

line-of-sight direction

Observer

Finger-of-God effect

1.5 Two-point correlation function

Theory vs Observation

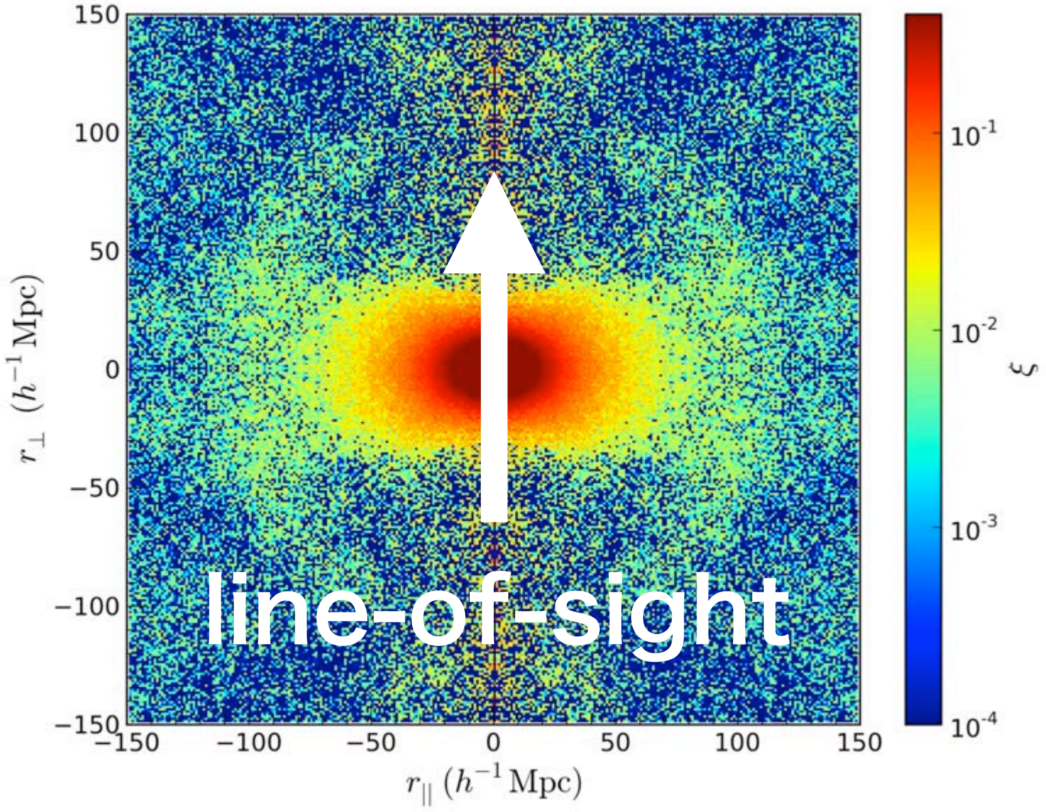
$$\xi^{(\mathrm{s})}(s,\mu) = \left\langle \delta^{(\mathrm{s})}(\boldsymbol{s}_1)\delta^{(\mathrm{s})}(\boldsymbol{s}_2) \right\rangle \quad \sim \text{galaxy number count with a fixed separation}$$

A specific direction: line-of-sight, breaks the statistical isotropy.

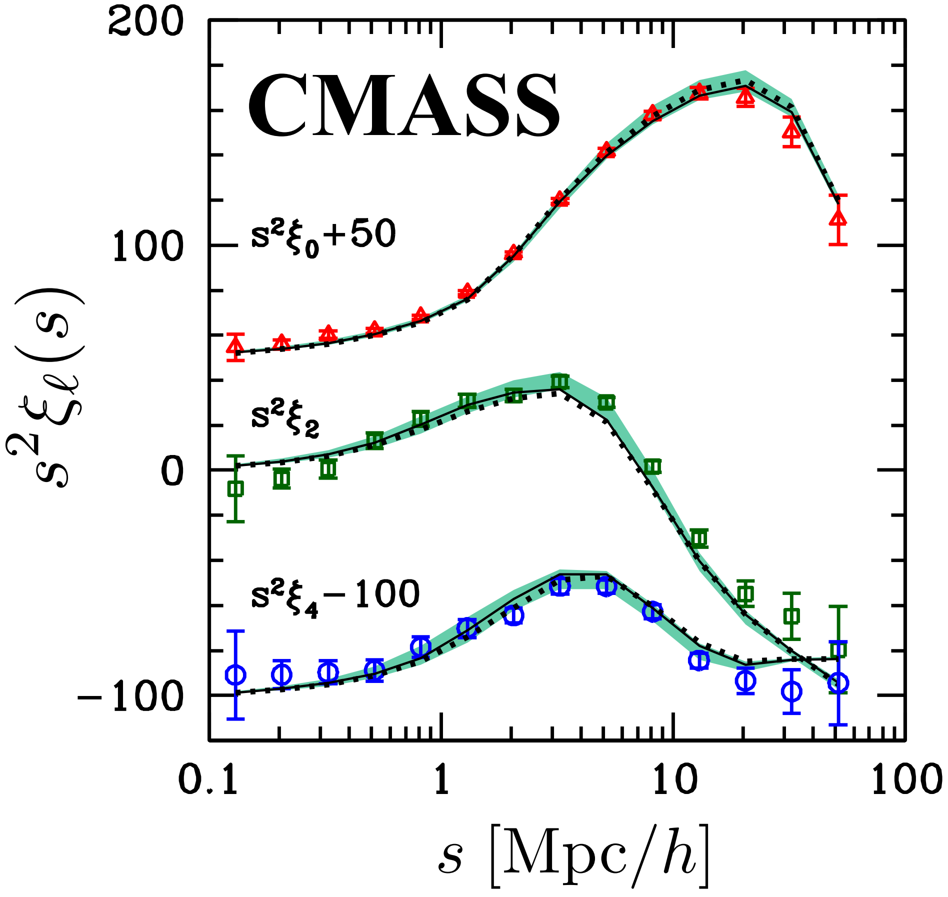
$$\xi_\ell(s) = \frac{2\ell+1}{2}\int_{-1}^{1} \mathrm{d}\mu\ \xi^{(\mathrm{s})}(s,\mu)\mathscr{P}_\ell(\mu) \qquad (\mathscr{P}_\ell(\mu)\text{: Legendre polynomial})$$

Two-point correlation function

SDSS DR11 #700,000, $0.43 < z < 0.7$



H.Guo et al. [1407.4811]



1.6 Cosmology in RSD

N. Kaiser (1987)

Redshift space ↔ Real space

redshift space $\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H(z)}\left(\boldsymbol{v}\cdot\hat{\boldsymbol{z}}\right)\hat{\boldsymbol{z}}$ (special relativity, $v \ll 1$)

real space

$\hat{z}$: constant line-of-sight vector

continuity equation (linear): $\dot{\delta}_{\rm L} + \frac{1}{a}\nabla\cdot\boldsymbol{v} \simeq 0$

conservation law: $\left(1+\delta^{(\rm s)}(\boldsymbol{s})\right){\rm d}^3s = \left(1+\delta(\boldsymbol{r})\right){\rm d}^3r$

Kaiser formula

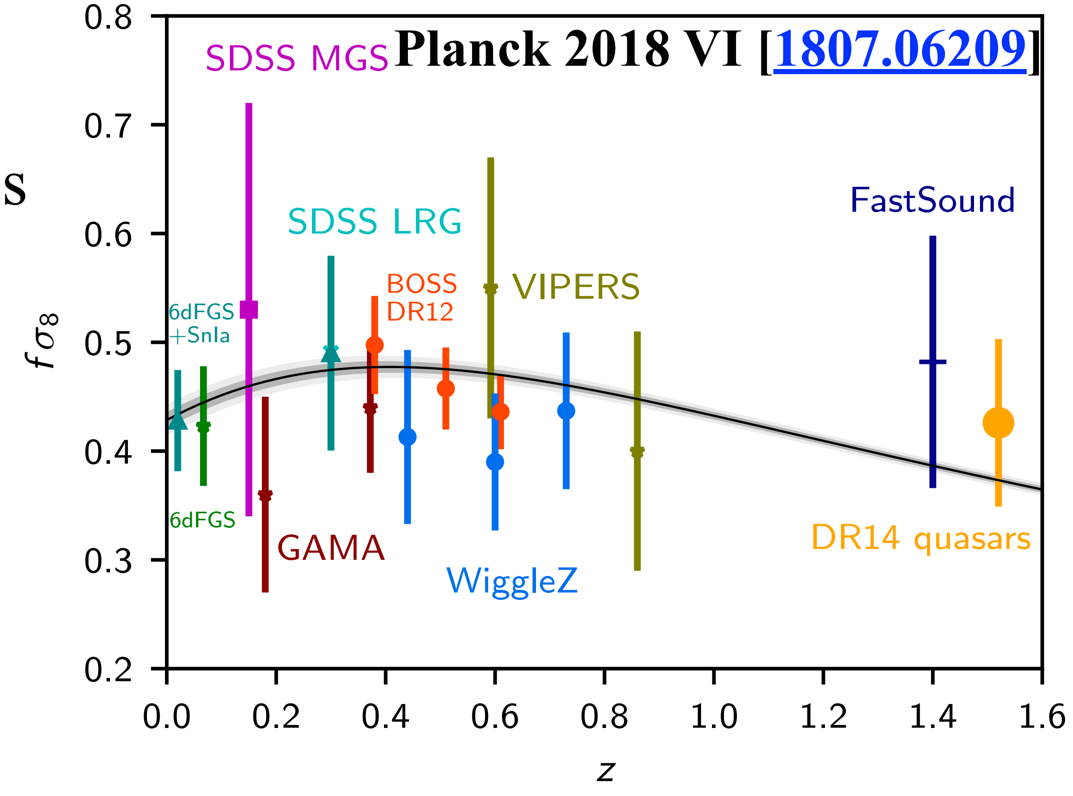
$$\delta^{(\rm s)}(\boldsymbol{k}) = \left(b + f(\hat{\boldsymbol{k}}\cdot\hat{z})^2\right)\delta_{\rm L}(\boldsymbol{k})$$

$f \equiv \frac{{\rm d}\ln\delta_{\rm L}}{{\rm d}\ln a}$: linear growth rate

Linear growth rate depends on the gravity theory

→ a **probe of gravity** on cosmological scales

For ΛCDM: $f \approx \left(\Omega_{\rm m}(a)\right)^{0.55}$



Planck 2018 VI [1807.06209]

Relativistic effects on redshift-space distortions at quasi-linear scales

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, **S.Saga** [1803.04294]
A.Taruya, **S.Saga**, M-A.Breton, Y.Rasera, T.Fujita [1908.03854]
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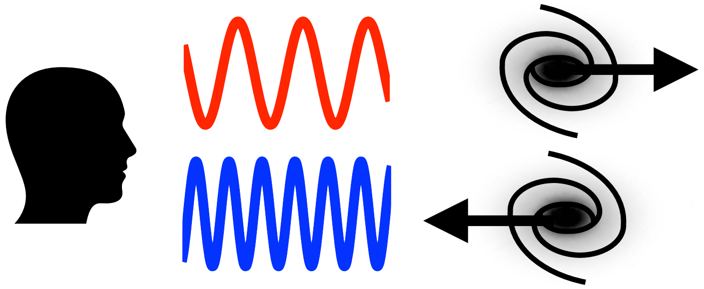
S.Saga et al. in prep.

2.1 General relativistic effects

Observed redshift

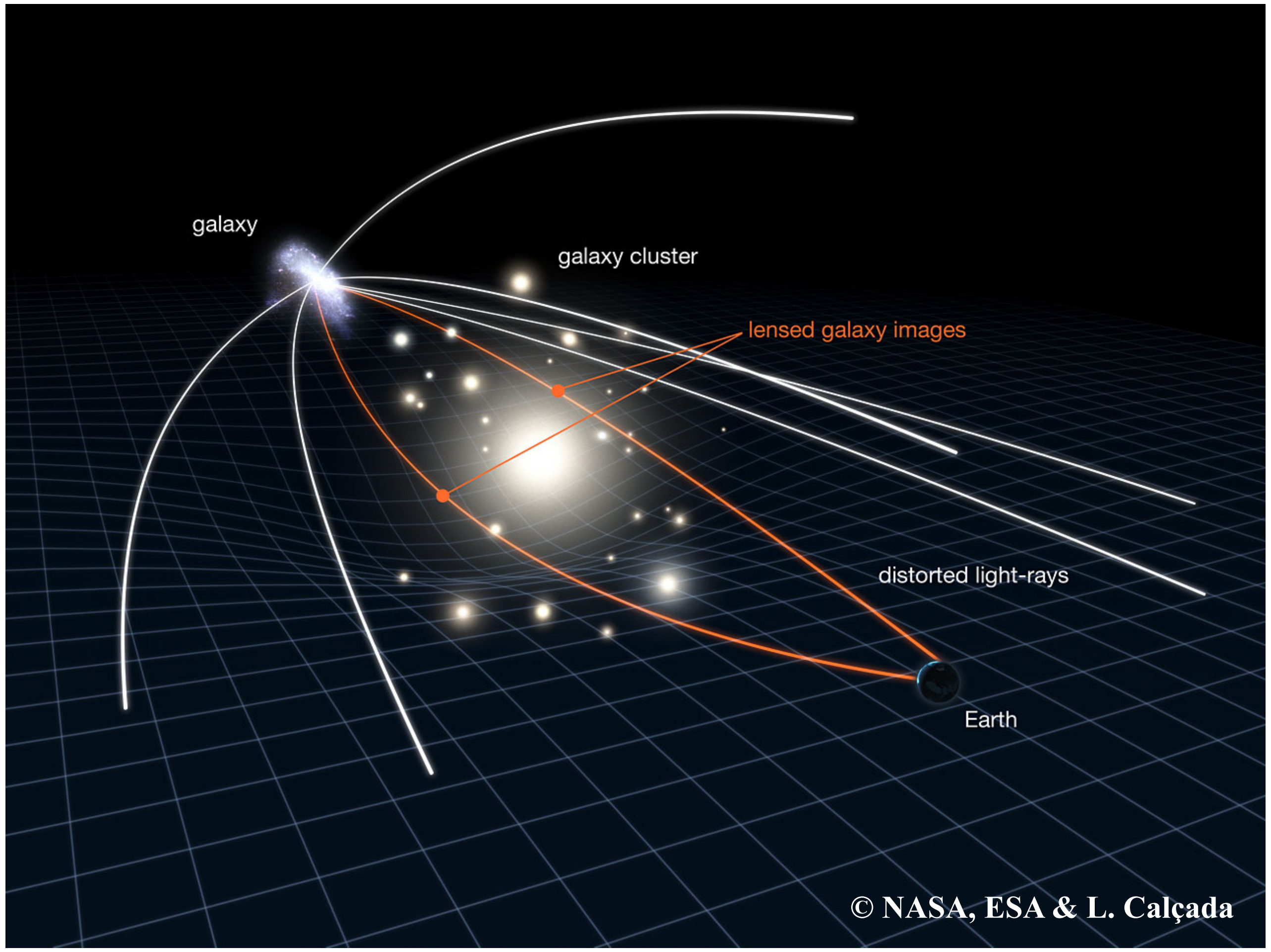
Cosmological redshift (Hubble flow)

+ **Doppler effect (peculiar velocity)**



Other relativistic effects:

- **+ gravitational redshift (Sachs-Wolfe)**
- **+ integrated Sachs-Wolfe**
- **+ Shapiro time delay**
- **+ gravitational lensing**
- **+ …**



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2.2 Relativistic RSD

A.Challinor and A.Lewis [1105.5292]
C.Bonvin and R.Durrer [1105.5280]
J.Yoo [1409.3223], and many works

How do relativistic effects imprint on redshift space?

Perturbed FLRW: $\mathrm{d}s^2 = \left[-(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Psi)\mathrm{d}x^2\right]$

Solve the geodesic eq.: $\dfrac{\mathrm{d}k^\mu}{\mathrm{d}\lambda} + \Gamma^\mu_{\alpha\beta}k^\alpha k^\beta = 0$

Define observed redshift including all effects: $1+z = \dfrac{(k_\mu u^\mu)_\mathrm{S}}{(k_\mu u^\mu)_\mathrm{O}}$

Redshift space including possible relativistic effects

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}$$ (Doppler effect)

(weak field approx.)

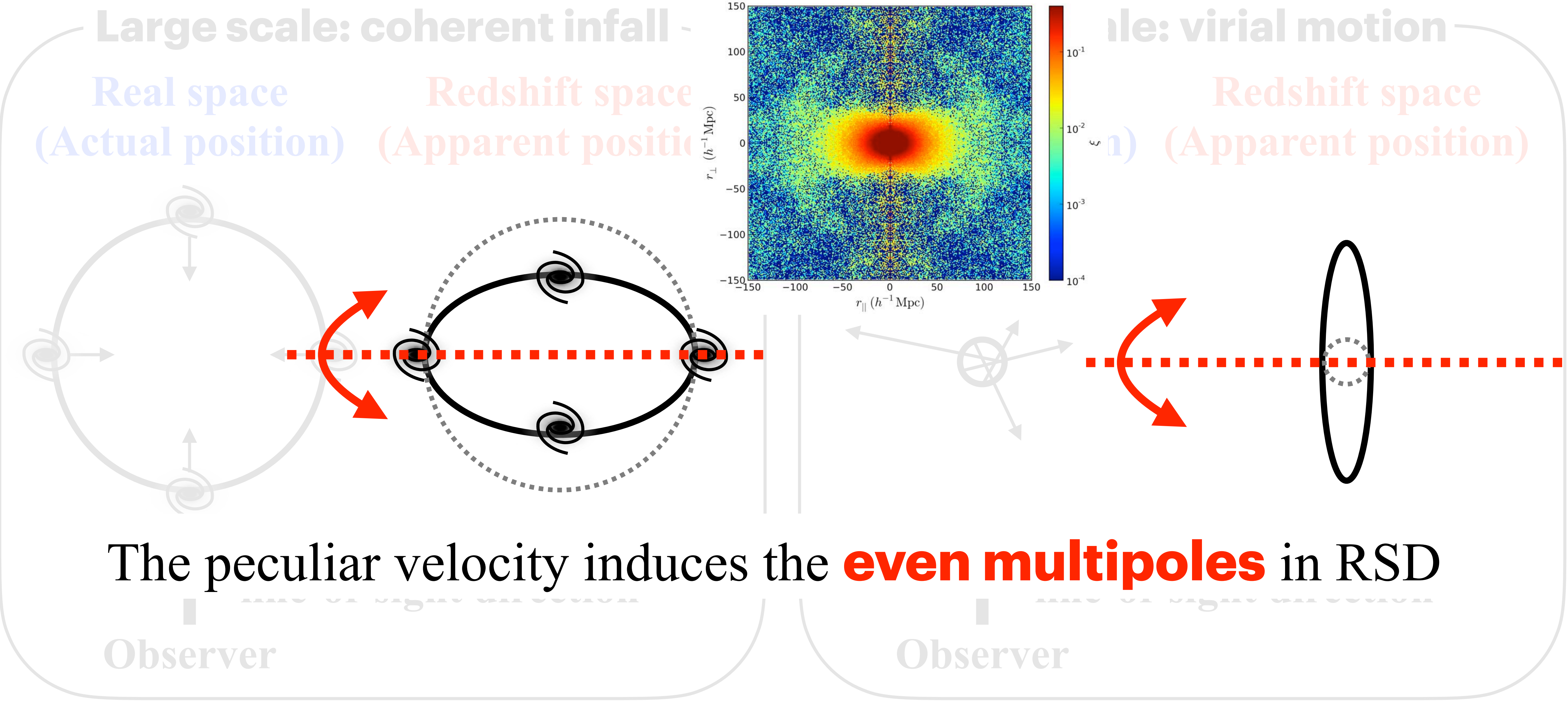
$$+\frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^\chi (\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^\chi (\chi-\chi')\,\nabla_\perp(\Phi+\Psi)\mathrm{d}\chi'$$

- gravitational redshift
- Transverse Doppler

- Shapiro time delay
- integrated Sachs-Wolfe
- gravitational lensing

What is the unique signature of relativistic effects ?

2.3 Recalling Doppler effects



The peculiar velocity induces the **even multipoles** in RSD

Kaiser formula

$$\delta^{(\mathrm{s})}(\boldsymbol{k}) = \left(b + f(\hat{\boldsymbol{k}}\cdot\hat{z})^2\right)\delta_{\mathrm{L}}(\boldsymbol{k}) \qquad f \equiv \frac{\mathrm{d}\ln\delta_{\mathrm{L}}}{\mathrm{d}\ln a} : \text{linear growth rate}$$

(line-of-sight vector)^2 → Quadrupole anisotropy

2.4 Linear theory of relativistic RSD

Observed redshift including possible relativistic effects

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}$$
$$+\frac{1+z}{H}\left(-\Phi+\frac{1}{2}v^2-\int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}}-\int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}}-\int_0^{\chi}(\chi-\chi')\,\nabla_{\perp}(\Phi+\Psi)\mathrm{d}\chi'$$

c.f. Kaiser formula

conservation law: $\left(1+\delta^{(\mathrm{S})}(s)\right)\mathrm{d}^3s=\left(1+\delta(r)\right)\mathrm{d}^3r$

(linear approximation)

Linear density field with relativistic effects

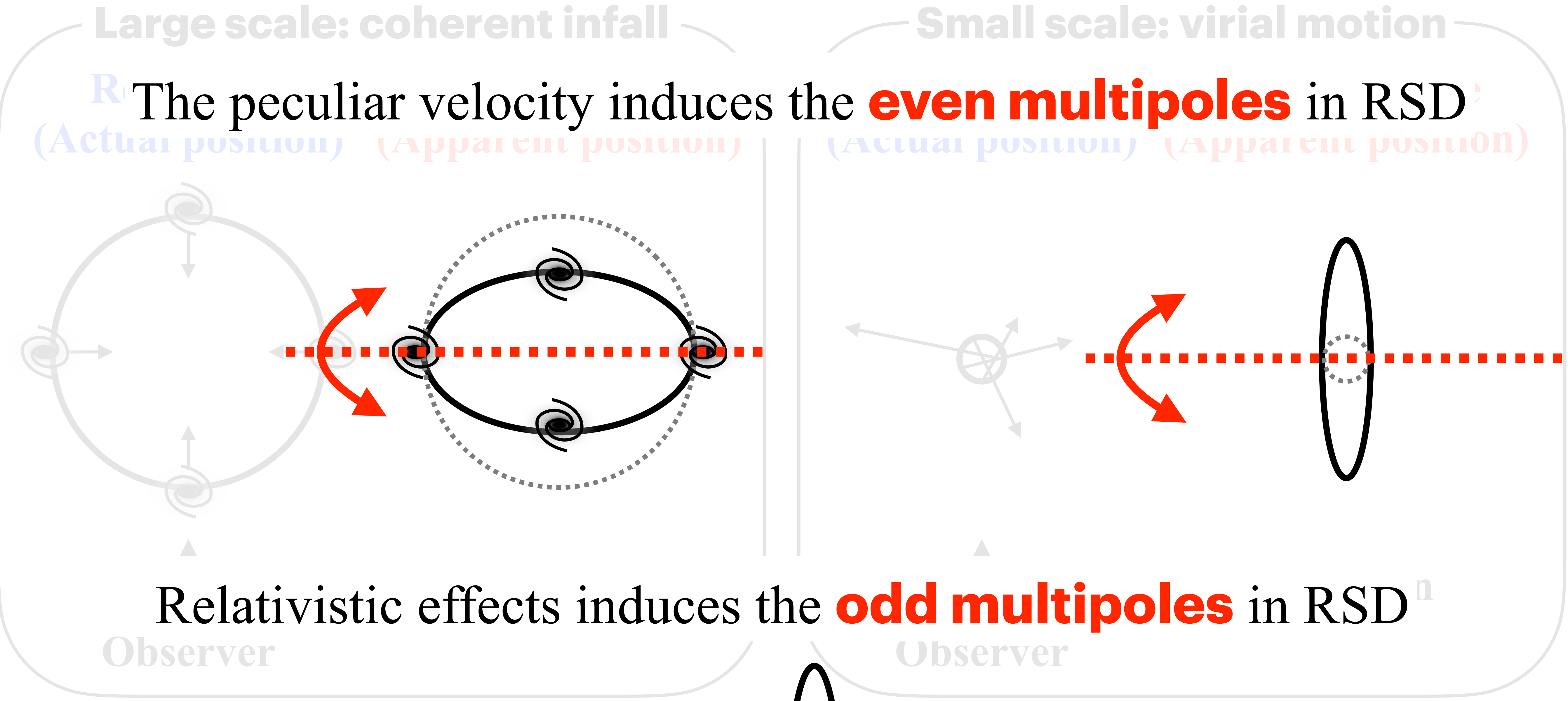
$$\delta^{(\mathrm{s})} = b\delta - \frac{1}{\mathcal{H}}\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial\boldsymbol{r}}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{v}\right)$$
$$-\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\hat{\boldsymbol{r}}\cdot\boldsymbol{v}+\frac{1}{\mathcal{H}}\left(\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial\boldsymbol{r}}\Psi+\mathcal{H}\hat{\boldsymbol{r}}\cdot\boldsymbol{v}+\hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{v}}\right)$$
$$-2\Phi+\Psi+\frac{\dot{\Phi}}{\mathcal{H}}+\frac{1}{r}\int_0^r\mathrm{d}r'\,\left(2-\frac{r-r'}{r'}\Delta_{\Omega}\right)(\Phi+\Psi)+\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\left(\Psi+\int_0^r\mathrm{d}r'\,\left(\dot{\Psi}+\dot{\Phi}\right)\right)$$

(the line-of-sight vector is highlighted by **red**)

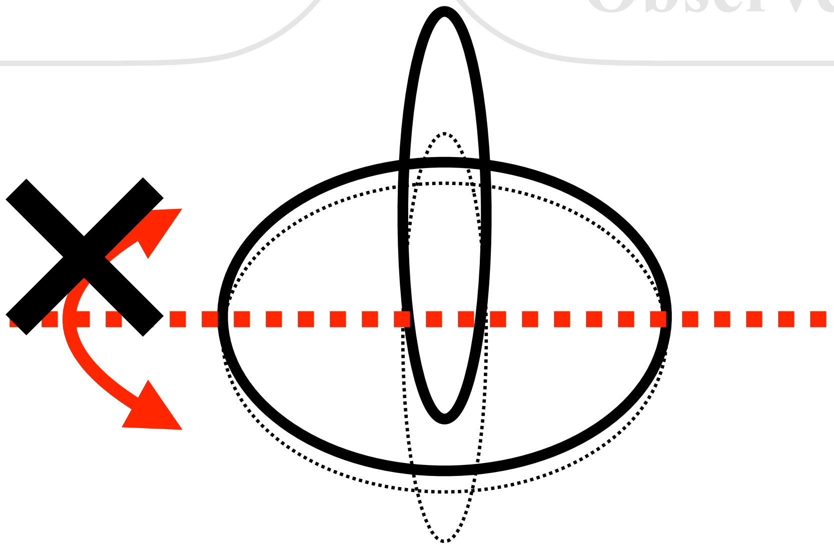
(line-of-sight vector)$^{\mathrm{odd}}$ ⇒ **odd multipole anisotropies**

2.5 Odd multipole anisotropies

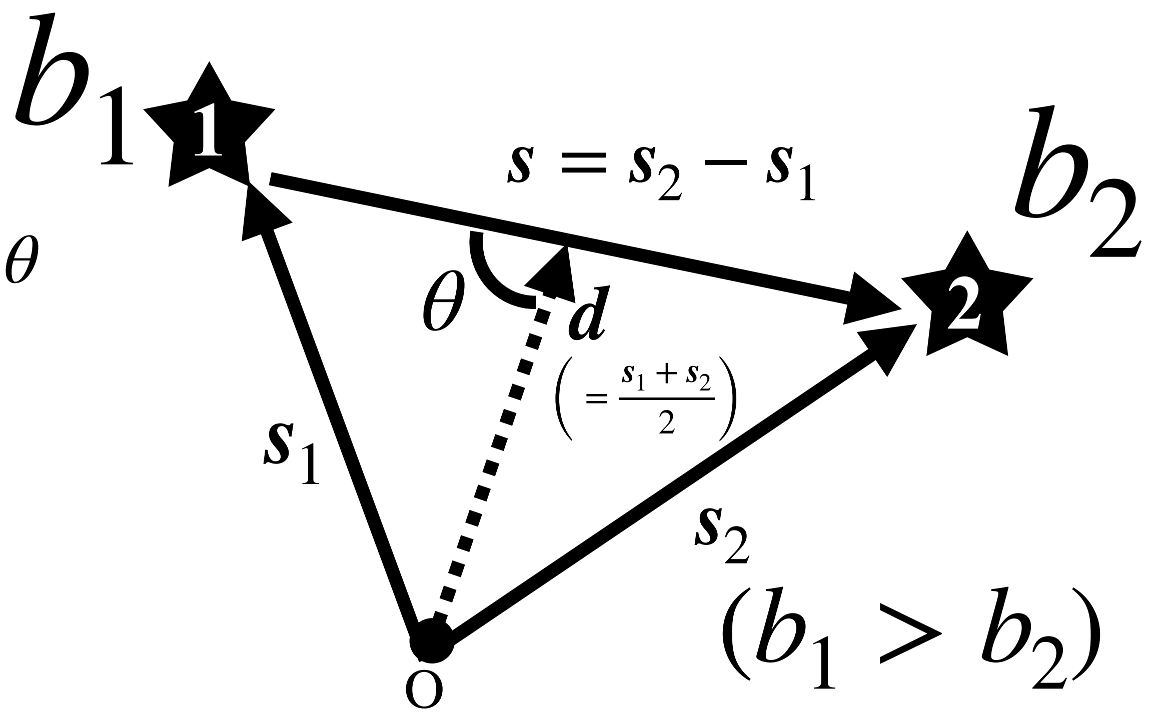
The peculiar velocity induces the **even multipoles** in RSD



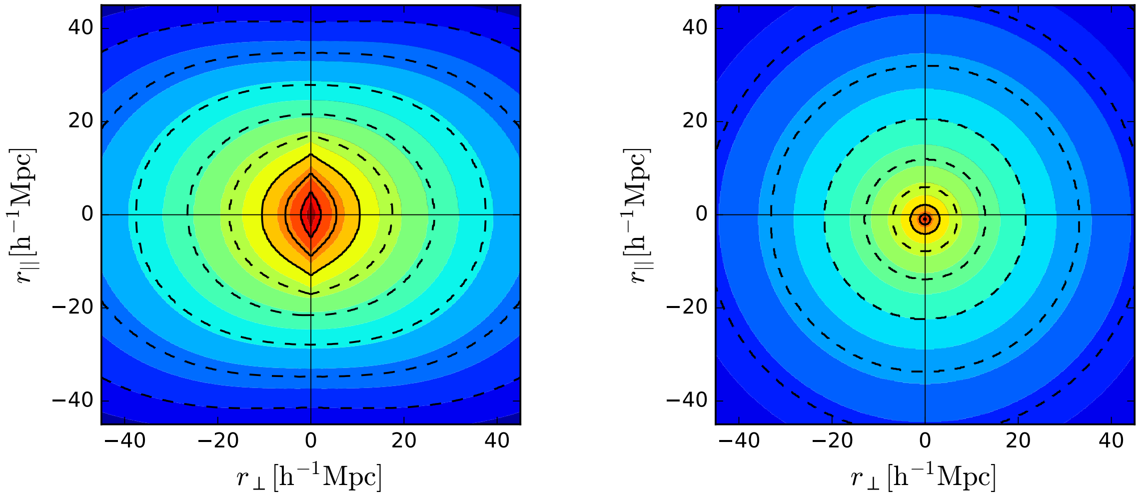
Relativistic effects induces the **odd multipoles** in RSD



2.6 Dipole moment



Dipole moment: $\xi_1 = \dfrac{3}{2}\displaystyle\int_{-1}^{1} \left(\xi^{(\mathrm{S})}(\boldsymbol{s}_1, \boldsymbol{s}_2)\ \cos\theta\right) \mathrm{d}\cos\theta$



Doppler effect **Relativistic effect**

E.Giusarma et al. [1709.07854]

Linear theory A.Challinor and A.Lewis [1105.5292], C.Bonvin and R.Durrer [1105.5280], J.Yoo [1409.3223], ...

implies $\xi_1 \propto (b_1 - b_2) \Rightarrow$ Cross-correlating with different biased object is essential.

Remark:

beyond the distant-observer limit, the Doppler effect also leads to the dipole, called wide-angle effect

Kaiser formula

$$\underset{\text{redshift space}}{\boldsymbol{s}} = \underset{\text{real space}}{\boldsymbol{r}} + \frac{1+z}{H(z)}\left(\boldsymbol{v}\cdot\overset{\hat{\boldsymbol{r}}}{\cancel{\hat{\boldsymbol{z}}}}\right)\overset{\hat{\boldsymbol{r}}}{\cancel{\hat{\boldsymbol{z}}}}$$

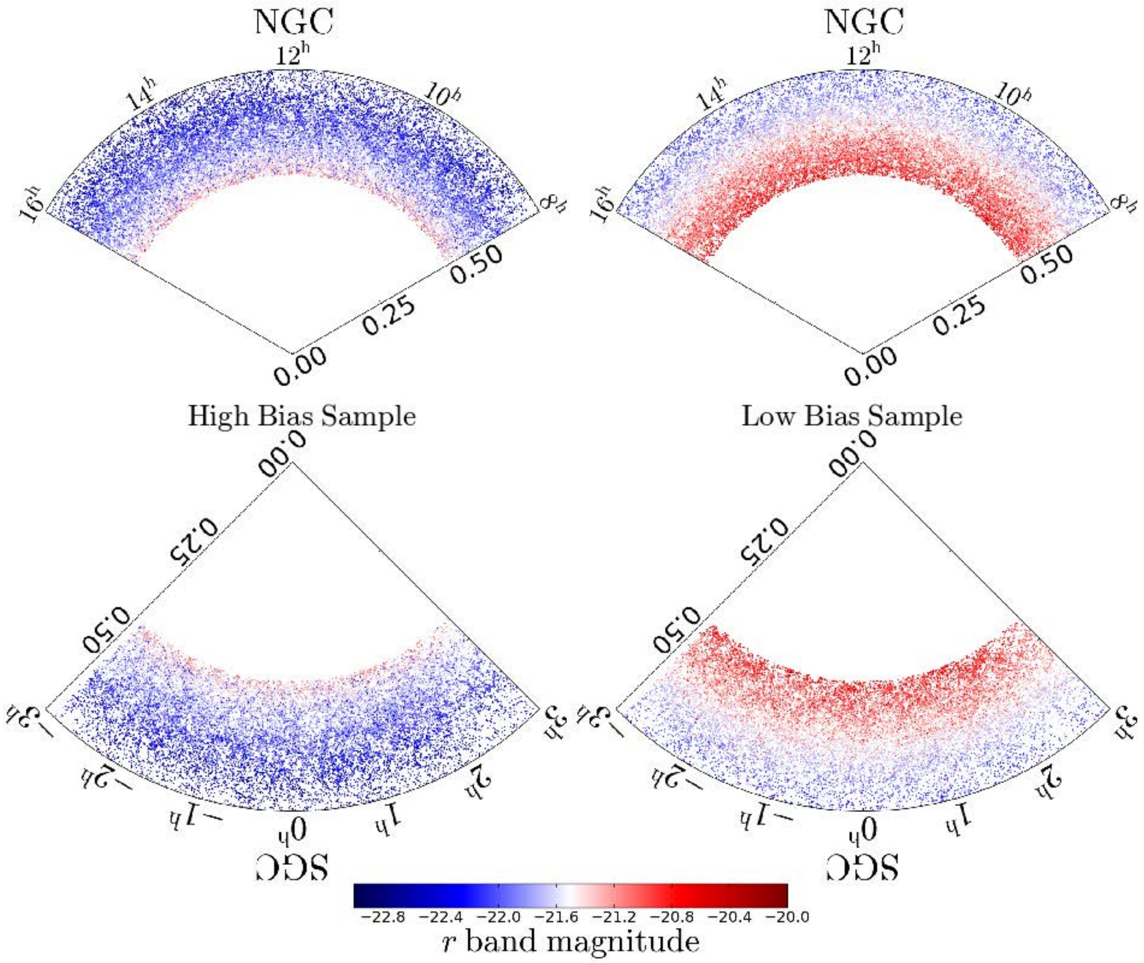
(special relativity, $v \ll 1$)

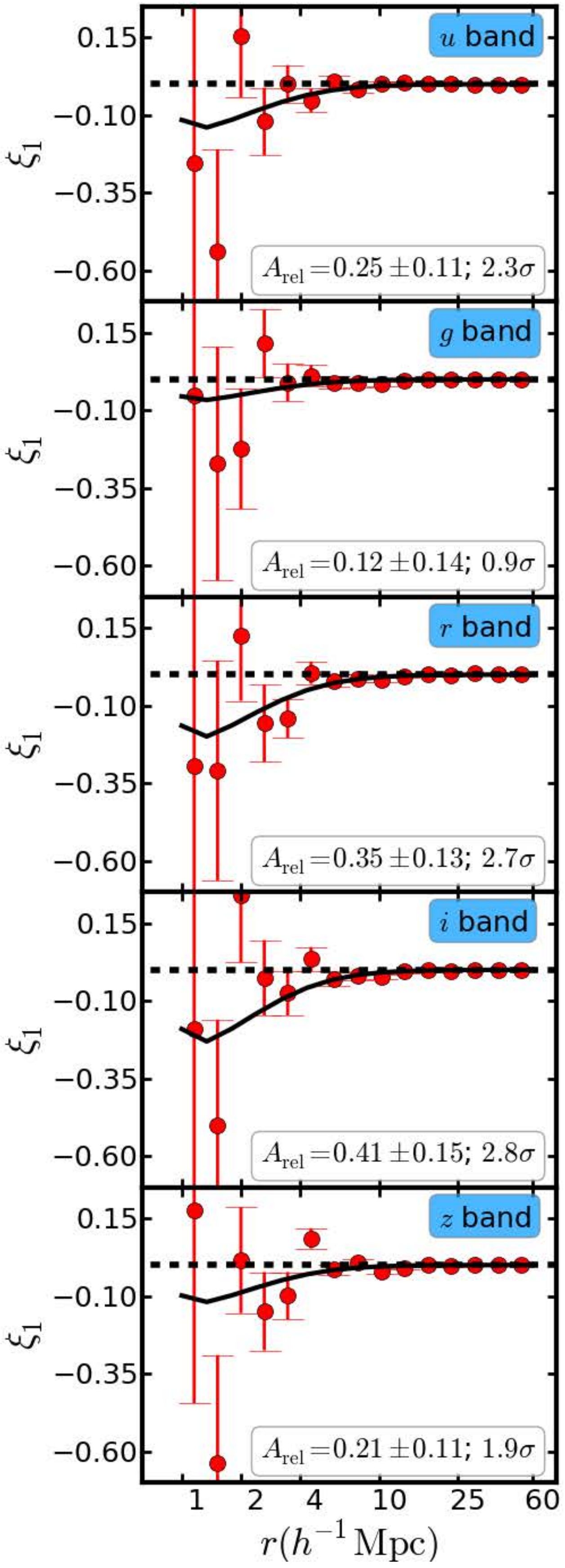
$\underset{\hat{\boldsymbol{r}}}{\cancel{\hat{\boldsymbol{z}}}}$: ~~constant~~ line-of-sight vector

2.7 Evidence of relativistic effects?

2.8σ detection of relativistic effects in SDSS DR12 CMASS galaxy sample

S. Alam et al. [1709.07855]





New cosmological probe of gravity!

Linear regime is well understood.

Q. non-linear (or quasi-linear) regime??

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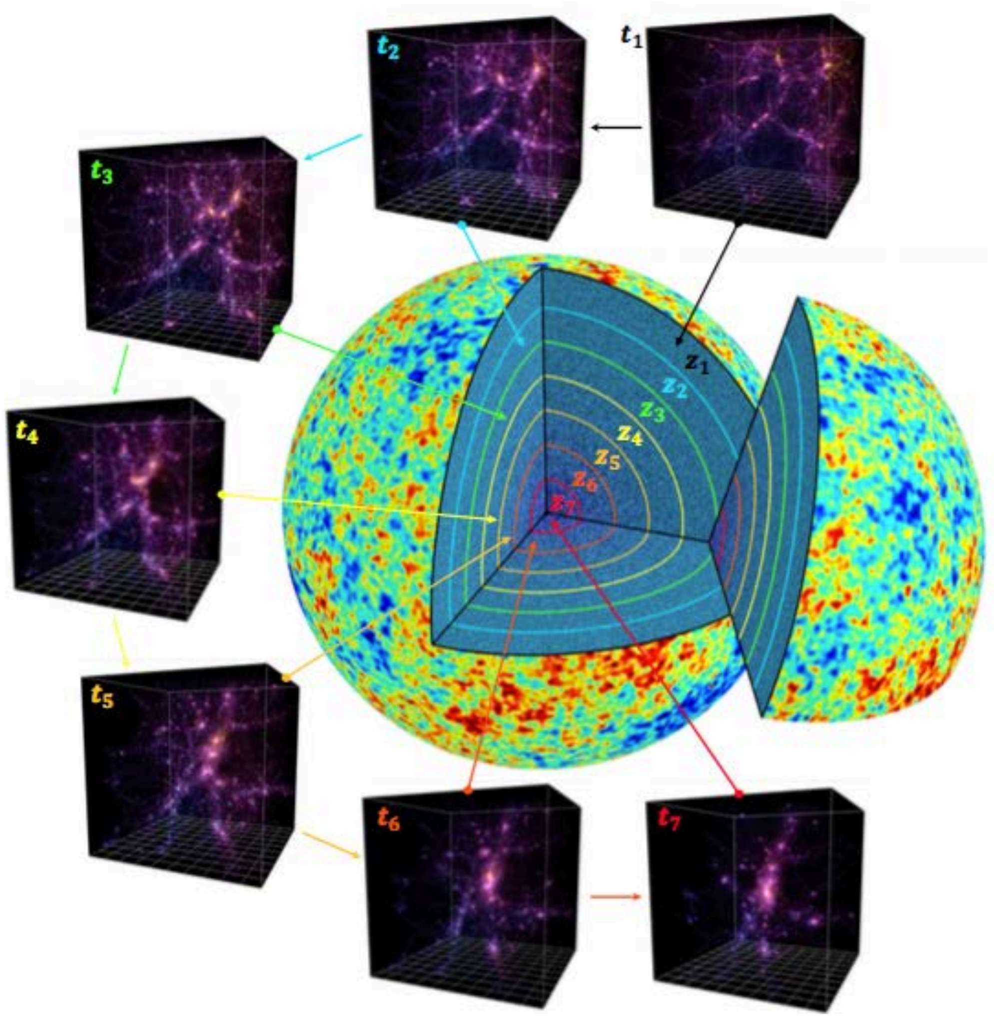
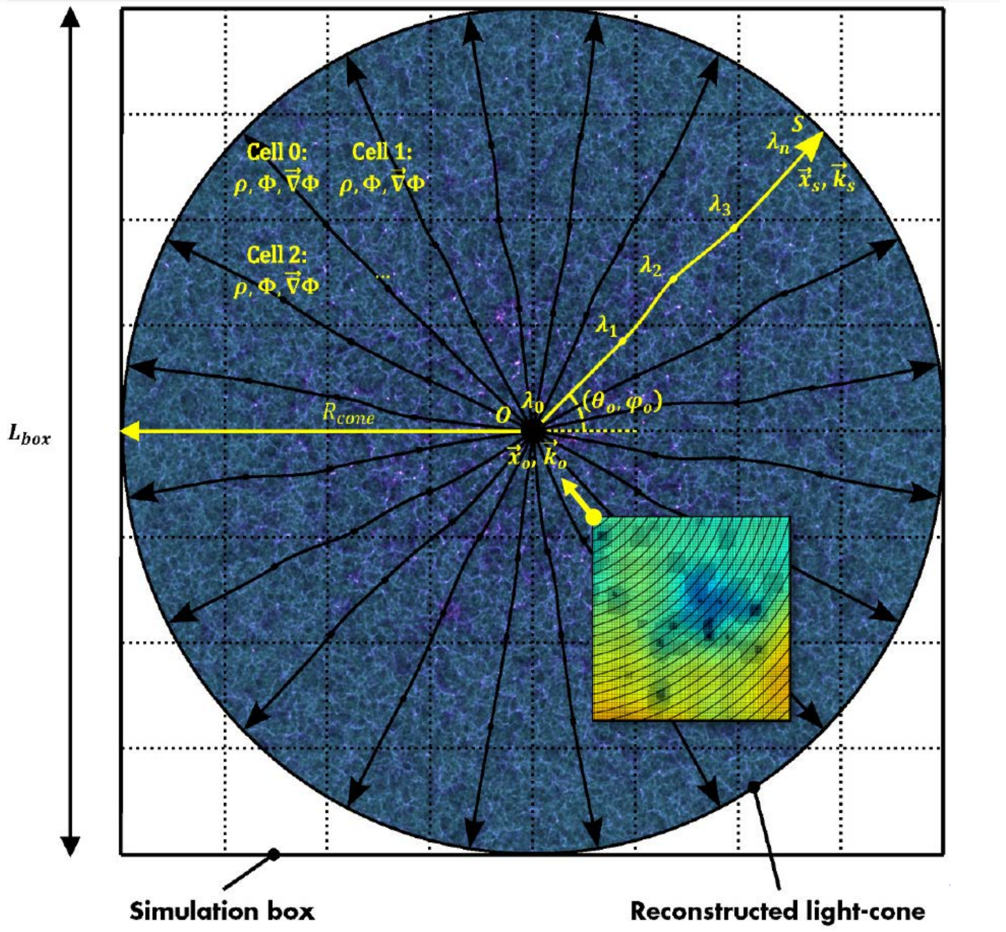
S.Saga et al. in prep.

3.1 N-body simulations

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]

- Using cosmological N-body code RAMSES.
- Storing gravitational potential data on light cone
- Tracing back the light ray to the source by direct integration of geodesic equation
- We obtain "Observed" position and redshift





(2.625 h^{-1}Mpc)^3, 4096^3 DM particles, assumed $\Phi=\Psi$

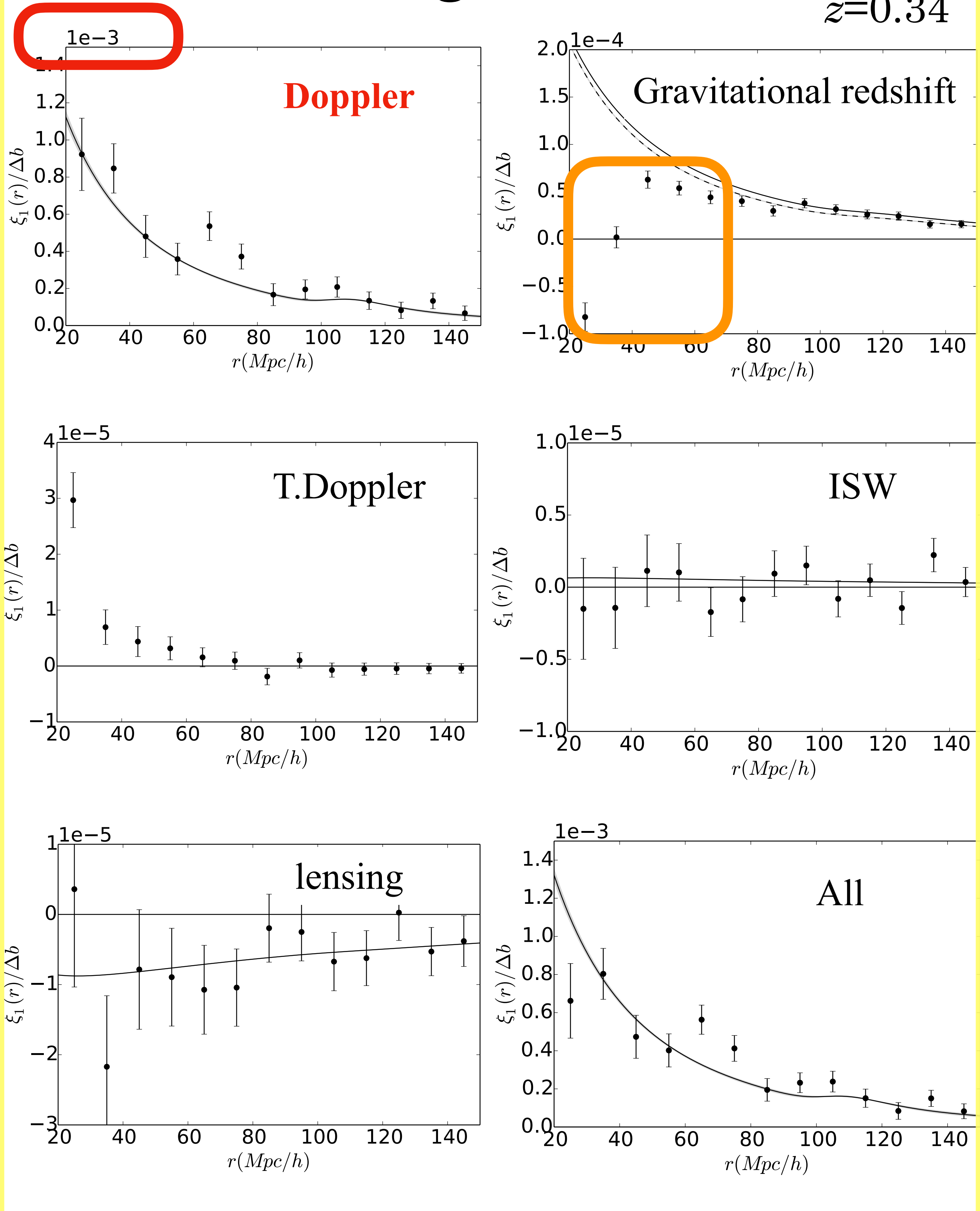
Then, all possible relativistic effects are taking into account

RayGalGroupSims **By Michel-Andres Breton and Yann Rasera**

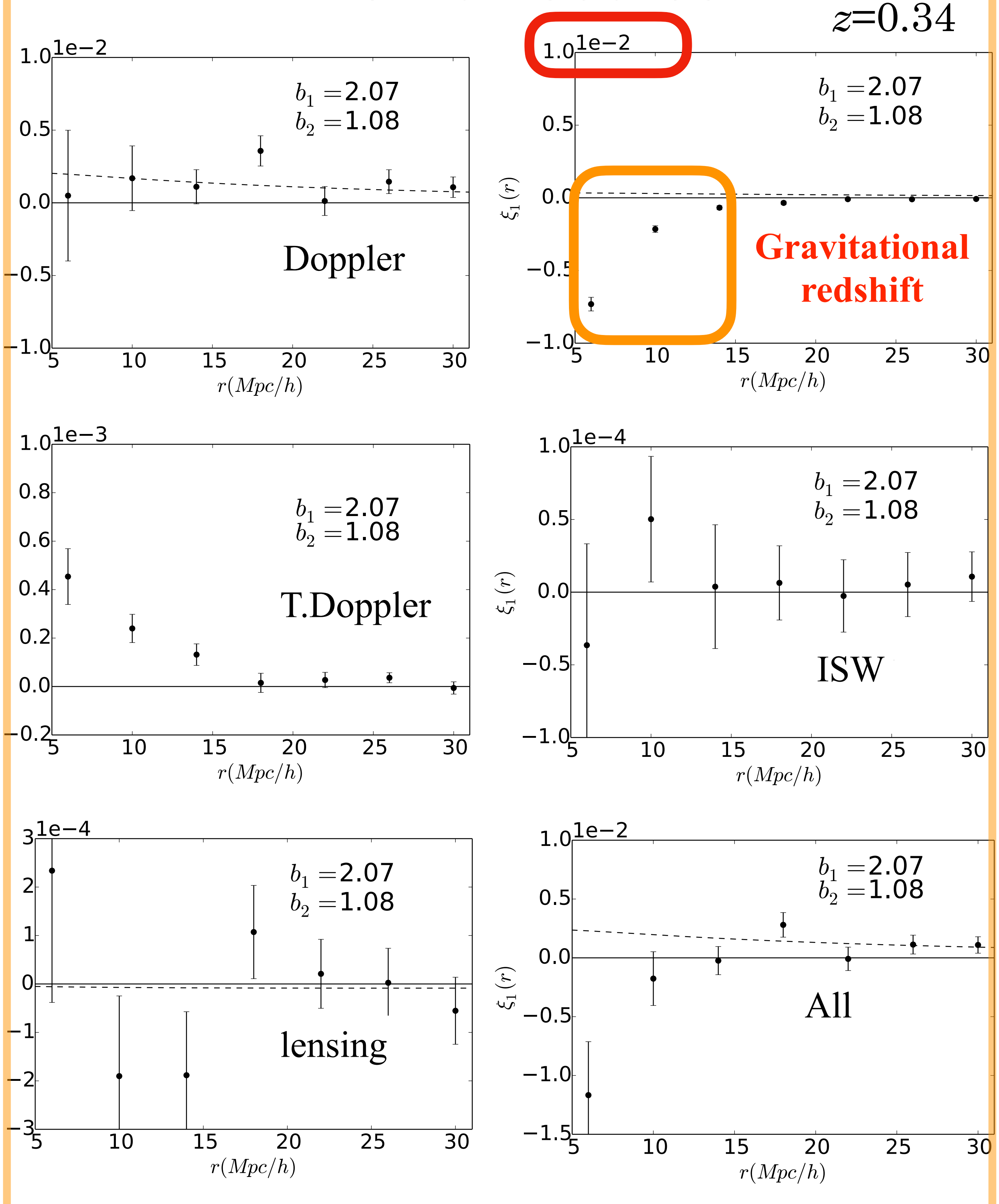
3.2 Measurements in RayGalGroupSims

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]

Large scales



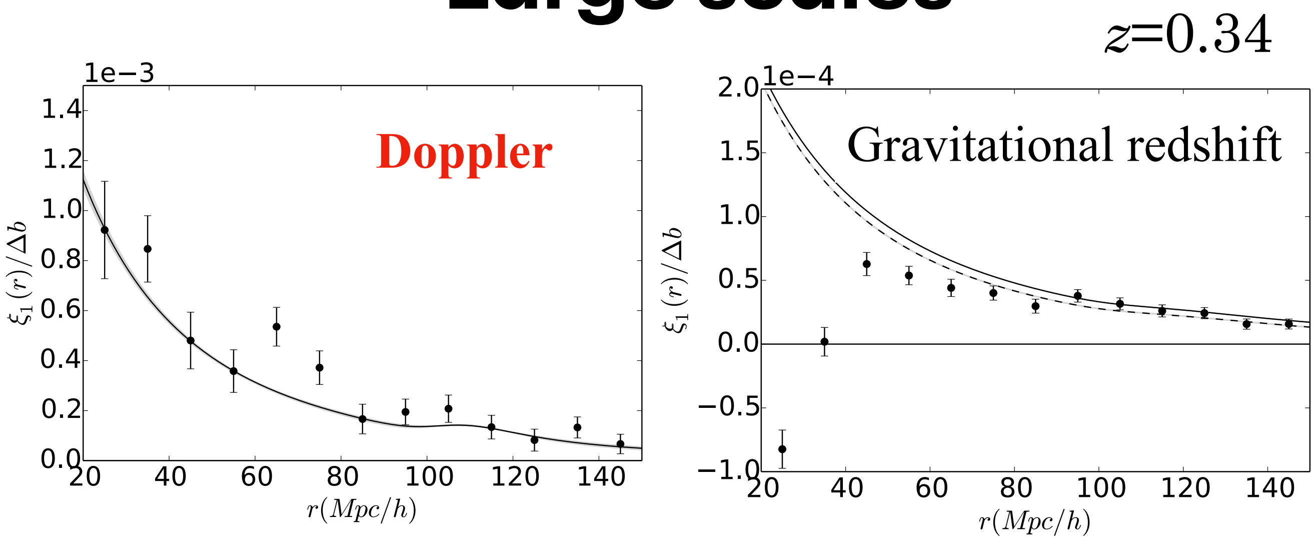
Small scales



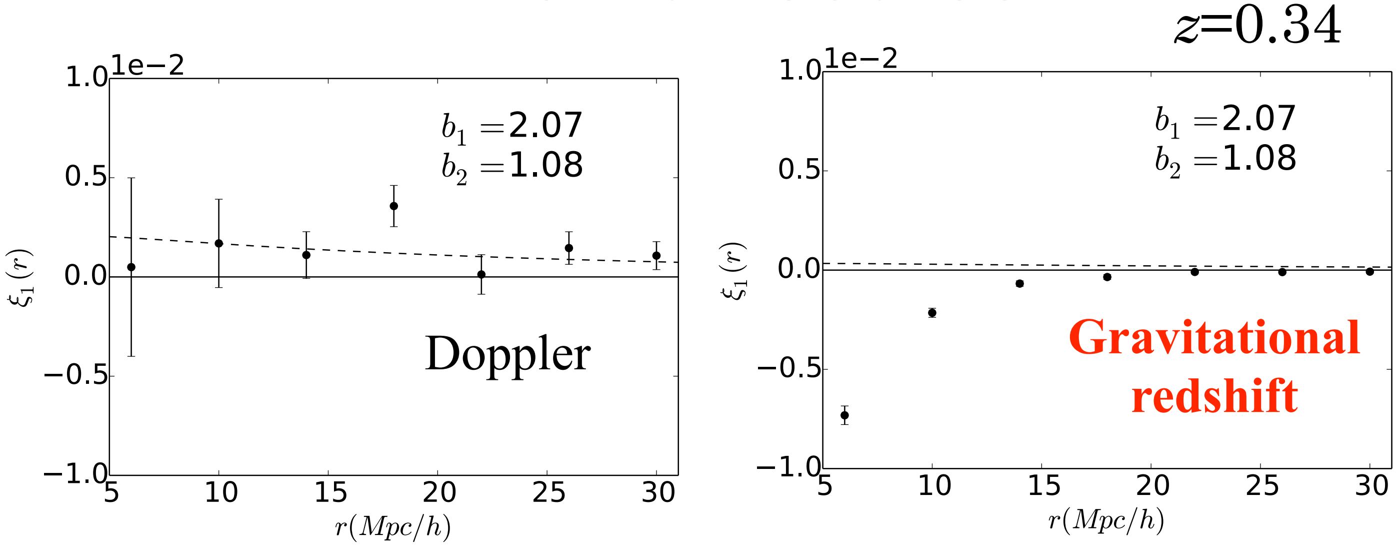
3.3 Measurements in RayGalGroupSims

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]

Large scales



Small scales



Based on RayGalGroupSims and/or linear theory...

Gravitational redshift | **Both effects** | **Doppler effect (wide-angle effect)**

Non-linear regime | **Quasi-linear regime** | **linear regime** → **scale**

We make a quasi-linear model taking into account both effects based on Lagrangian PT (Zel'dovich approx.)

A.Taruya, S.Saga, M-A.Breton, Y.Rasera, T.Fujita [1908.03854]
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

Related works:
Castorina & White [1803.08185]
E. Di Dio & Seljak [1811.03054]
F.Beutler & E. Di Dio [2004.08014]

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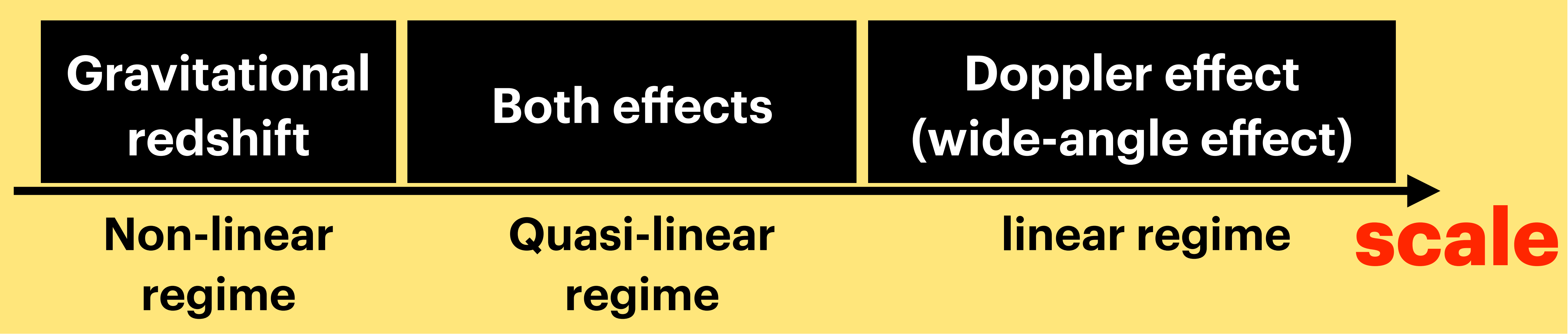
S.Saga et al. in prep.

4.1 Quasi-linear modelling

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

All relativistic effects:

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} \quad + \frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^{\chi}(\chi-\chi')\,\nabla_{\perp}(\Phi+\Psi)\mathrm{d}\chi'$$



Gravitational redshift

Both effects

Doppler effect (wide-angle effect)

Non-linear regime

Quasi-linear regime

linear regime

scale

Pick up the dominant contributions

✓ Doppler effect

✓ Gravitational redshift effect

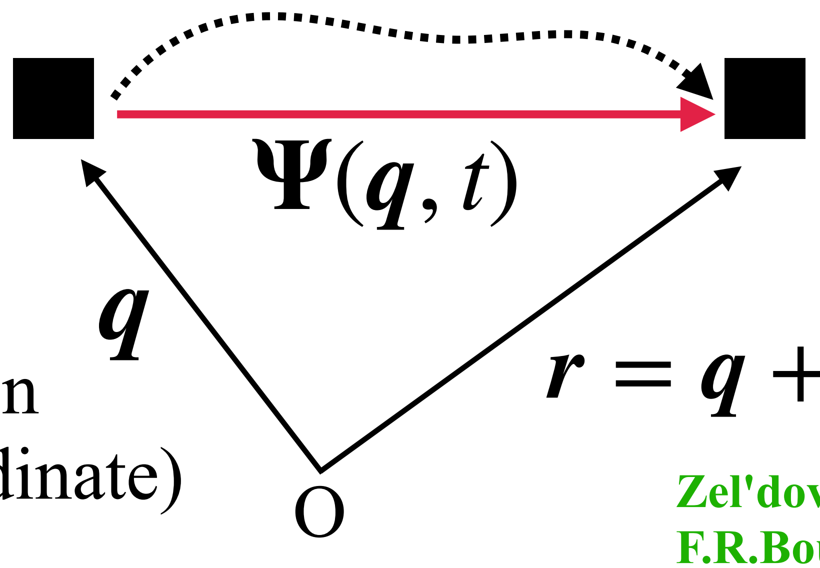
Dominant term:

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} \quad + \frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^{\chi}(\chi-\chi')\,\nabla_{\perp}(\Phi+\Psi)\mathrm{d}\chi'$$

Doppler effect **Gravitational redshift**

4.2 Lagrangian Perturbation Theory (LPT)

– analytical approach to motion of fluid element via Lagrangian picture



trajectory of a fluid element

initial position
(Lagrangian coordinate)

$\boldsymbol{r} = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q}, t)$

$\boldsymbol{\Psi}$: displacement field
(displacement of a fluid element
from the initial position)

Zel'dovich (1970), Shandarin and Zel'dovich (1989), F.R.Bouchet et al, (1992), T.Buchert (1992), F.R.Bouchet et al. [astro-ph/9406013], F.Bernardeau [astro-ph/9311066], ...

Equation of Motion: $\ddot{\boldsymbol{\Psi}} + 2H(z)\dot{\boldsymbol{\Psi}} = -\frac{1}{a^2}\nabla_x \phi(\boldsymbol{x})$

Poisson equation: $\Delta_x \phi(\boldsymbol{x}) = 4\pi G\bar{\rho}a^2\delta(\boldsymbol{x}, z)$

Zel'dovich approximation (1st order LPT):

$$\nabla_q \cdot \boldsymbol{\Psi}_{\rm ZA}(\boldsymbol{q}, t) = -D_+(t)\delta_{\rm L}(\boldsymbol{q}) \quad (D_+(t)\text{: Linear growth factor})$$

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \frac{1+z}{H}\Phi \qquad \boldsymbol{r} = \boldsymbol{q} + \boldsymbol{\Psi}_{\rm ZA}(\boldsymbol{q}, t)$$

4.3 Modelling gravitational potential

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

Our model:

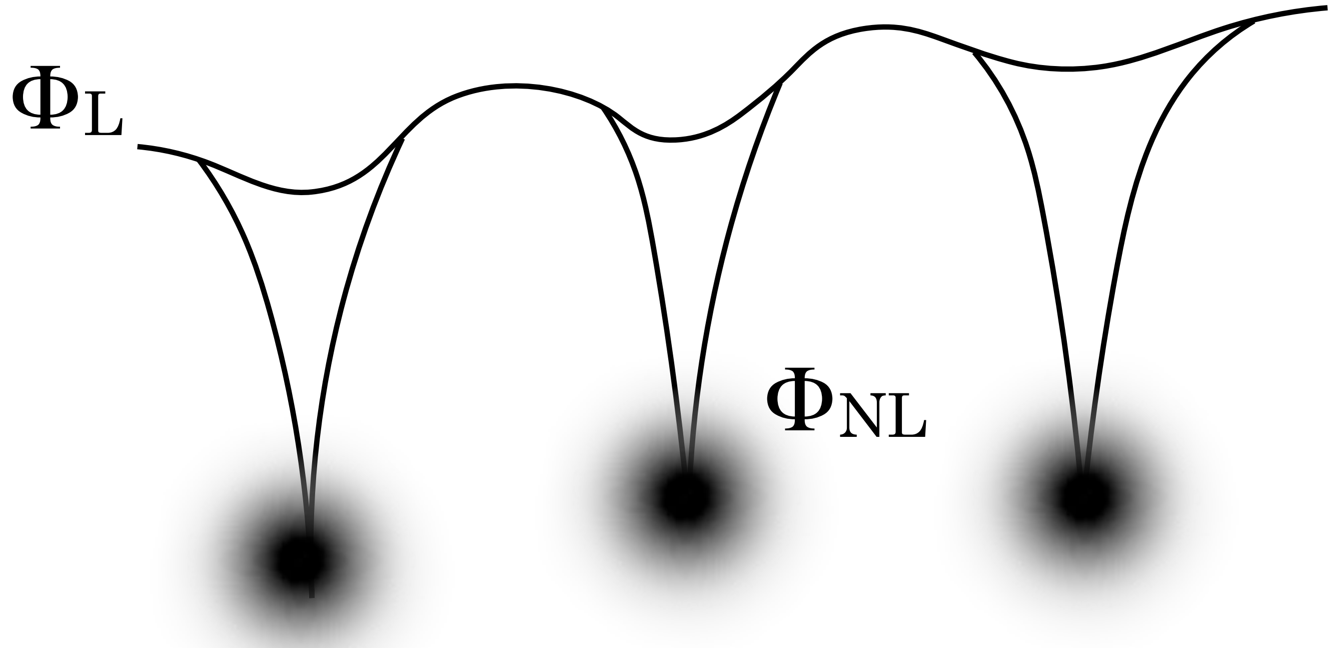
$$s = r + \frac{1+z}{H}(v \cdot \hat{r})\hat{r} - \frac{1+z}{H}\Phi \qquad r = q + \Psi_{\rm ZA}(q, t)$$

The gravitational potential Φ must be modified by the gravitational potential of haloes

Modelling non-linear potential

$$\Phi = \underbrace{\Phi_{\rm L}}_{\text{linear potential}} + \underbrace{\Phi_{\rm NL}}_{\text{non-linear halo potential}}$$

$$\nabla_x^2 \Phi_{\rm L}(x) = -4\pi G a^2 \bar{\rho}(\nabla \cdot \Psi_{\rm ZA})$$



We estimate $\Phi_{\rm NL}$ using

- **NFW profile**
- **N-body simulations**

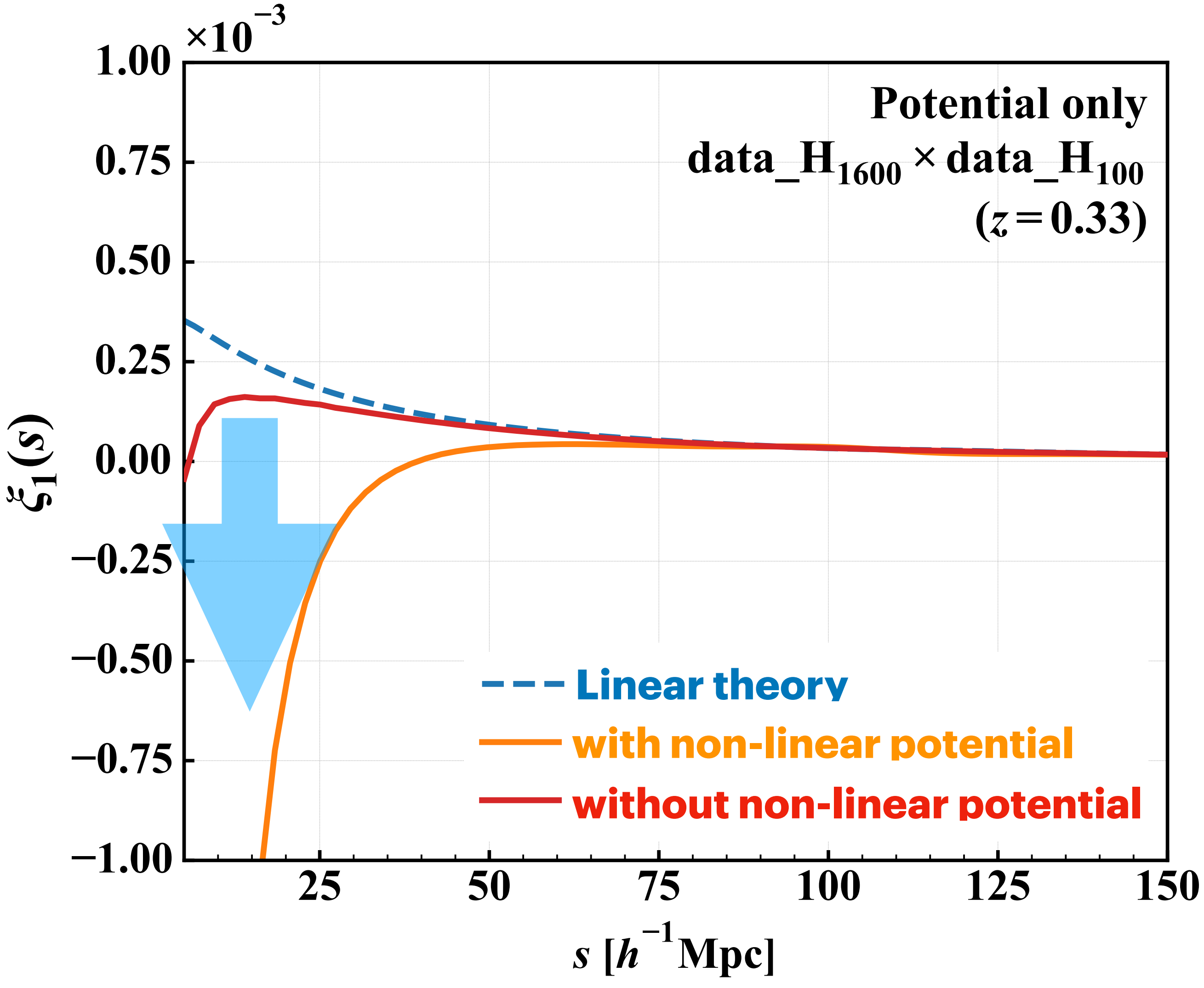
Assumptions: $\Phi_{\rm NL}$ is a constant value determined by halo masses and redshifts

4.4 Analytical prediction

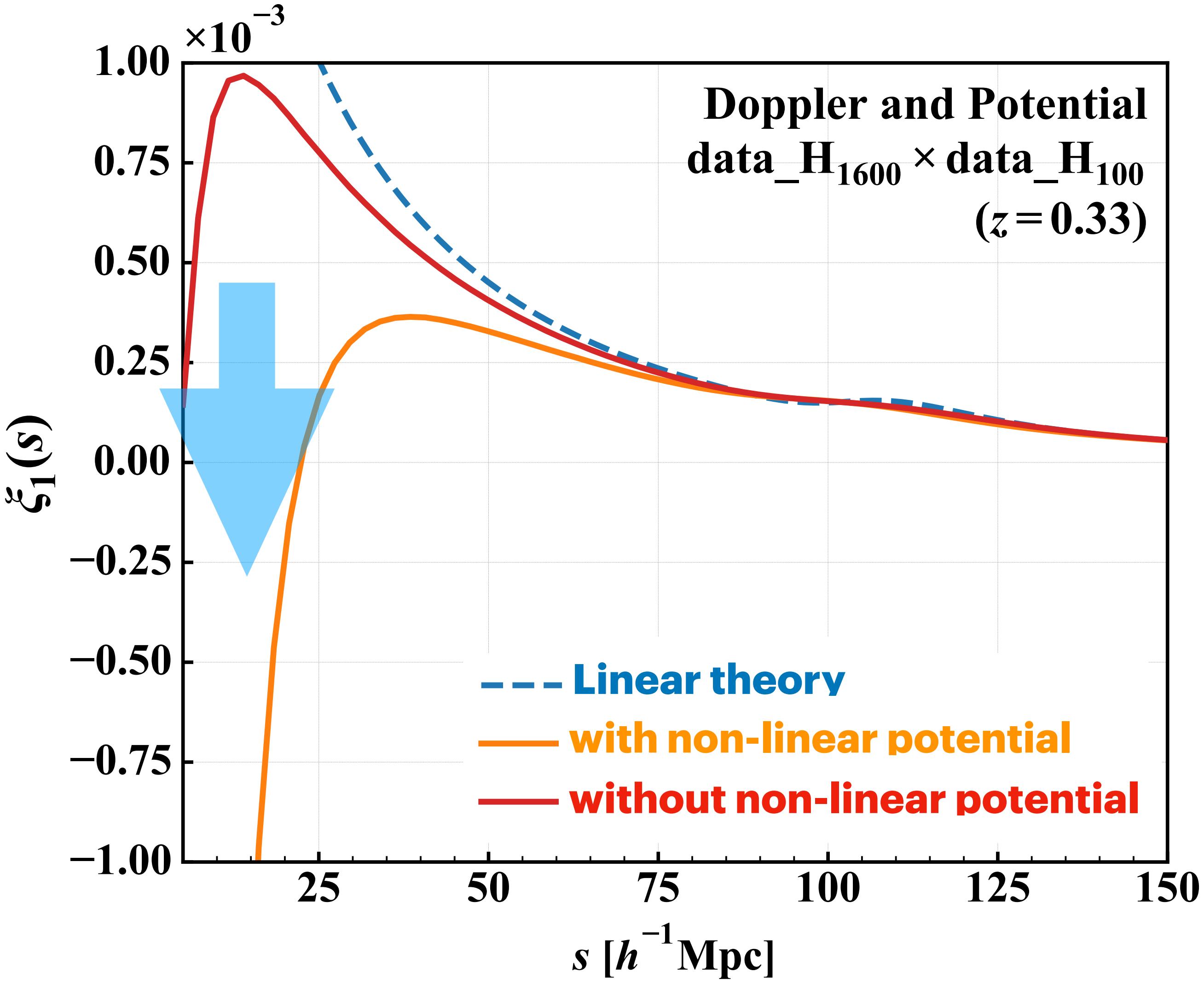
Our model:

$$s = r + \frac{1+z}{H}(v \cdot \hat{r})\hat{r} - \frac{1+z}{H}\Phi \qquad r = q + \Psi_{\rm ZA}(q,t)$$

$$\Phi = \Phi_{\rm L} + \Phi_{\rm NL}$$



Gravitational redshift



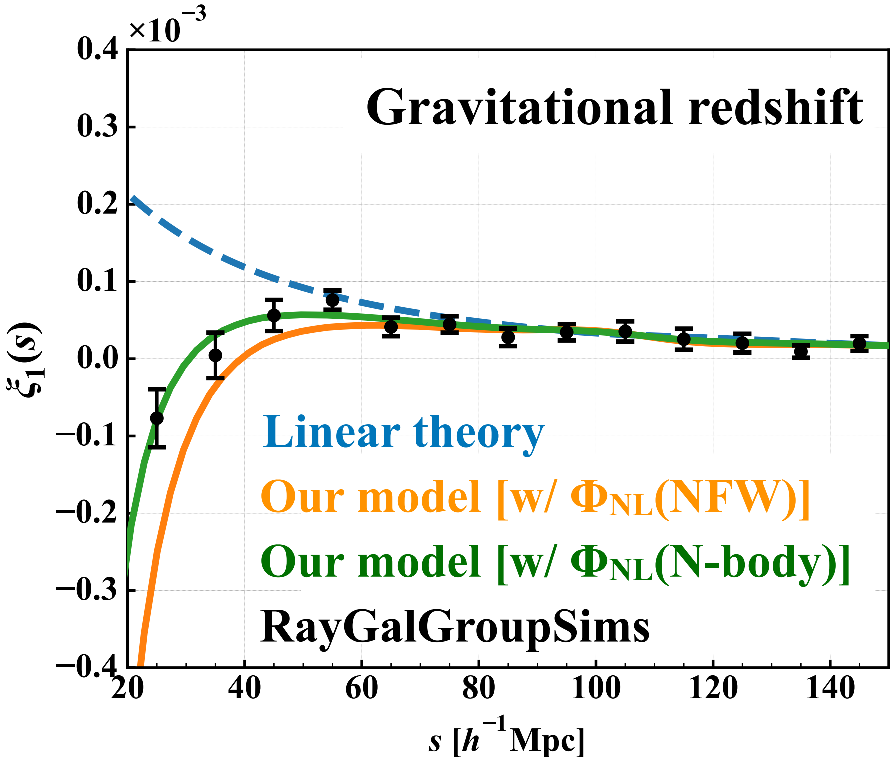
Doppler + Gravitational redshift

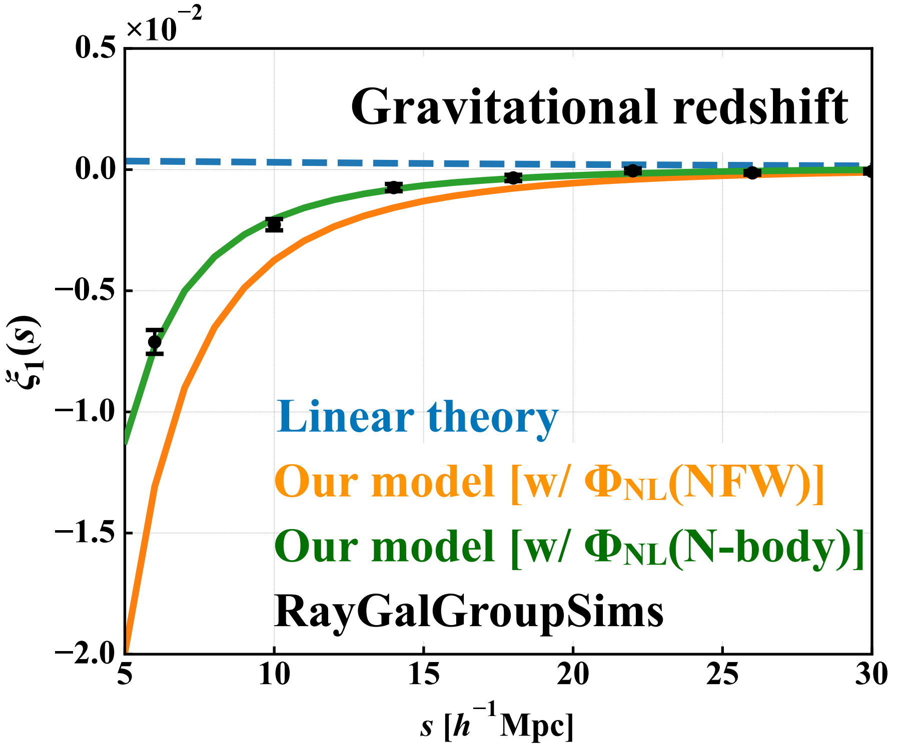
4.5 Results (z=0.33)

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

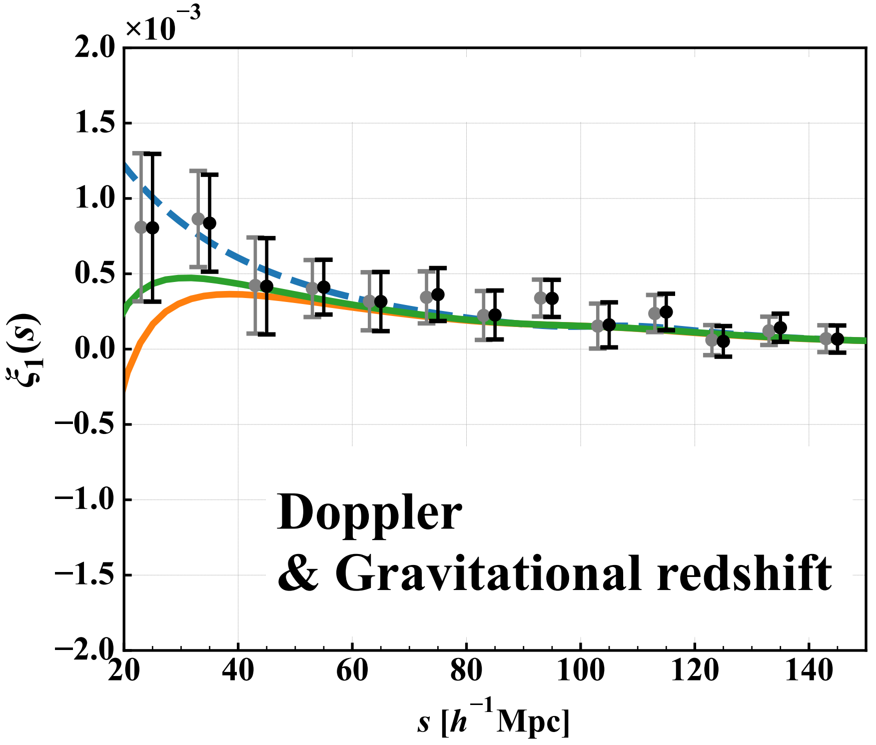
Large scales **Small scales**

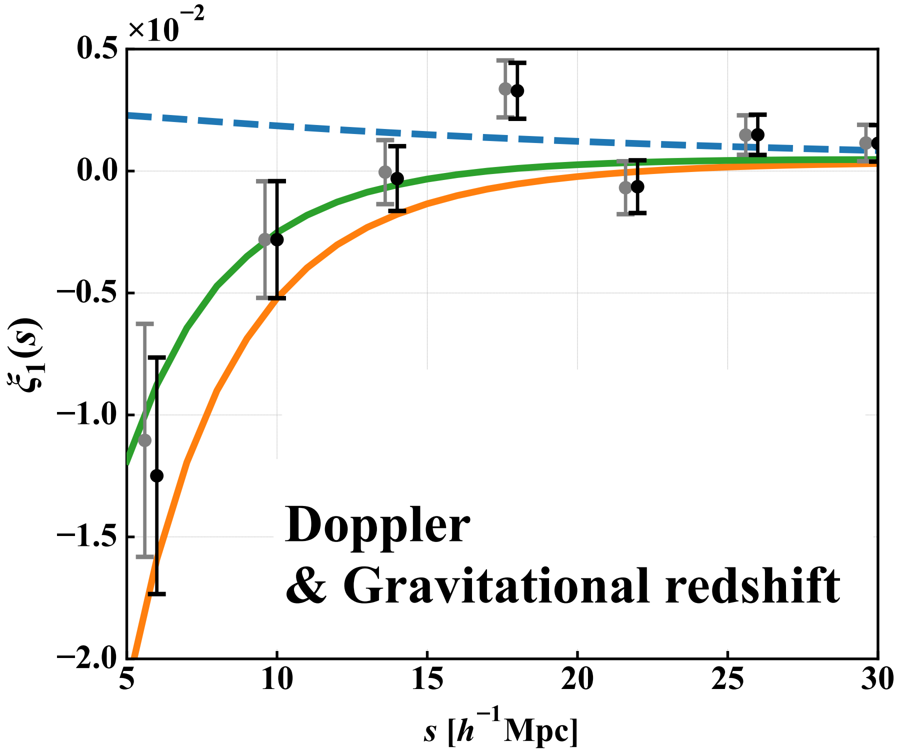
Gravitational redshift





Doppler
+
Gravitational redshift

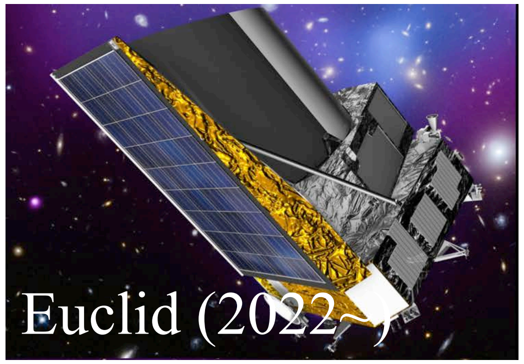


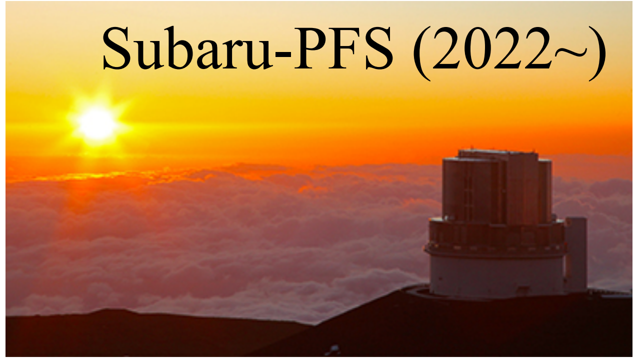


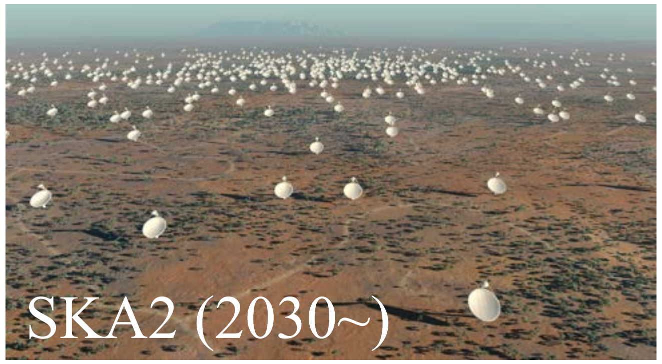
4.6 Detectability

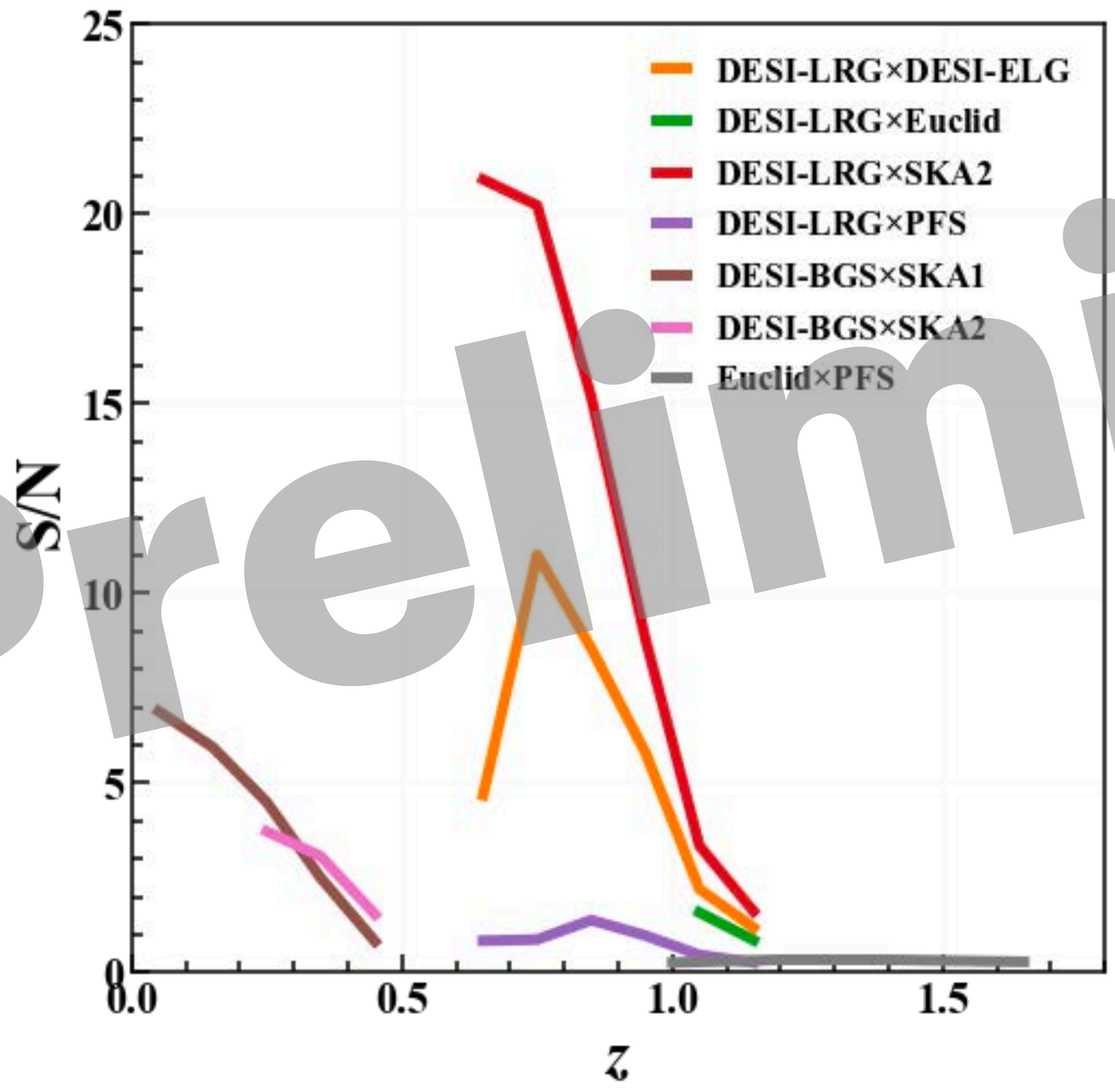
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]
S.Saga et al. in prep

Future surveys:









Synergy between two
observations is quite important

5. Summary

Our model:

Lagrangian perturbation theory + modelling of non-linear potential

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \frac{1+z}{H}\Phi \qquad \boldsymbol{r} = \boldsymbol{q} + \boldsymbol{\Psi}_{\rm ZA}(\boldsymbol{q}, t)$$

$$\Phi = \Phi_{\rm L} + \Phi_{\rm NL}$$

this **quasi-linear model** taking into account both **Doppler** and **gravitational redshift** effects

We found

- ✓ our model describes RayGalGroupSims results
- ✓ linear theory is recovered at large scales.
- ✓ non-linear halo potential plays important role at small scales

We expect

- ✓ the dipole signal can be a new probe to explore gravity theory
- ✓ we can propose the "best" survey strategy, based on the feasibility study