Extracting parameters of binary black holes with LISA

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work in collaboration with J. Baker (NASA GSFC), T. Dal Canton (NASA GSFC), S. Babak (APC), A. Toubiana (APC)

[Marsat&Baker arXiv/1806.10734]
[Marsat, Baker, Dal Canton arXiv/2003.00357]

Outline

- Introduction and motivation
- The duration of Black Hole Binary signals in LISA
- The LISA response in the Fourier domain
- Methods for Bayesian parameter estimation
- Parameter estimation for Massive Black Hole Binaries
- Parameter estimation for Stellar-mass Black Hole Binaries
- Conclusions and outlook

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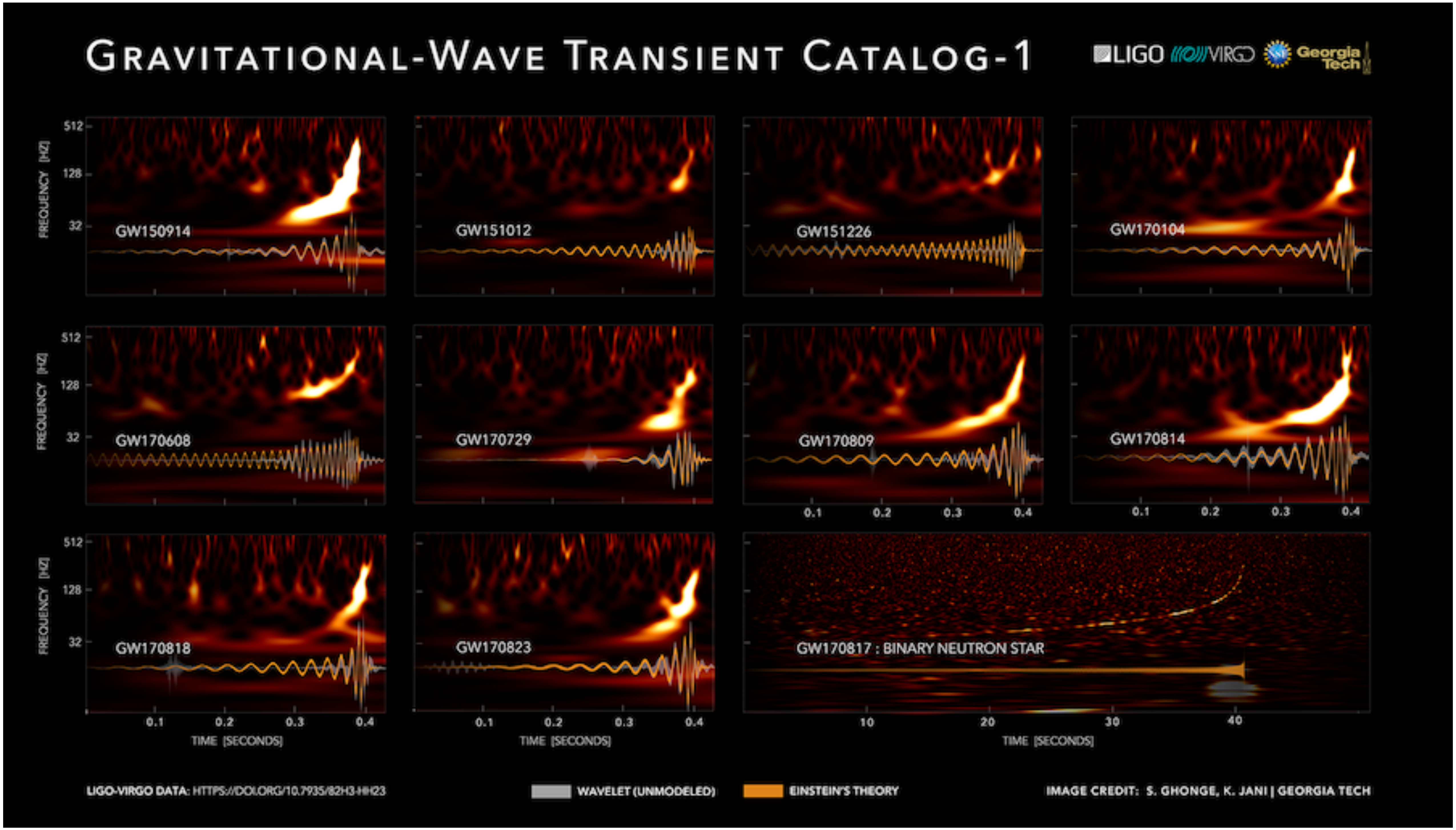
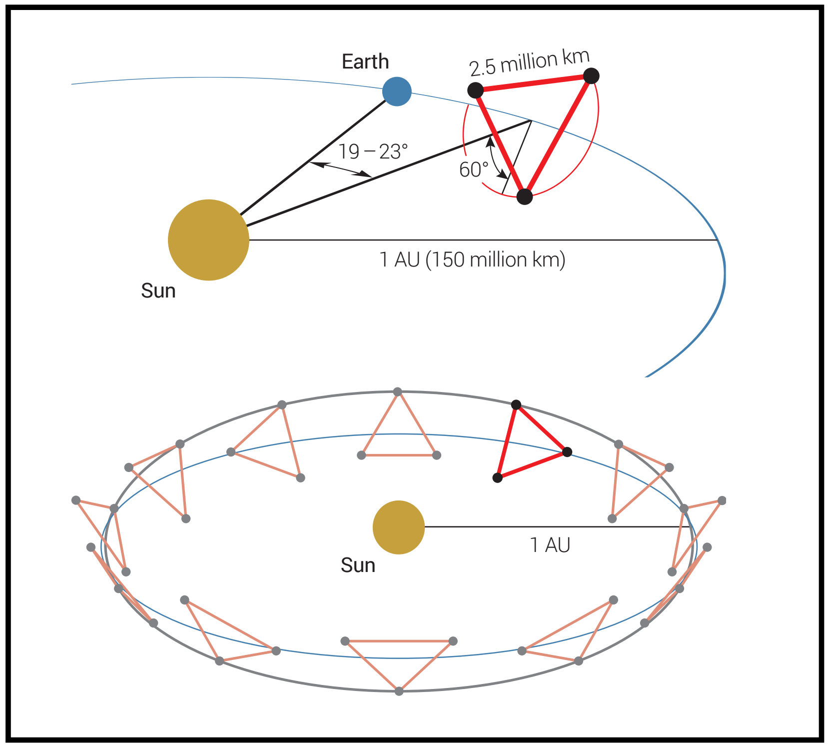
LIGO/Virgo O1-O2 detections

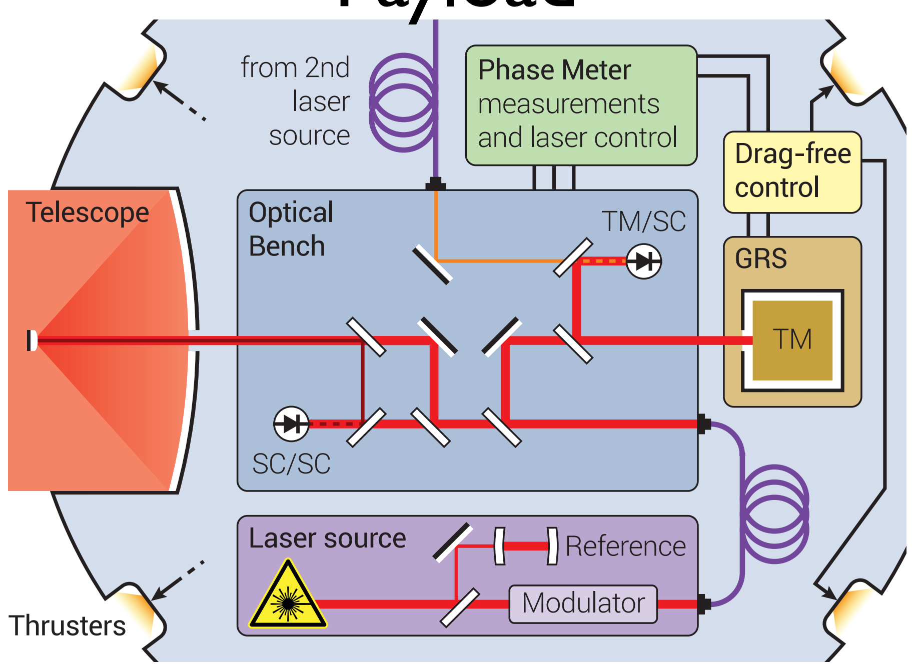


LISA mission - 2034

Orbits

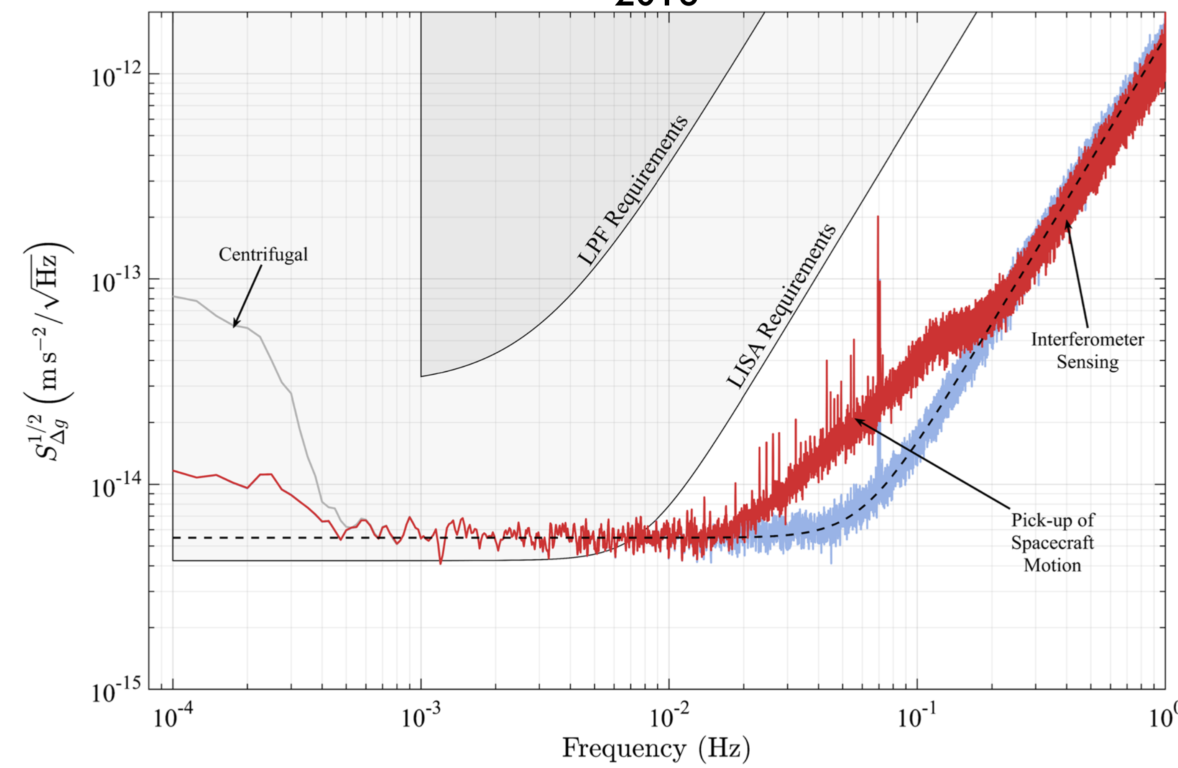


Payload

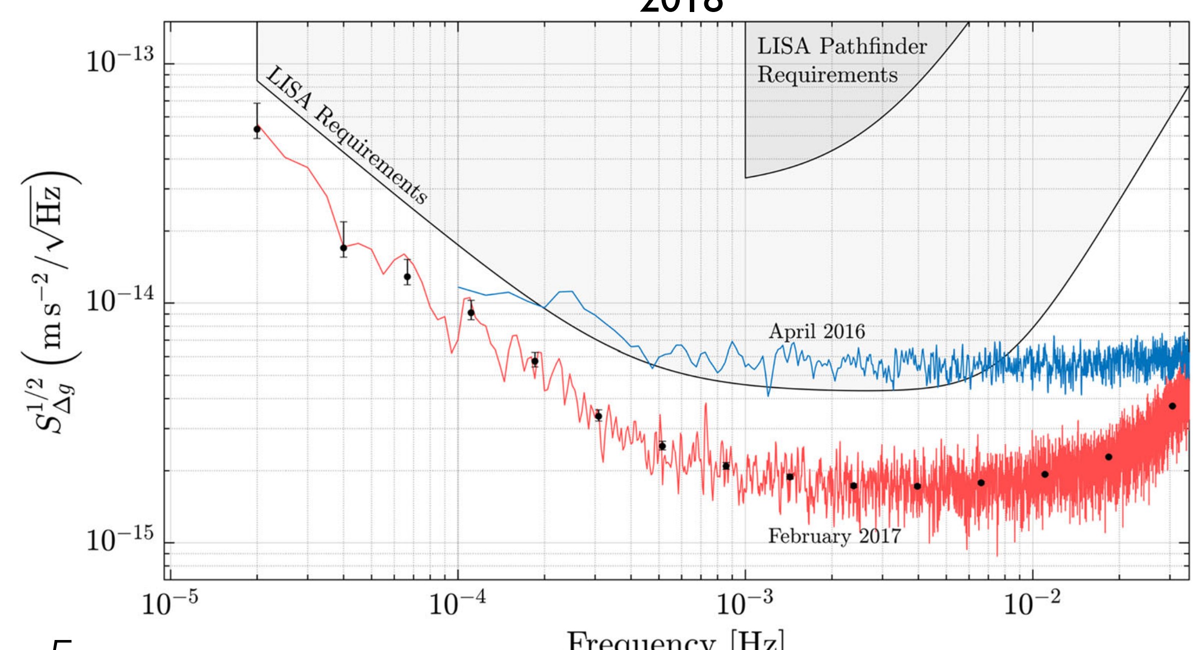


LISA Pathfinder success !

2016

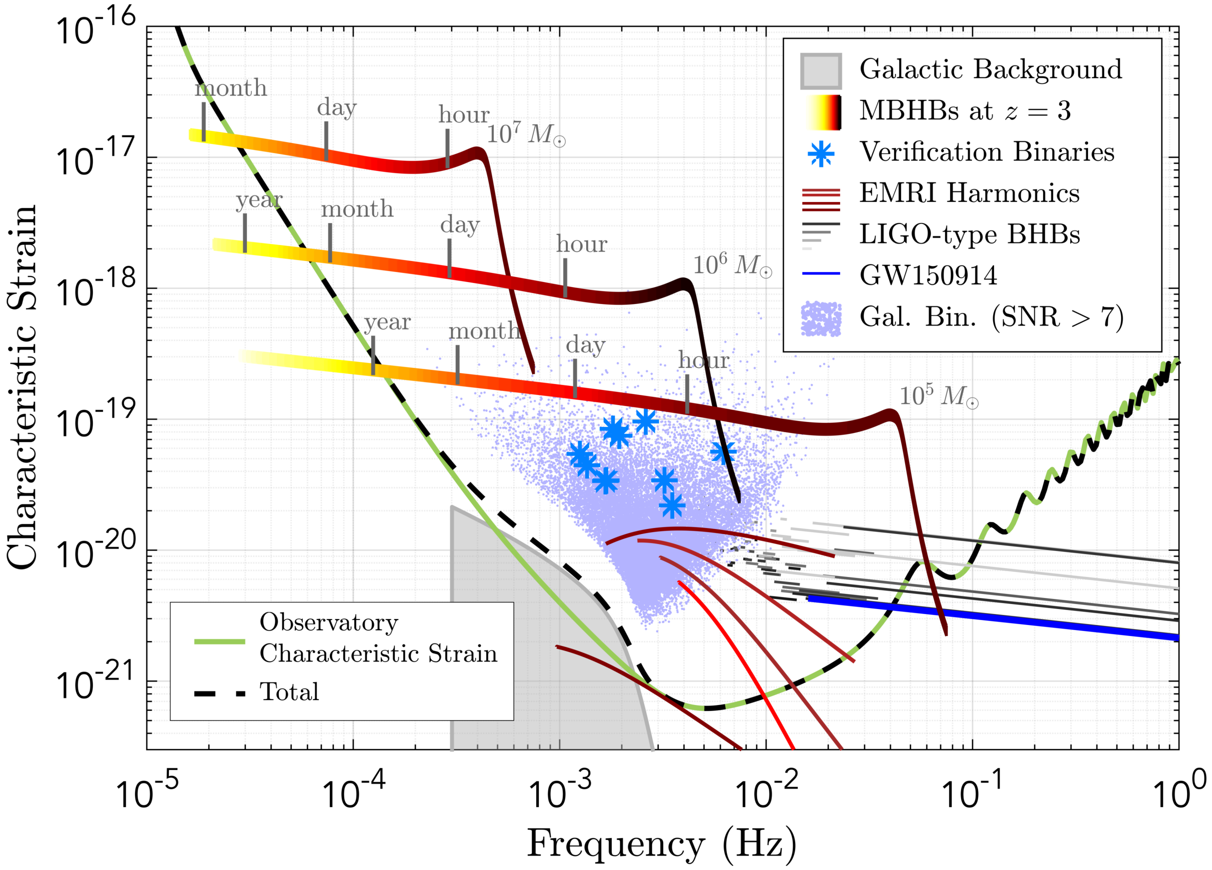


2018

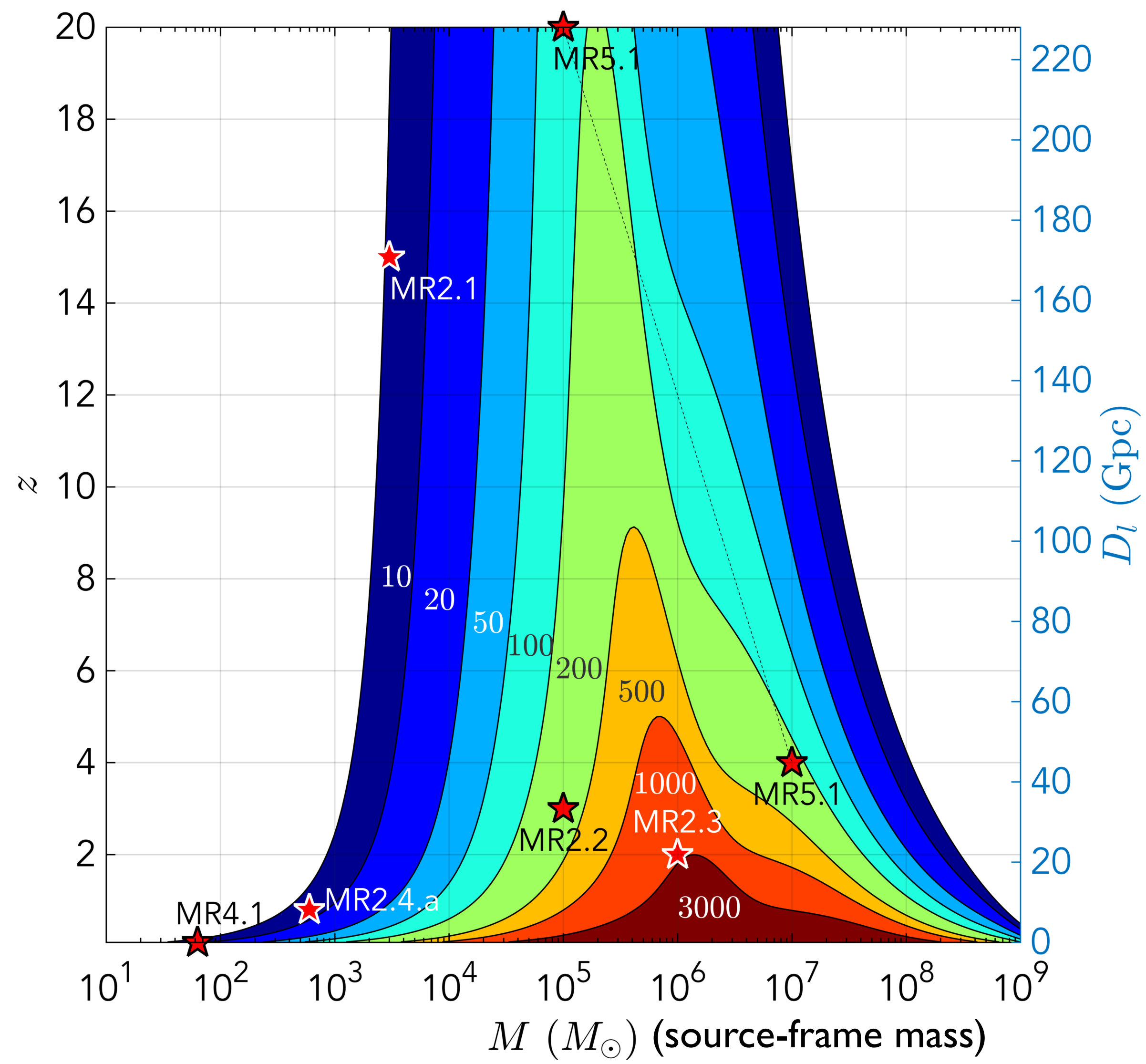


LISA science - overview

LISA sources



MBHB/SBHB SNR



Terminology:

- Massive black holes binaries (MBHBs)
- Stellar-mass black hole binaries (SBBHs): masses observable by ground-based detectors [Sesana 2016]

Contrasting LIGO/Virgo and LISA responses: LIGO/Virgo

Pattern functions

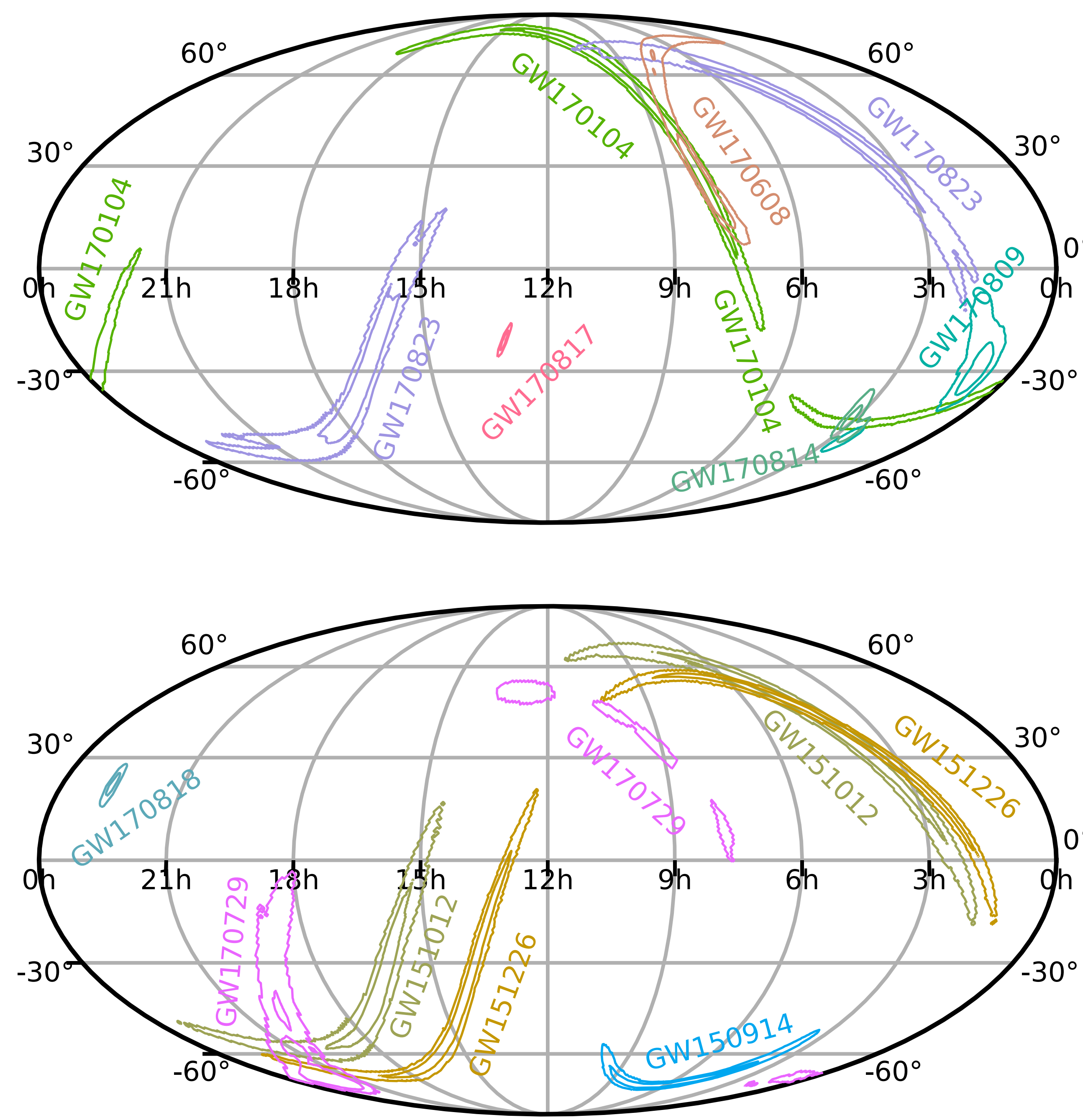
Simple multiplicative response

$$s = F_+ h_+ + F_\times h_\times$$

Angular dependence:

$$F_+ = \frac{1}{2}\left(1+\cos^2\theta\right)\cos\left(2\phi\right) \ ,$$
$$F_\times = \cos\theta\sin\left(2\phi\right)$$

GWTC-1 sky localisation



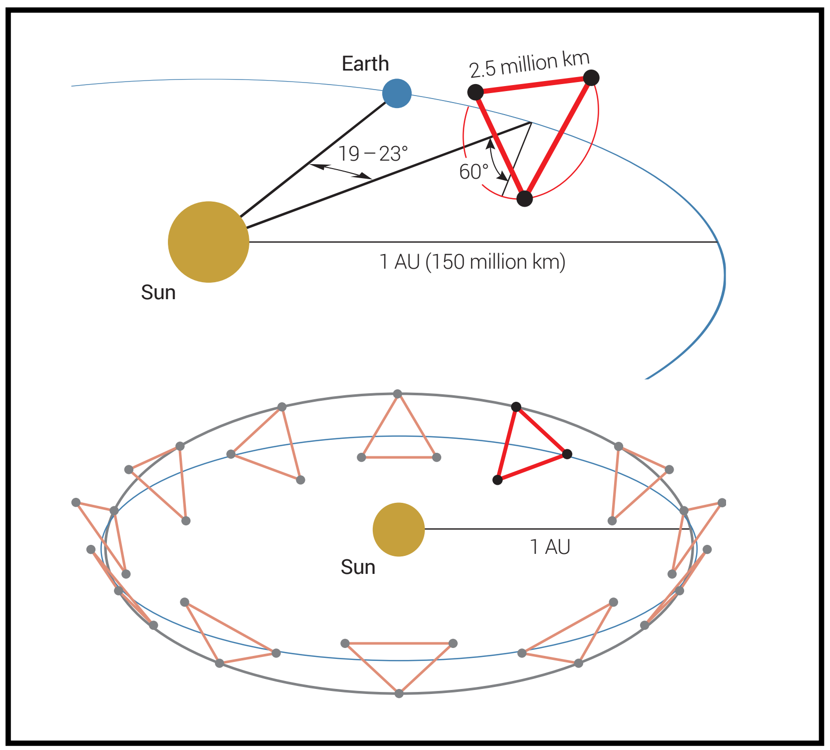
Time-of-arrival triangulation

- Two detectors: ~ring on the sky
- Better localization for 3 or more detectors (even low SNR!)

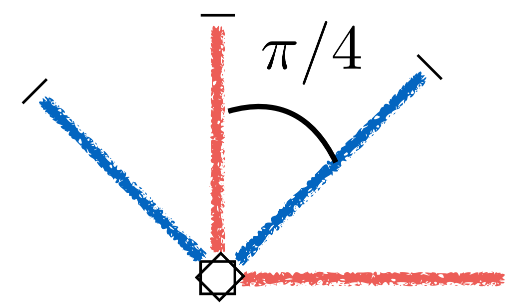
Contrasting LIGO/Virgo and LISA responses: LISA

LISA-frame

SSB-frame: global view of the orbits

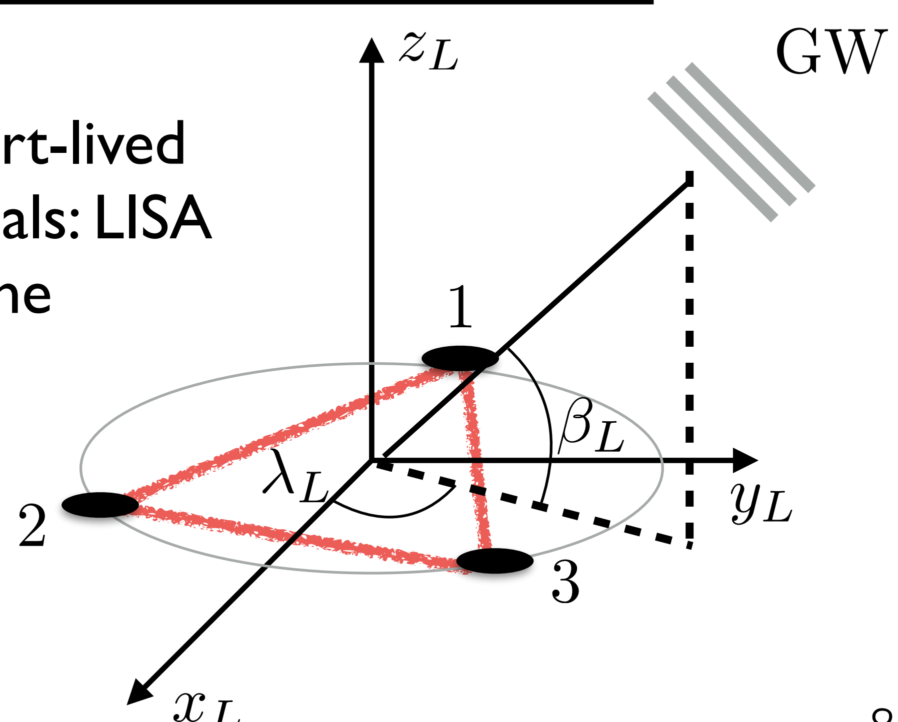


Low-f approximation: **two LIGO-type detectors** in motion [Cutler 1997]



High-f: **three channels** with complicated frequency-dependence

Short-lived signals: LISA frame



Sky localisation from the modulations induced by the orbits for long-lived signals

Main sky degeneracy for MBHBs: **reflection by the LISA plane**

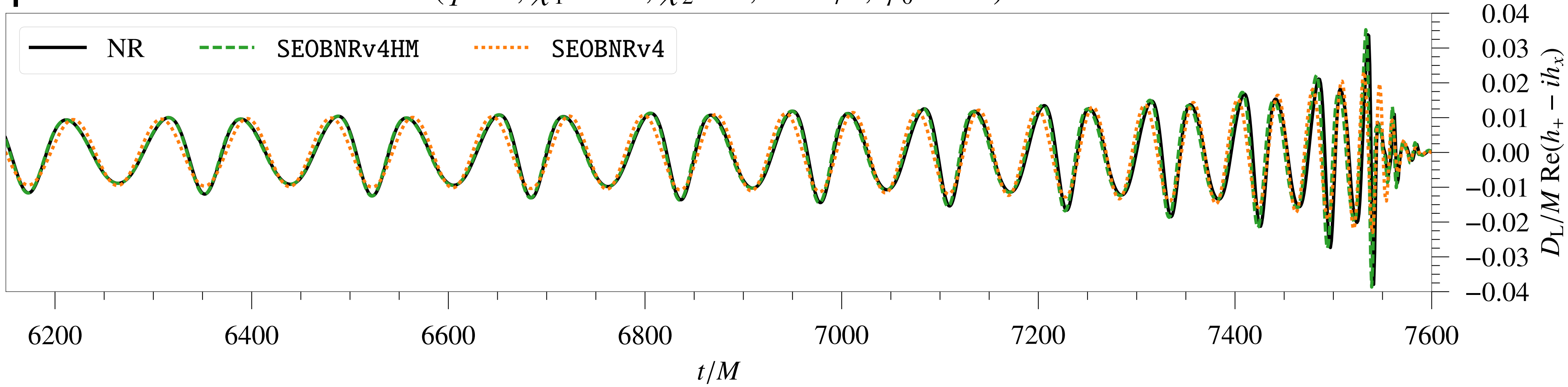
Higher harmonics in the waveform

Higher harmonics

$$h_+ - ih_\times = \sum_{\ell\geq 2}\sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\iota,\varphi)h_{\ell m}$$

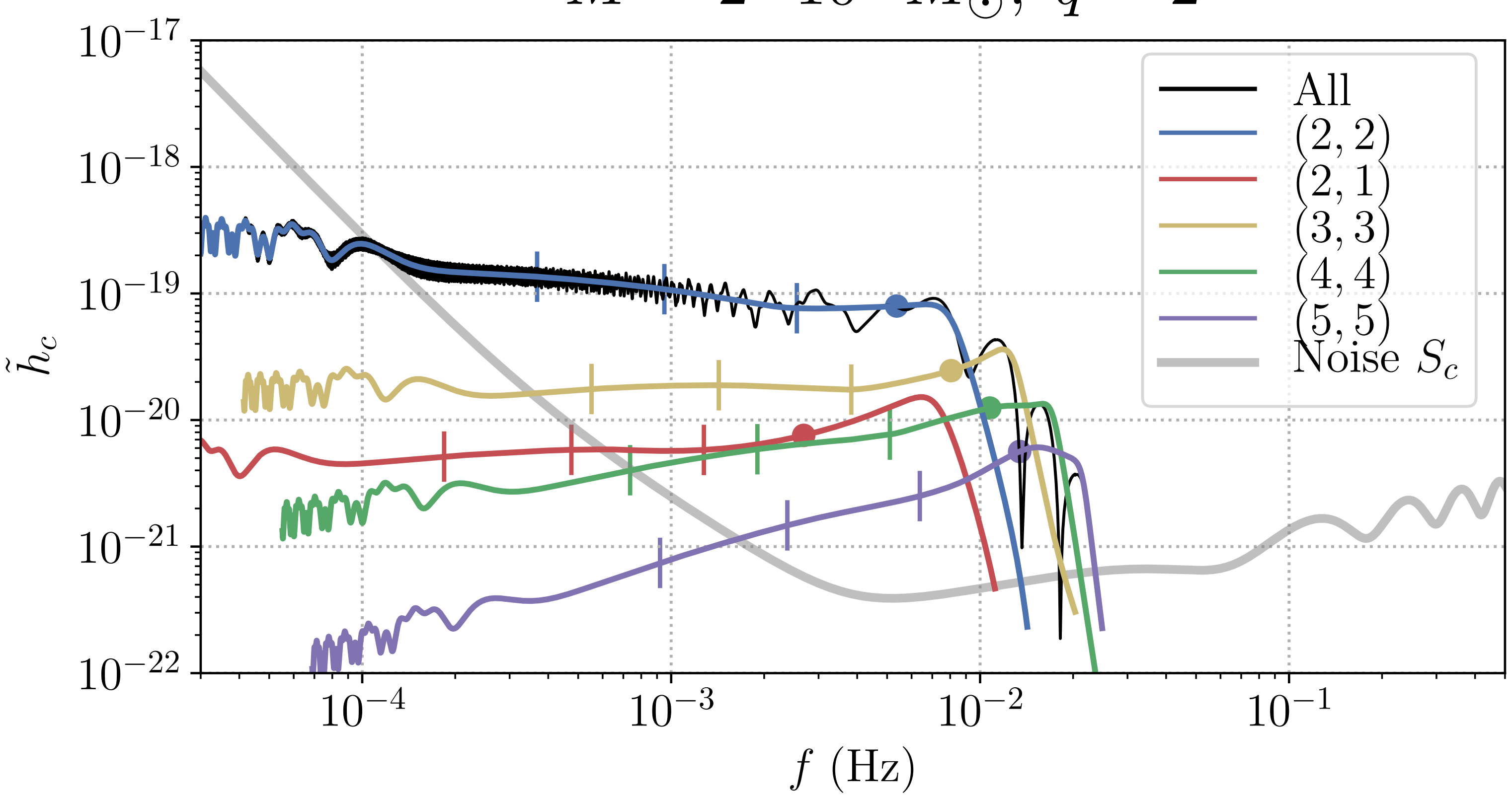
- Dominant harmonic h22
- Higher modes more important for high q and edge-on

Example in time domain: $(q = 8, \chi_1 = 0.5, \chi_2 = 0, \iota = \pi/2, \varphi_0 = 1.2)$



MBHB with higher modes

$M = 2\cdot 10^6\ M_\odot,\ q = 2$



Ticks:

- SNR/64 (40h)
- SNR/16 (2.5h)
- SNR/4 (7min)
- merger

Challenges of parameter estimation for LISA

Accurate waveforms needed to extract physical information without bias

MBHB features

- Large (>1000) SNR: accurate waveforms needed
- Large SNR for merger/ringdown and higher harmonics (HM)
- Wide range of mass ratios and spins
- Possible significant eccentricity in triplets
- Signal length: from days to months for IMBHs
- Observations not SNR-limited: edge-on common

This study:

Non-spinning, q=3
Inspiral-Merger-Ringdown
Higher Harmonics

SBHB features

- Small SNR (<20), long signals (years), at high frequencies
- Very deep inspiral; chirping signals and slowly-chirping signals
- Masses and spins: cf LIGO/Virgo!
- Possible significant eccentricity if formation in clusters

This study:

Aligned spin
Chirping signals
No eccentricity

Instrument response

- Instrument response is time- and frequency-dependent, carrying information about the sky position

This study:

Full FD response

Data analysis challenges

- Signal superposition requiring global fit
- Non-stationarity, glitches, gaps…

This study:

Idealized noise

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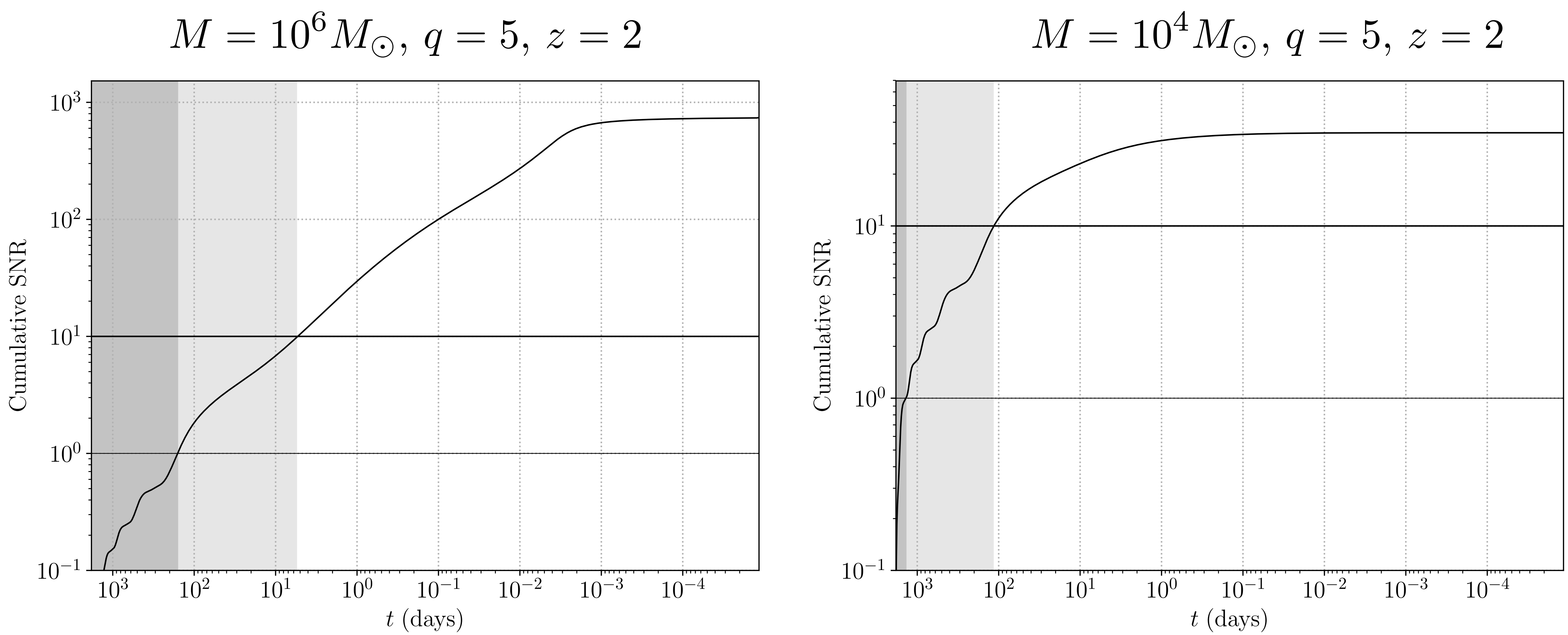
Accumulation of SNR with time for MBHB/IMBHB

Accumulation of SNR as time left before merger diminishes
Shaded areas: thresholds SNR=1 and SNR=10



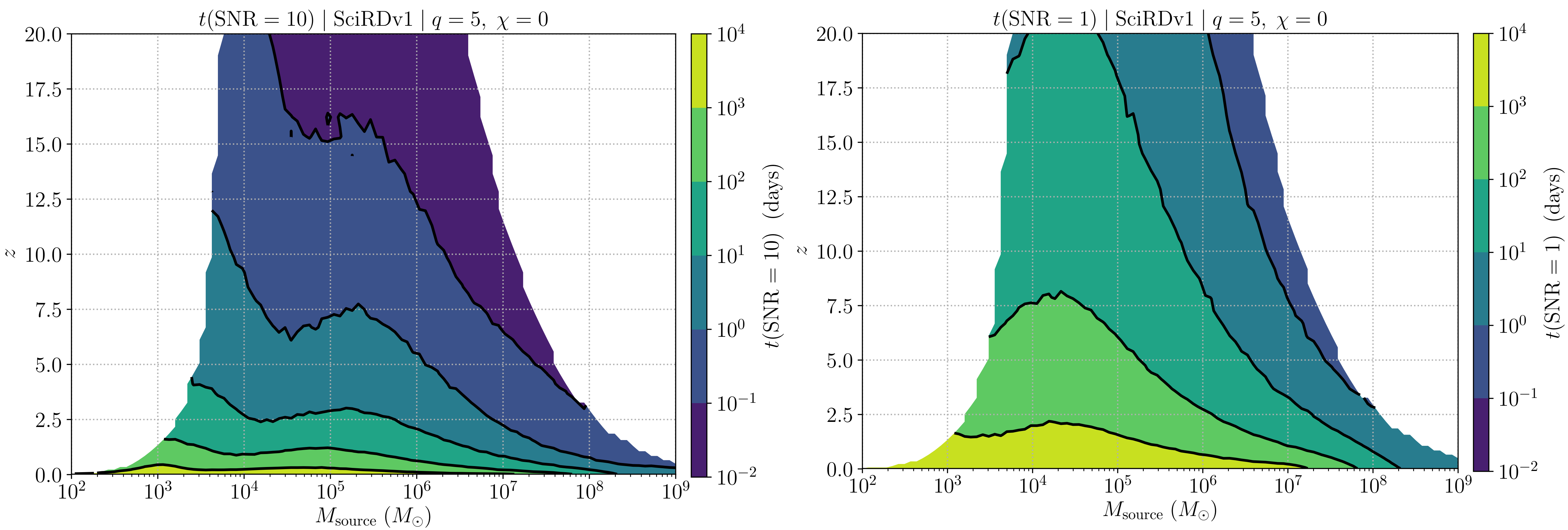
Two different definitions of "signal duration":

- Looking back in time from merger, when is the signal negligible ? Here SNR=1
- Accumulating signal towards merger, when is the signal detected ? Here SNR=10

For MBHBs, SNR accumulates shortly before merger (days)

Length of MBHB LISA signals: for the observer

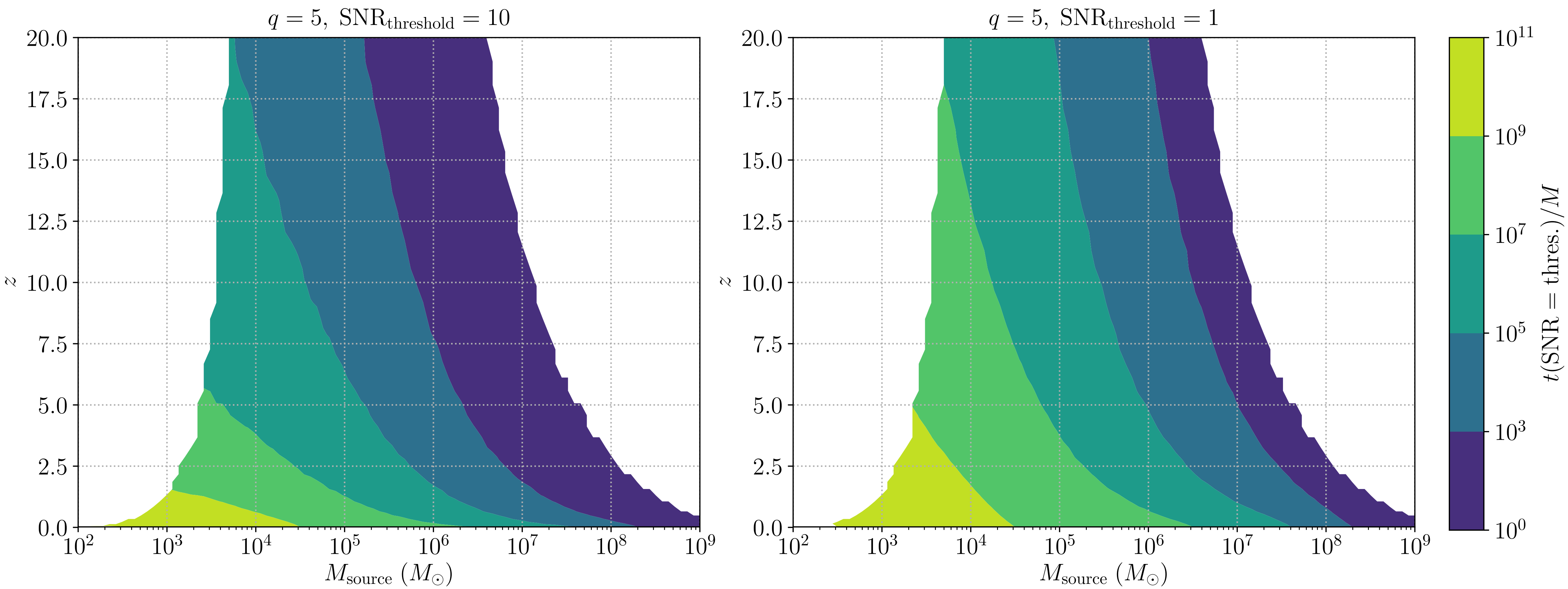
t(SNR): time to merger left when the signals has accumulated a given SNR



- SNR=10 as the time to merger left when we can claim detection
- SNR=1 assuming everything before that point can be neglected in PE

Length of MBHB LISA signals: for waveform models

t(SNR)/M: same length of signal, but seen in geometric units for waveforms models (longest NR simulation: t/M=10^5)



- SNR=10 as the time to merger left when we can claim detection
- SNR=1 assuming everything before that point can be neglected in PE

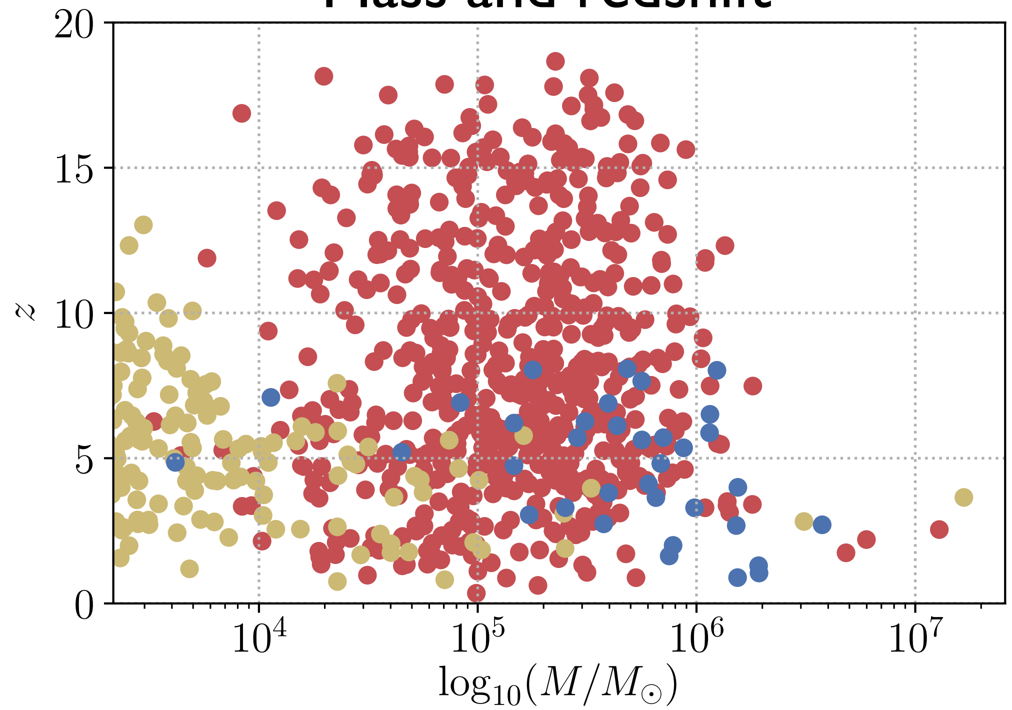
LISA: simulated catalog for MBHB astrophysical models

[Barausse 2012]

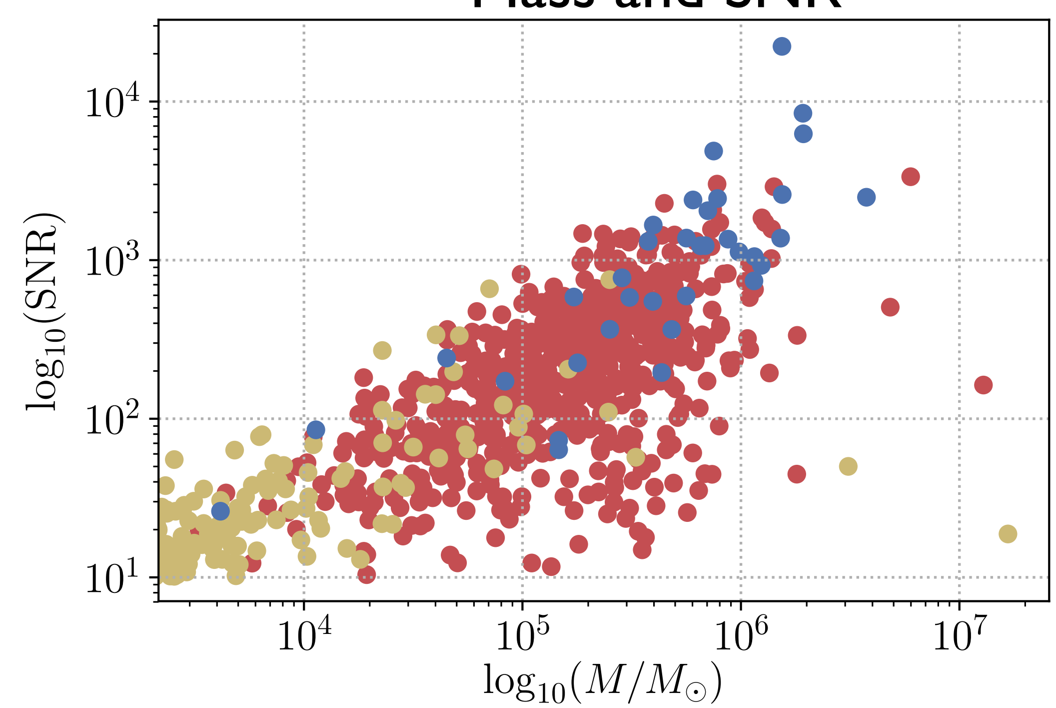
Astrophysical models:

- Heavy seeds - delay
- Light seeds - no delay
- PopIII seeds - delay

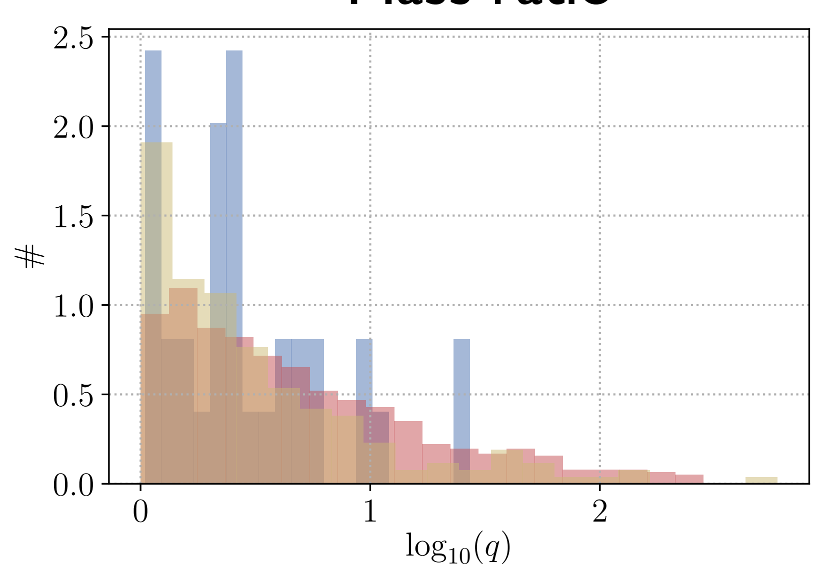
Mass and redshift



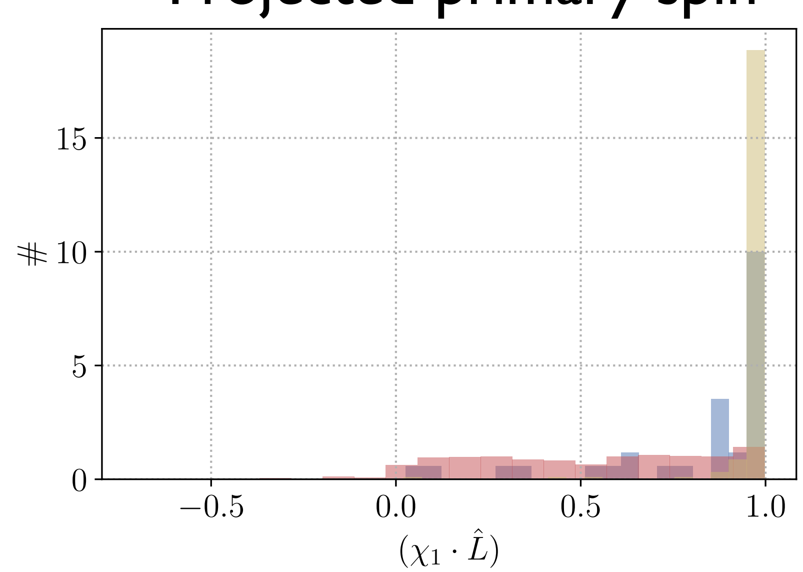
Mass and SNR



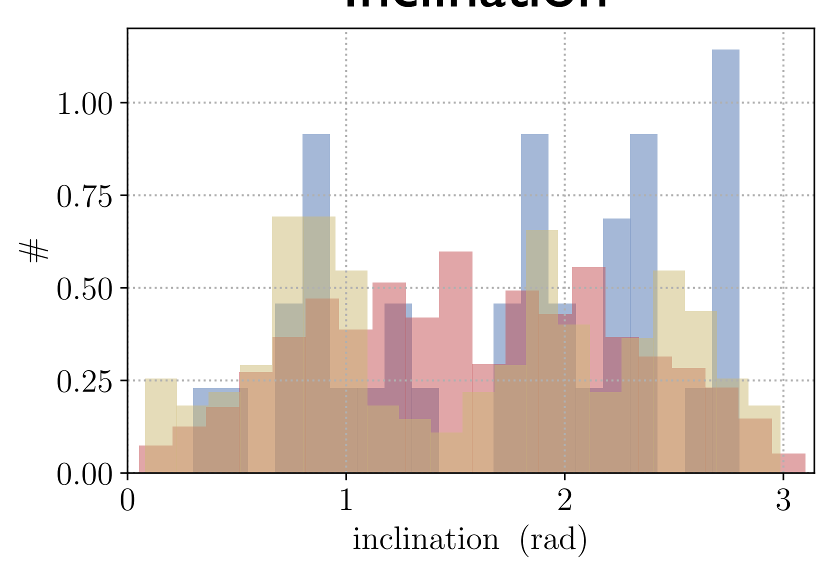
Mass ratio



Projected primary spin



Inclination



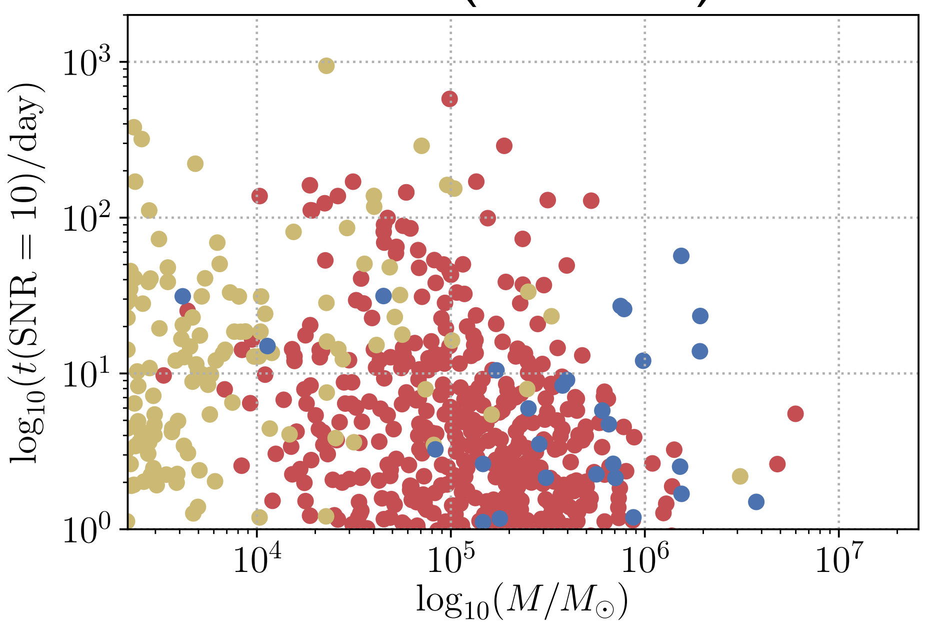
LISA: simulated catalog for MBHB astrophysical models

[Barausse 2012]

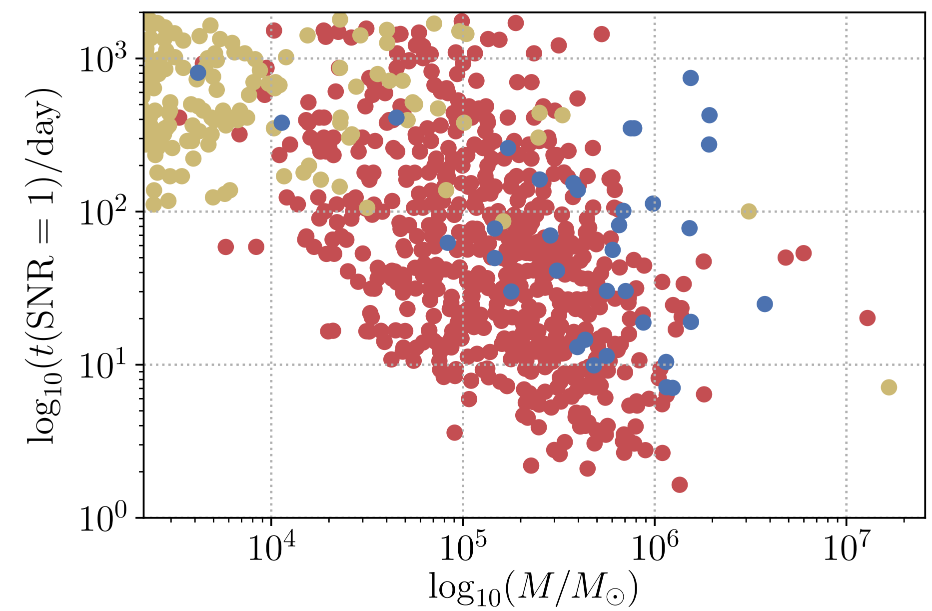
Astrophysical models:
- Heavy seeds - delay
- Light seeds - no delay
- PopIII seeds - delay

MBHB detected signals:
Bulk shorter than ~10days
Tail extending to ~3months

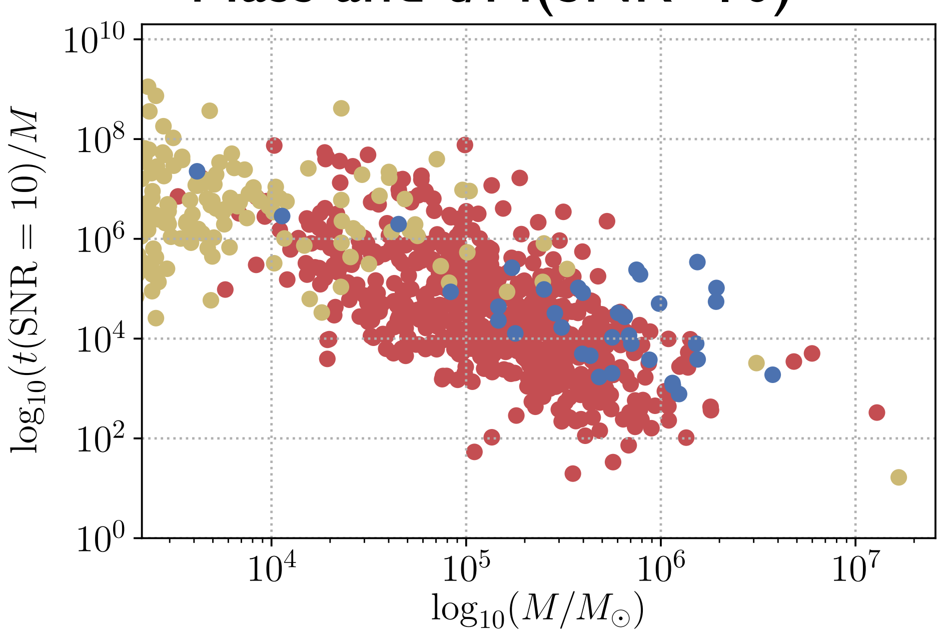
Mass and t(SNR=10)



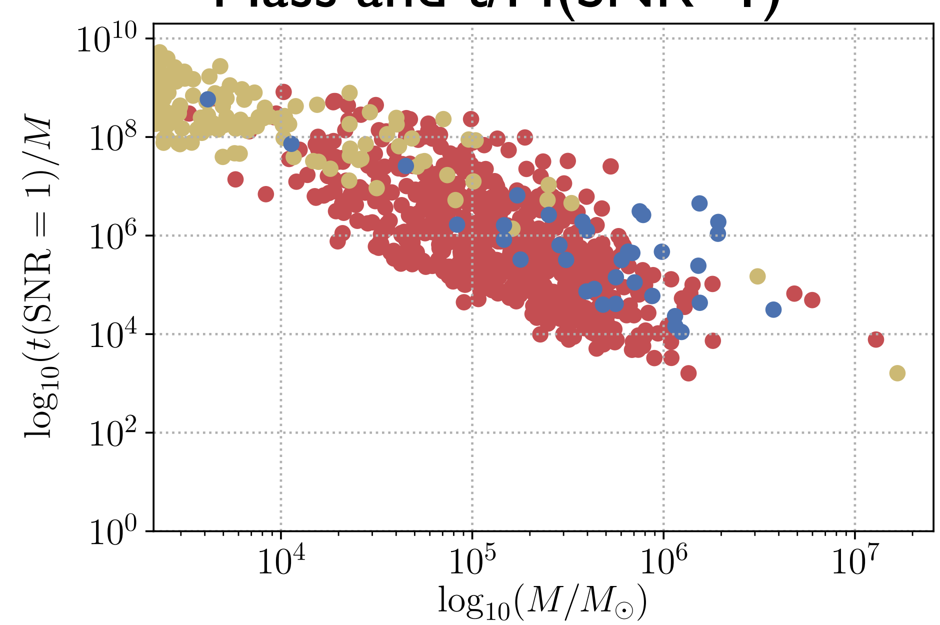
Mass and t(SNR=1)



Mass and t/M(SNR=10)



Mass and t/M(SNR=1)



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LISA instrument response

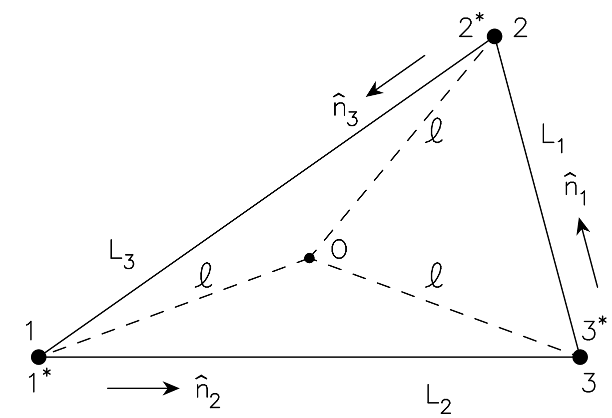
One-arm frequency observables

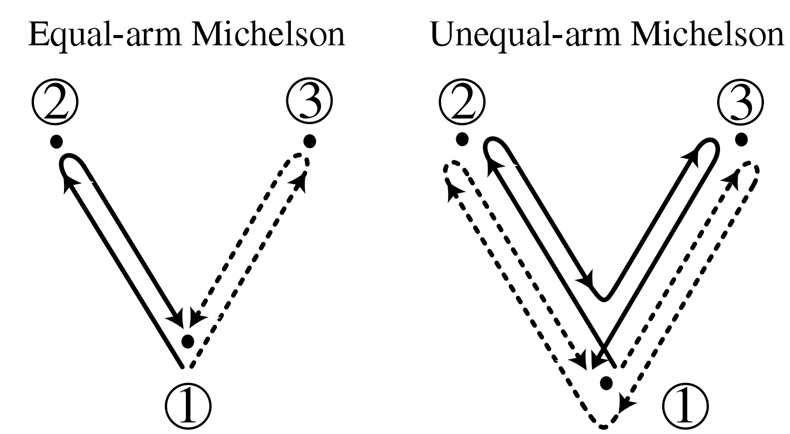
From spacecraft s to spacecraft r
through link s: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

$$t_s = t - L - \hat{k}\cdot p_s, \quad t_r = t - \hat{k}\cdot p_r$$

$$h = h_+ P_+(\hat{k}) + h_\times P_\times(\hat{k})$$ GW at SSB





Time-delay interferometry (TDI)

- Crucial to cancel laser noise
- First generation: unequal arms
- Second generation: propagation and flexing
- Michelson X,Y,Z - Uncorrelated noises A,E,T

$$X_1^{\rm GW} = \underbrace{\left[(y_{31}^{\rm GW} + y_{13,2}^{\rm GW}) + (y_{21}^{\rm GW} + y_{12,3}^{\rm GW})_{,22} - (y_{21}^{\rm GW} + y_{12,3}^{\rm GW}) - (y_{31}^{\rm GW} + y_{13,2}^{\rm GW})_{,33}\right]}_{X^{\rm GW}(t)} - \underbrace{\left[(y_{31}^{\rm GW} + y_{13,2}^{\rm GW}) + (y_{21}^{\rm GW} + y_{12,3}^{\rm GW})_{,22} - (y_{21}^{\rm GW} + y_{12,3}^{\rm GW}) - (y_{31}^{\rm GW} + y_{13,2}^{\rm GW})_{,33}\right]_{,2233}}_{X^{\rm GW}(t-2L_2-2L_3)\simeq X^{\rm GW}(t-4L)} .$$

Approximations

- Long-wavelength approximation: two moving LIGOs rotated by $\pi/4$ + orbital delay
- **Rigid approximation** (order of the delays does not matter, delay=L simple in Fourier domain)

LISA FD response - motivation

Motivation

- Aim: computationally intensive applications (PE)
- Take advantage of recent FD IMR waveform models
- Response directly in the Fourier domain
- Keep a compact representation (~1000 pts)
- Assess errors of FD processing

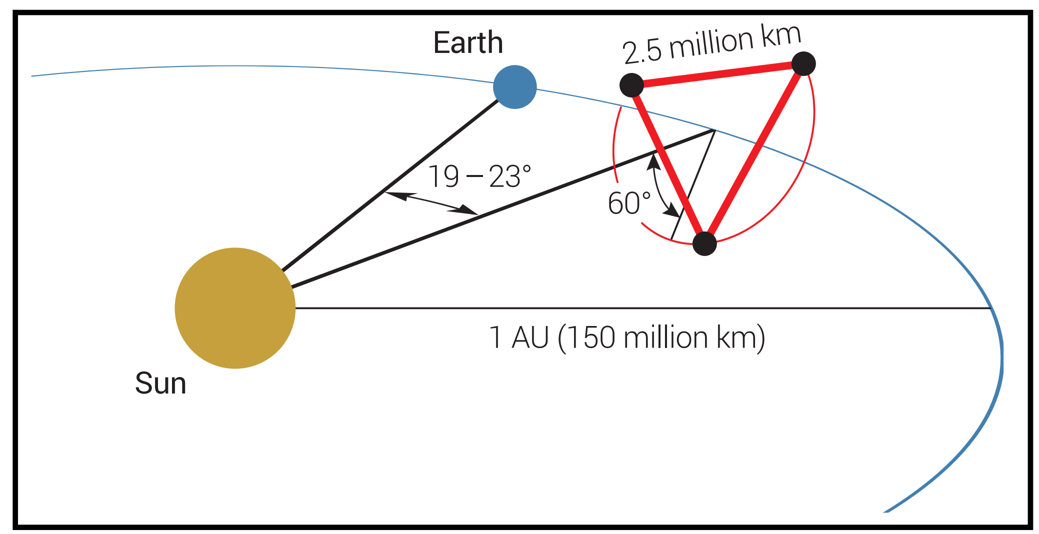
Terminology:

- **Orbital**: main motion around the Sun
- **Constellation**: other motion and inter-spacecrafts delays

Frequency observables $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

TDI: combination of delayed y_{slr}



Decomposition of the response:

- Orbital delay
- Time-varying orientation
- Inter-spacecrafts delays

Transfer function for modulated and delayed signal

$$\mathrm{FT}\left[F(t)h(t+d(t))\right] = \mathcal{T}(f)\tilde{h}(f)$$

The timescales in the problem

Instrumental timescales

- Motion (approximately) periodic $f_0 = 1/\mathrm{yr} \simeq 3.10^{-8}\mathrm{Hz}$
- Transfer frequencies for the delays: when the baseline is one wavelength
  Orbital : $f_R = 3.2 \times 10^{-4}\mathrm{Hz}$
  Constellation: $f_L = 1.9 \times 10^{-2}\mathrm{Hz}$

GW timescales

- Wave frequency $f \gg f_0$
- Radiation-reaction timescale $T_{\mathrm{RR}} \sim 1/\sqrt{\dot{\omega}}$

Separation of timescales

- Conditioned by $T_{\mathrm{RR}}/T_0 \ll 1$
- **Also** dimensionless factors $2\pi f d$

Guessing...

Separation will be good for chirping binaries but breaks in the quasi-monochromatic limit

Inspiral will be harder than merger-ringdown — opposite of the SPA assumptions

Separation of timescales becomes a frequency-dependent statement due to the presence of delays

A local time-to-frequency map

Convolution with f-dependent kernel

Input: $\tilde{h}(f) = A(f) e^{-i\Psi(f)}$

$$s(t) = F(t) h(t + d(t))$$

$$\tilde{s}(f) = \int df' \, \tilde{h}(f - f') \tilde{G}(f - f', f')$$

$$G(f, t) \equiv e^{-2i\pi f d(t)} F(t)$$

Separation of timescales: if F, d have only frequencies <<f, local convolution - expand h(f-f') in f'

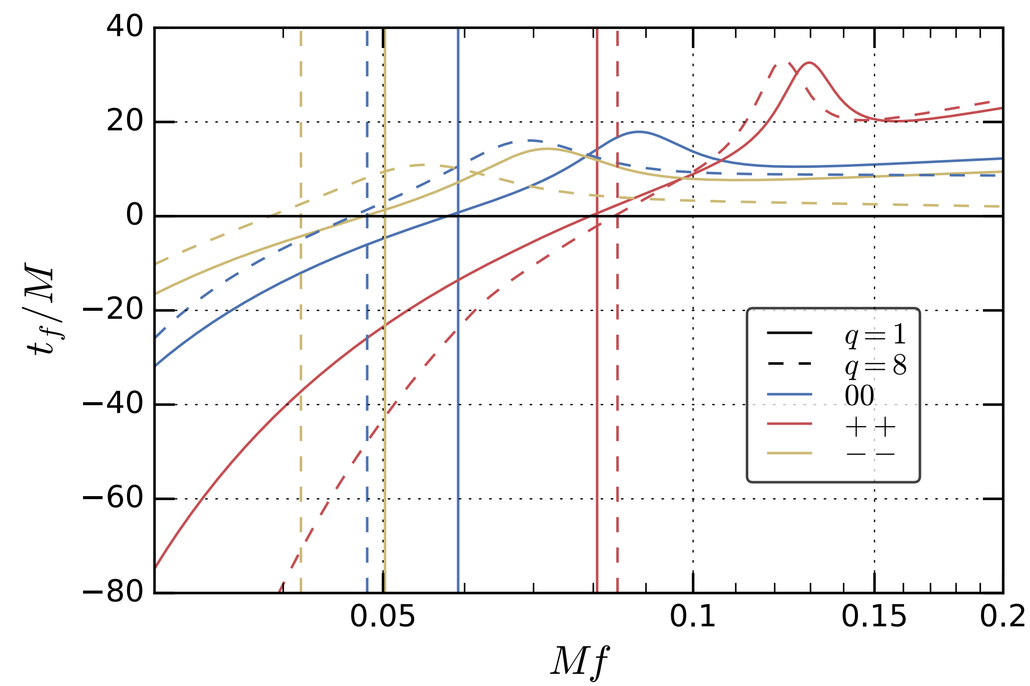
Leading-order: time-of-frequency

Keeping linear term in the phase:

$$t_f \equiv -\frac{1}{2\pi} \frac{\mathrm{d}\Psi}{\mathrm{d}f}$$

$$\tilde{s}(f) = \mathcal{T}(f) \tilde{h}(f)$$
$$\mathcal{T}(f) = G(f, t_f)$$

Close to the SPA - but extends through MRD



Leading-order one-arm transfer function:

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \mathrm{sinc}\left[\pi f L \left(1 - \hat{k} \cdot n_l\right)\right] \exp\left[i\pi f \left(L + \hat{k} \cdot (p_1 + p_2)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

Beyond leading order: [Marsat&Baker]

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Bayesian analysis

Bayesian formalism

- Matched-filtering overlap: $$(h_1|h_2) = 4\mathrm{Re}\int df\, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$$
- For Gaussian, stationary noise, for independent channels:

$$\ln \mathcal{L}(d|\theta) = -\sum_{\text{channels}} \frac{1}{2}(h(\theta)-d|h(\theta)-d)$$

$$d = s(\theta_0) + n$$

- Bayes theorem defines the posterior: $$p(\theta|d) = \frac{\mathcal{L}(d|\theta)p_0(\theta)}{p(d)}$$

h template
θ parameters
d data
s signal
θ_0 signal params.
n noise
S_n noise PSD

$p_0(\theta)$ prior
$p(d)$ evidence

Fisher matrix analysis

- Quadratic expansion of log-likelihood around injection

$$\ln \mathcal{L} = -\frac{1}{2}\Delta\theta_i F_{ij}\Delta\theta_j + \mathcal{O}(\Delta\theta^3)$$

$$F_{ij} = (\partial_i h|\partial_j h)$$

- Matrix inversion to get to the covariance of the Gaussian

$$C = F^{-1}$$

- Valid at high SNR, and misses degeneracies

0-noise parameter estimation

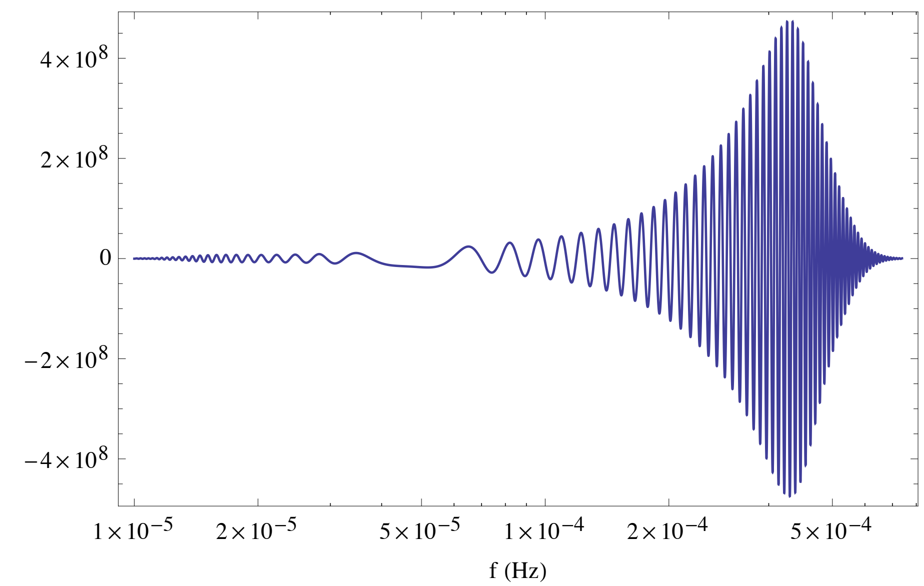
- Simply put the noise realisation to 0, otherwise sample from the posterior
- Allows to explore the full likelihood
- Likelihood automatically peaks at injection

Fast likelihoods and implementation

Accelerated zero-noise overlaps

- Sparse grids: amplitude/phase and response
- Cubic spline representation 300-800 pts
- Mode-by-mode overlaps: significant cost increase with higher modes
- Much simpler than Reduced Order Quadratures, but cannot handle noise

Overlaps: oscillatory integrands



$$(h_1|h_2) = 4\mathrm{Re}\int df\, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} \longrightarrow \int_{f_i}^{f_{i+1}} P(f)e^{i[af+bf^2]} \longrightarrow \int_{f_i}^{f_{i+1}} e^{i[af+bf^2]}$$

Waveforms

MBHB: EOBNRv2HM waveforms

- Non-spinning model, includes modes (22, 21, 33, 44, 55)
- Reduced Order Model implementation for sub-millisecond sparse waveform evaluation

SBHB: PhenomD waveforms

- Aligned spins, 22 mode
- Analytic ansatz, sub-millisecond sparse waveform

Likelihood cost
Single mode h22: 1-3ms
5 modes hlm: 15ms

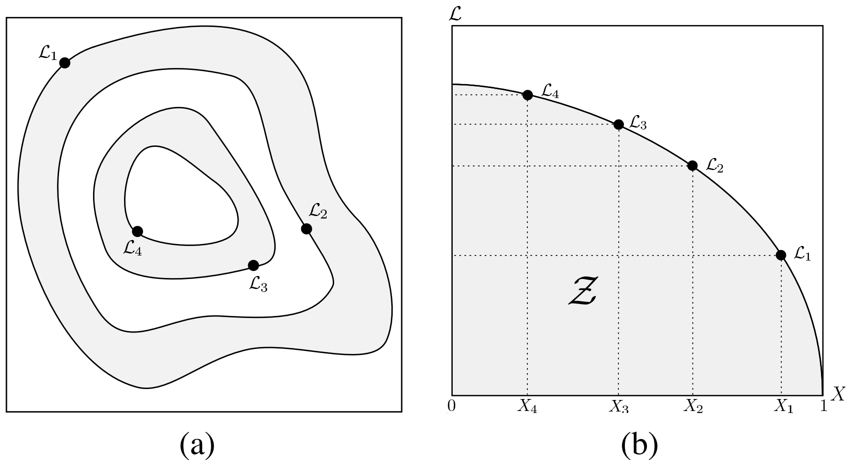
[Katz&al]: PhenomHM waveforms, fast GPU computation of likelihoods with noise

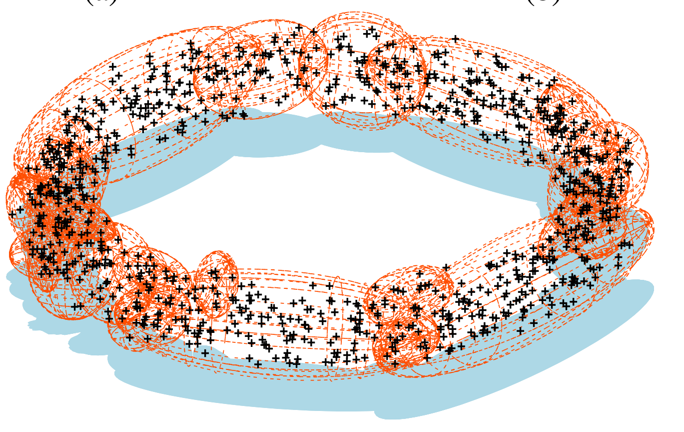
Bayesian samplers

MultiNest [Feroz&al 2009]

- Implements Nested Sampling [Skilling 2006]
- Evolves a population of live points by replacements from within isolikelihood contours
- Evaluates the evidence
- Drawing from within a set of ellipsoids, clustering

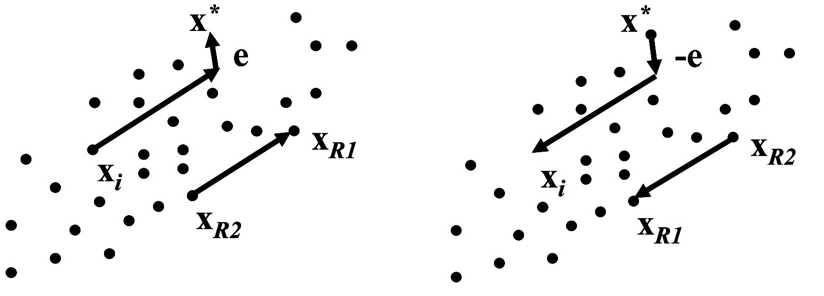
- Available as off-the-shelf sampler
- Less flexible than MCMC (jumps, …)

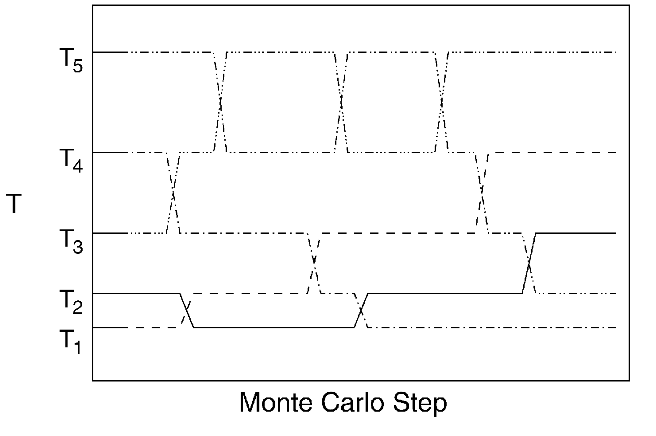




PTMCMC

- Custom code
- Parallel tempering [Swendsen&al 1986]
- Differential evolution [Braak&al 2008]
- Can be informed with proposal jumps
- Can be used as brute-force method to resolve all degeneracies





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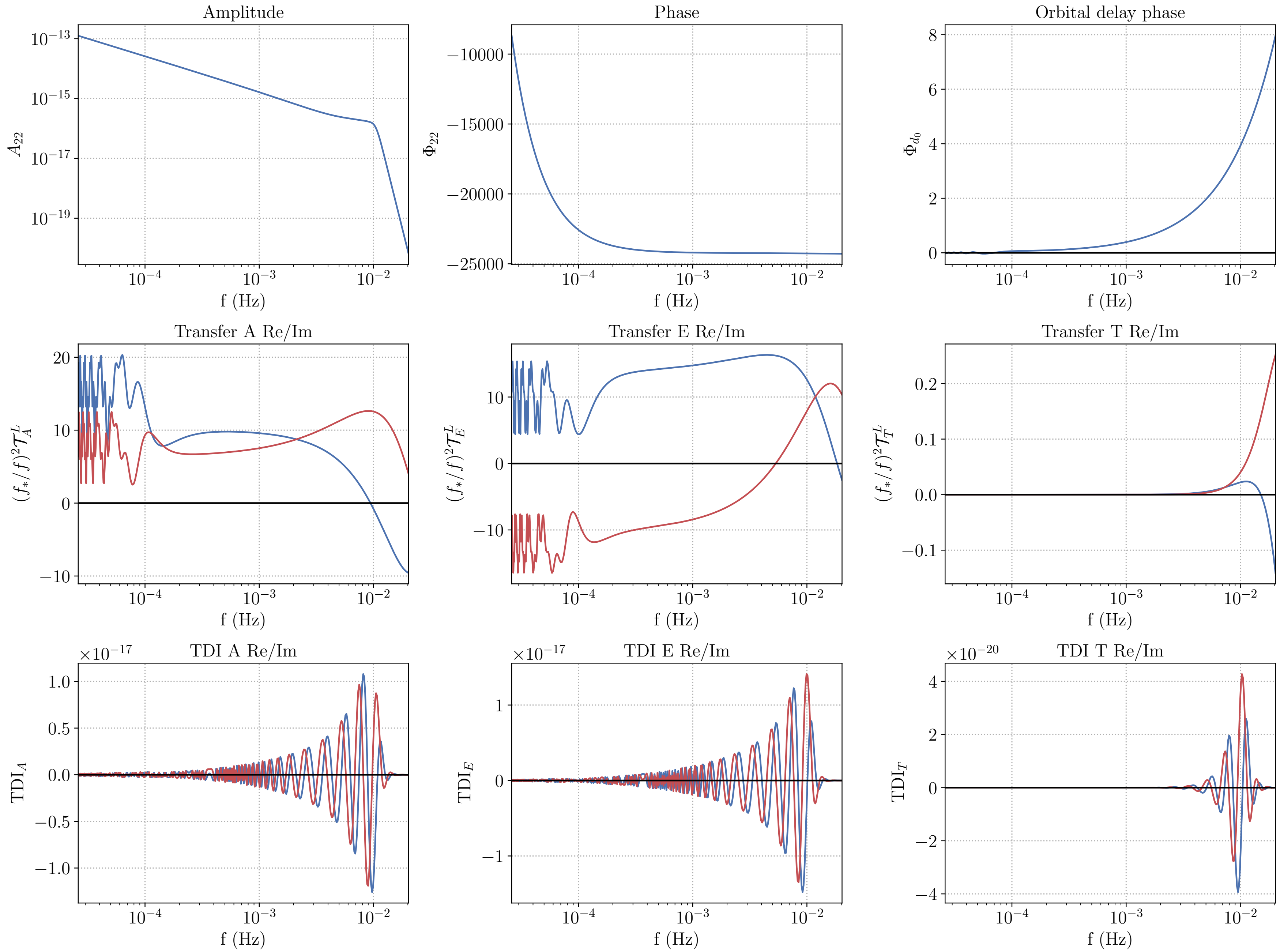
SMBH analysis setting



Sources

- Plausible SMBH sources at z=4
- Masses $M = 2 \cdot 10^6\ M_\odot,\ q = 2$
- Vary orientation

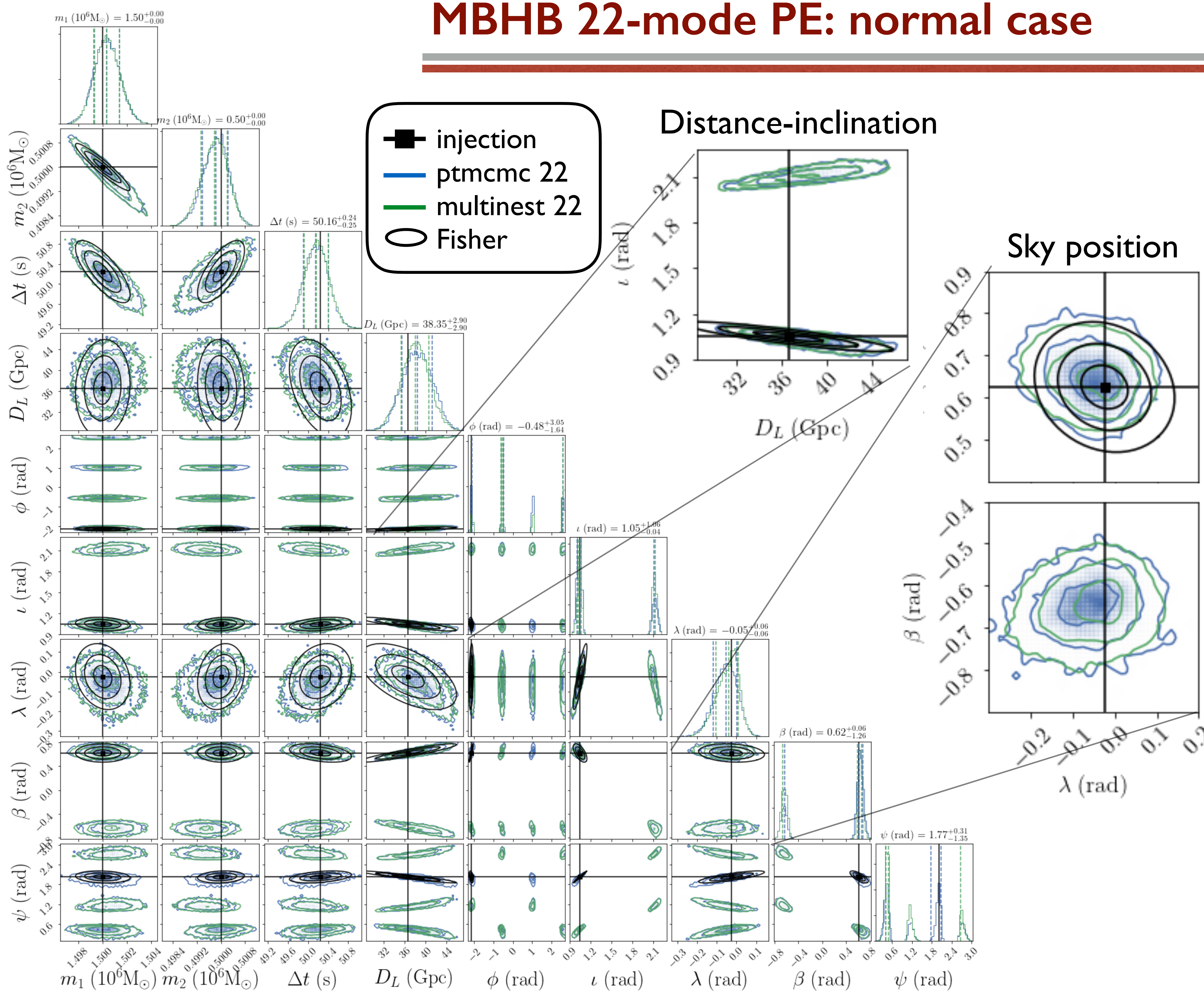
SNR

	I	II
22	857	645
HM	945	666

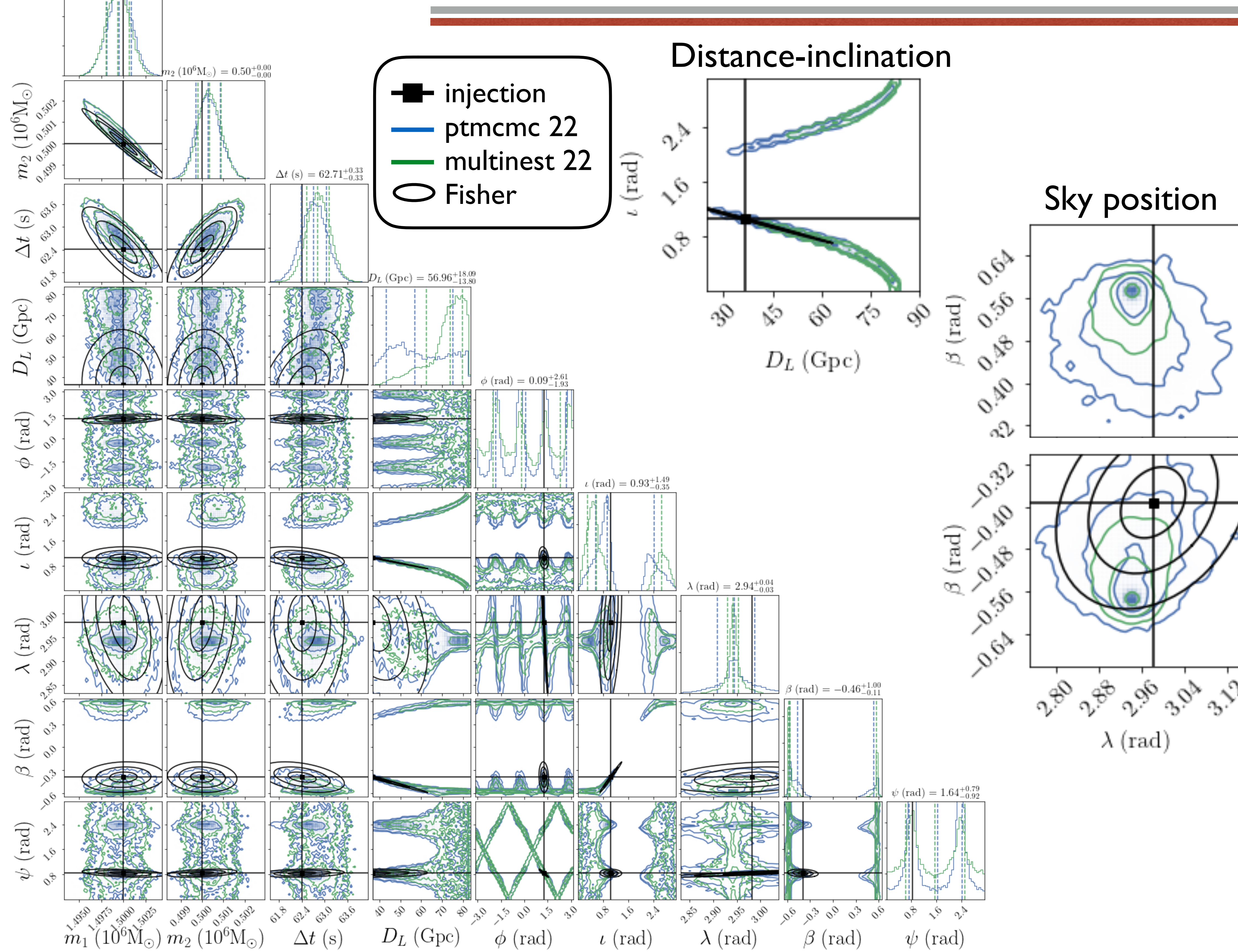
Evaluations

Multinest: $120 \cdot 10^6$
PTMCMC: $400 \cdot 10^6$

MBHB 22-mode PE: normal case



MBHB 22-mode PE: degenerate case



Understanding degeneracies in the likelihood

The frozen LISA approximation

- Neglect all LISA motion for the duration of the signal, take low-frequency response
- Neglect weak correlations between intrinsic and extrinsic parameters - fix masses, time, and vary extrinsic parameters only

Two independent channels A and E:

$$\ln \mathcal{L} = -\frac{1}{2}\Lambda \left(\left| s_a - s_a^{\mathrm{inj}} \right|^2 + \left| s_e - s_e^{\mathrm{inj}} \right|^2 \right)$$

Λ normalisation constant

Explicit pattern functions

$$\begin{aligned}
F_a^+ &= \frac{1}{2}\left(1+\sin^2\beta_L\right)\sin\left(2\lambda_L + \frac{\pi}{6}\right) ,\\
F_a^\times &= -\sin\beta_L \cos\left(2\lambda_L + \frac{\pi}{6}\right) ,\\
F_e^+ &= \frac{1}{2}\left(1+\sin^2\beta_L\right)\cos\left(2\lambda_L + \frac{\pi}{6}\right) ,\\
F_e^\times &= \sin\beta_L \sin\left(2\lambda_L + \frac{\pi}{6}\right) .
\end{aligned}$$

Single-channel response for A:

$$\begin{aligned}
s_a = &\frac{3i}{4D_L}\sqrt{\frac{5}{\pi}}\cos^4\frac{\iota}{2} e^{2i(-\varphi-\psi_L)}\left(D_a^+ + iD_a^\times\right)\\
&+ \frac{3i}{4D_L}\sqrt{\frac{5}{\pi}}\sin^4\frac{\iota}{2} e^{2i(-\varphi+\psi_L)}\left(D_a^+ - iD_a^\times\right) .
\end{aligned}$$

and similarly for E.

Analogous roles of $(\lambda_L, \beta_L) \leftrightarrow (\varphi_L, \iota)$

Understanding degeneracies in the likelihood

The face-on / face-off limit

- Two branches: close to face-on or face-off
- Effective amplitude and phase degenerate in distance/inclination and in phase/polarization

$$\mathcal{A}(D_L, \iota) \sim \cos^4(\iota/2)/D_L$$
$$\xi(\varphi_L, \psi_L) = -\varphi_L - \psi_L$$

For example for $\sin^4 \frac{\iota}{2} \ll 1$

$$s_a \simeq i\mathcal{A}e^{2i\xi}\left(F_a^+ + iF_a^\times\right) \,,$$
$$s_e \simeq i\mathcal{A}e^{2i\xi}\left(F_e^+ + iF_e^\times\right) \,,$$

Explicit solution for the degeneracy

Reproduce s_a, s_e of injection if condition on sky position is met:

$$r = \frac{s_a^{\rm inj}}{s_e^{\rm inj}} = \frac{F_a^+ + iF_a^\times}{F_e^+ + iF_e^\times}(\lambda_L, \beta_L)$$

Then **line degeneracy** for both (φ_L, ψ_L) and (D_L, ι)

Solution : $\rho = \sqrt{\left|\frac{1+ir}{1-ir}\right|}$

$$\sin\beta_L^* = \frac{\rho - 1}{\rho + 1}$$
$$\lambda_L^* = -\frac{\pi}{12} + \frac{1}{4}\mathrm{Arg}\frac{1+ir}{1-ir} + \frac{k\pi}{2} \,.$$

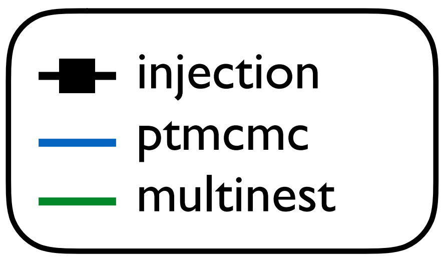
+ approximate symmetry

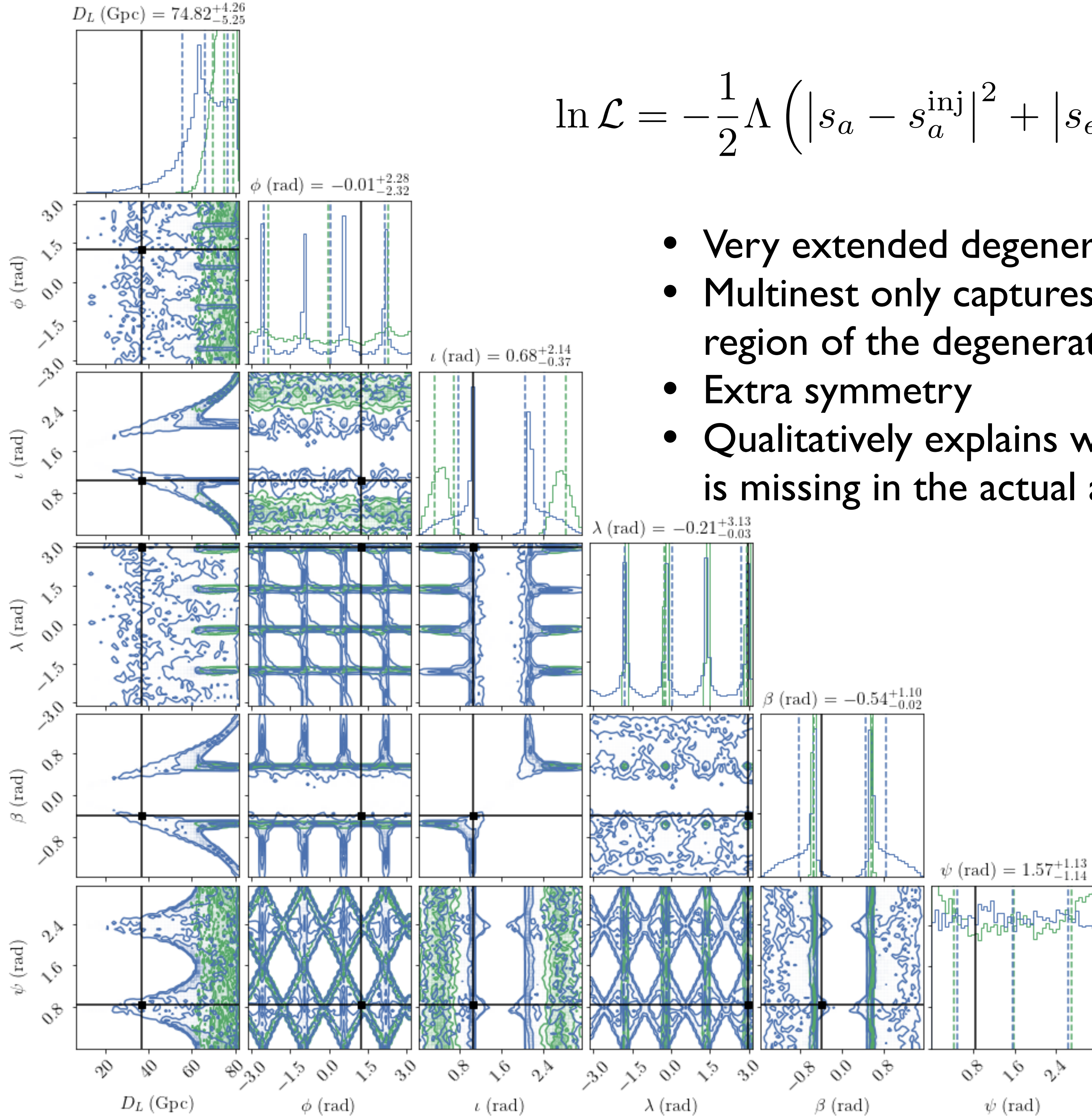
$$(\lambda_L, \beta_L) \leftrightarrow (\varphi_L, \iota)$$

Exploring the analytic simplified extrinsic likelihood

$$\ln \mathcal{L} = -\frac{1}{2}\Lambda\left(\left|s_a - s_a^{\mathrm{inj}}\right|^2 + \left|s_e - s_e^{\mathrm{inj}}\right|^2\right)$$

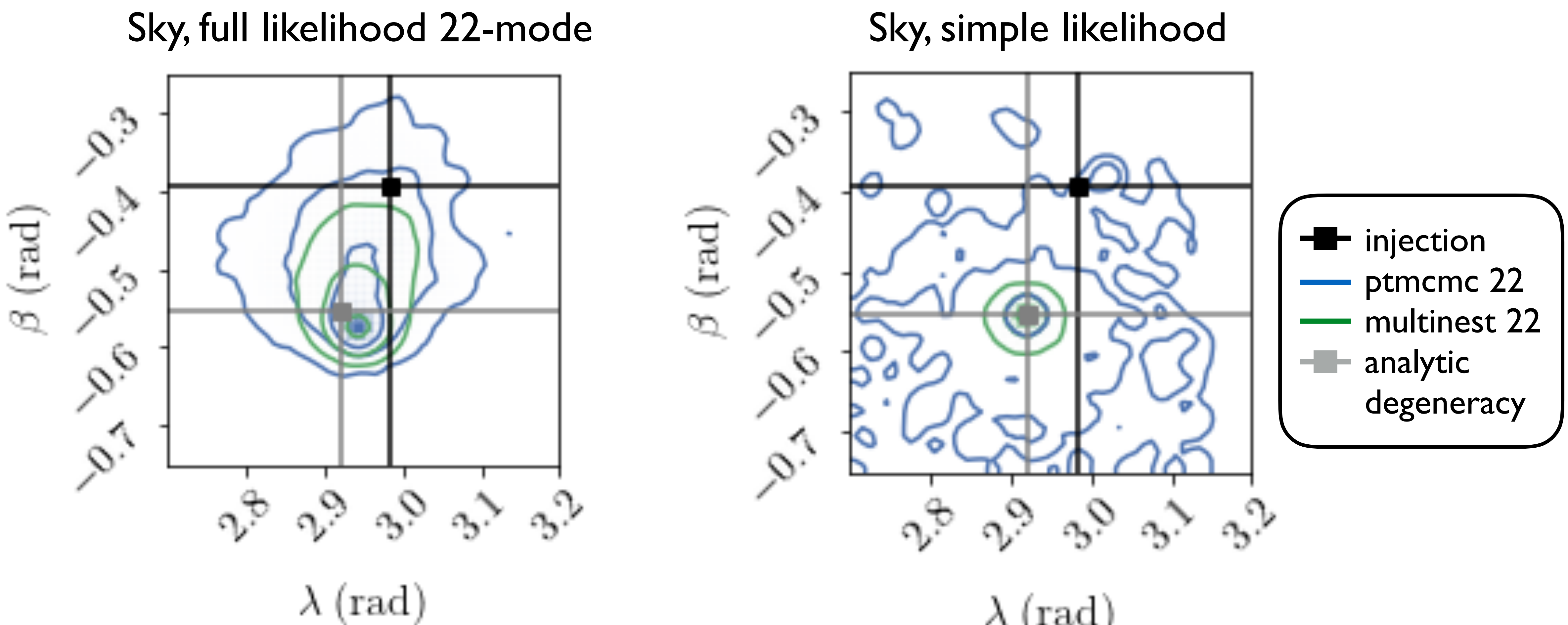
- Very extended degeneracies
- Multinest only captures a small region of the degenerate likelihood
- Extra symmetry
- Qualitatively explains what multinest is missing in the actual analysis





Understanding degeneracies

A projection effect for the marginal posterior



The role of higher harmonics

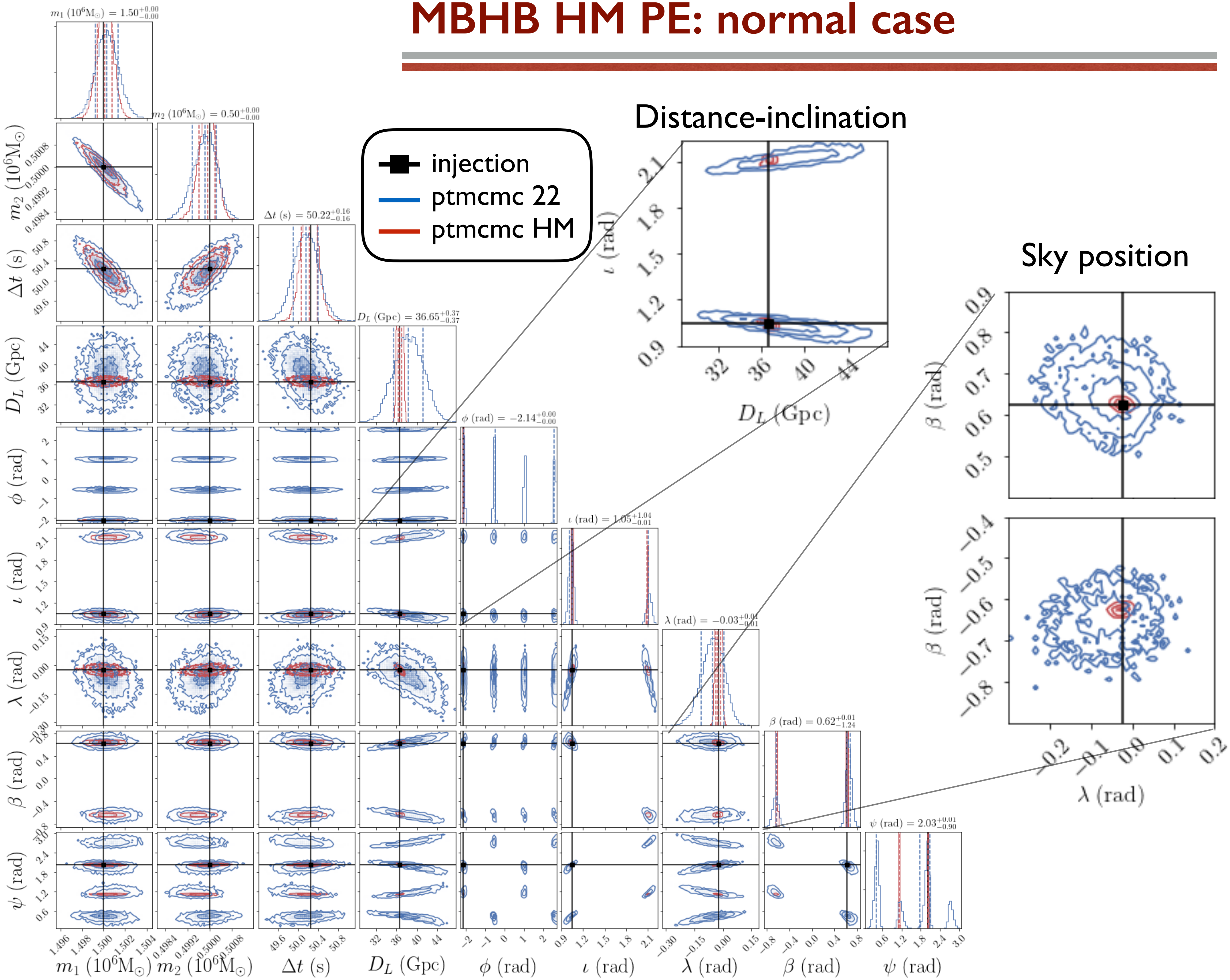
$$h_+ - ih_\times = \sum {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

$${}_{-2}Y_{\ell m}(\iota, \varphi) \propto e^{im\varphi}$$

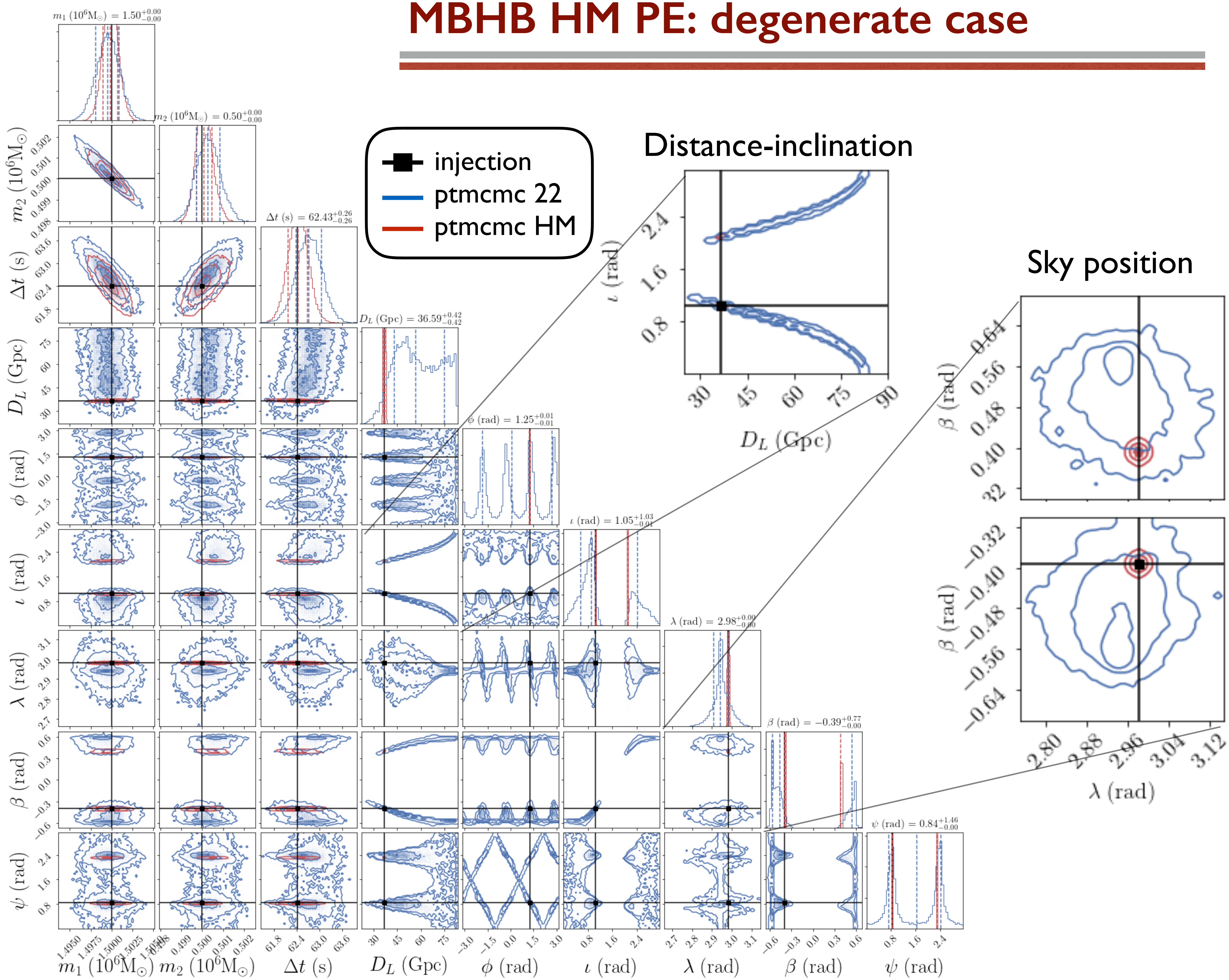
Different modes have different inclination and phase dependence

- Measuring relative amplitude of two modes gives the inclination
- Distance is then fixed by the amplitude
- Phase affects modes differently, not degenerate with polarization anymore

MBHB HM PE: normal case



MBHB HM PE: degenerate case



SMBH PE: accumulation of information with time

Method

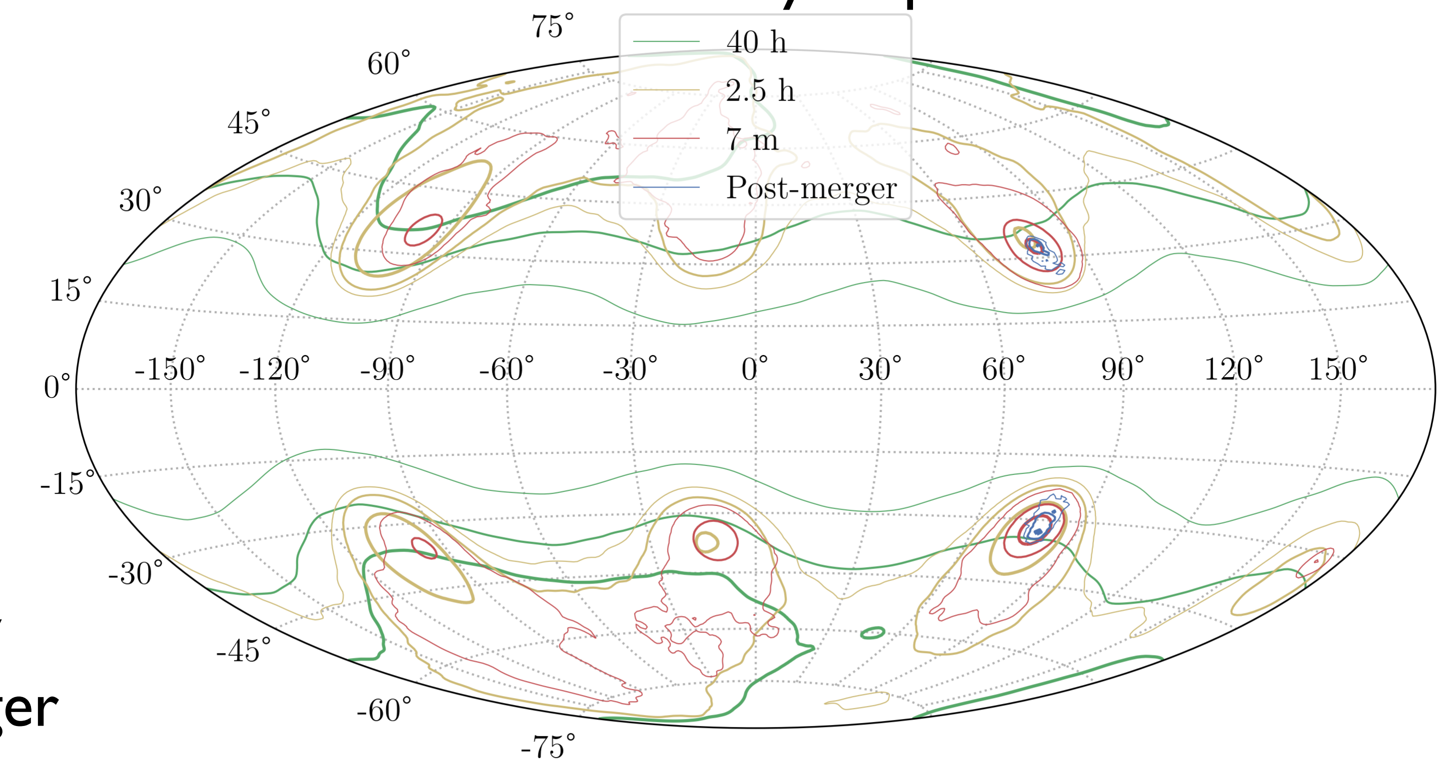
- Represent a cut in time-to-merger by a cut in frequency, becomes inaccurate at merger
- Use Multinest and PTMCMC with and without higher harmonics

8-maxima sky degeneracy
only broken shortly before merger
2-maxima sky degeneracy
survives after merger

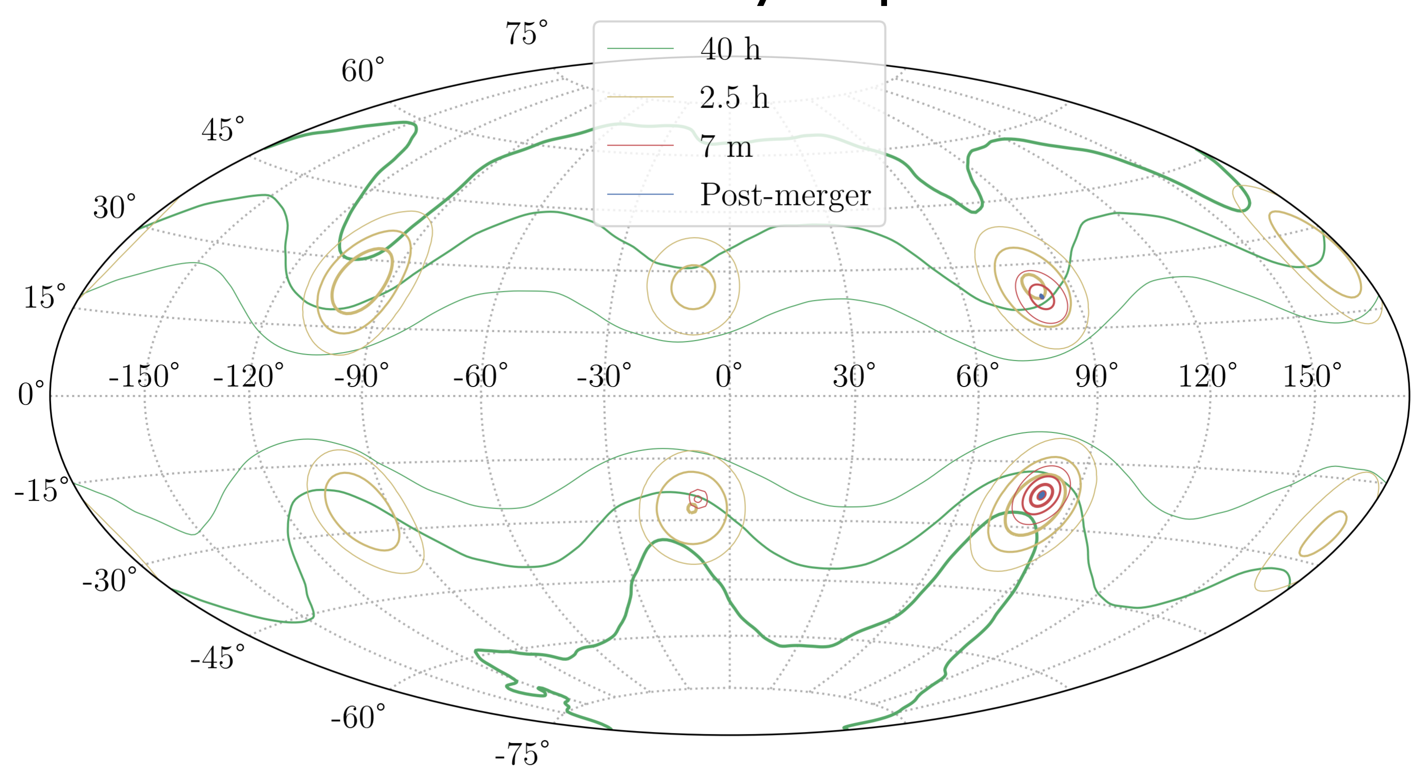
SNR-based time cuts:

SNR	DeltaT
10	40h
42	2.5h
167	7min
666	-

LISA-frame sky map 22



LISA-frame sky map hm



MBHB PE: accumulation of information with time

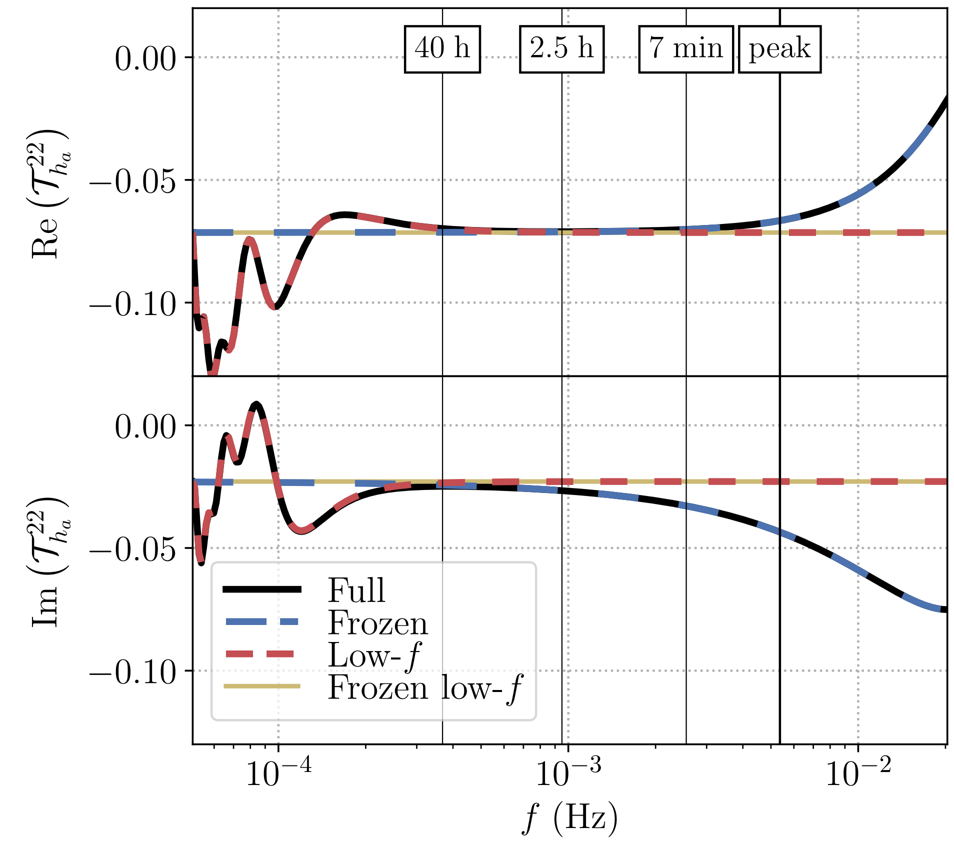
Decomposing the response

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \text{sinc}\left[\pi f L \left(1 - k \cdot n_l\right)\right] \exp\left[i\pi f \left(L + k \cdot (p_r + p_s)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

Time and frequency-dependency in transfer functions
Time: motion of LISA on its orbit
Frequency: departure from long-wavelength approx.

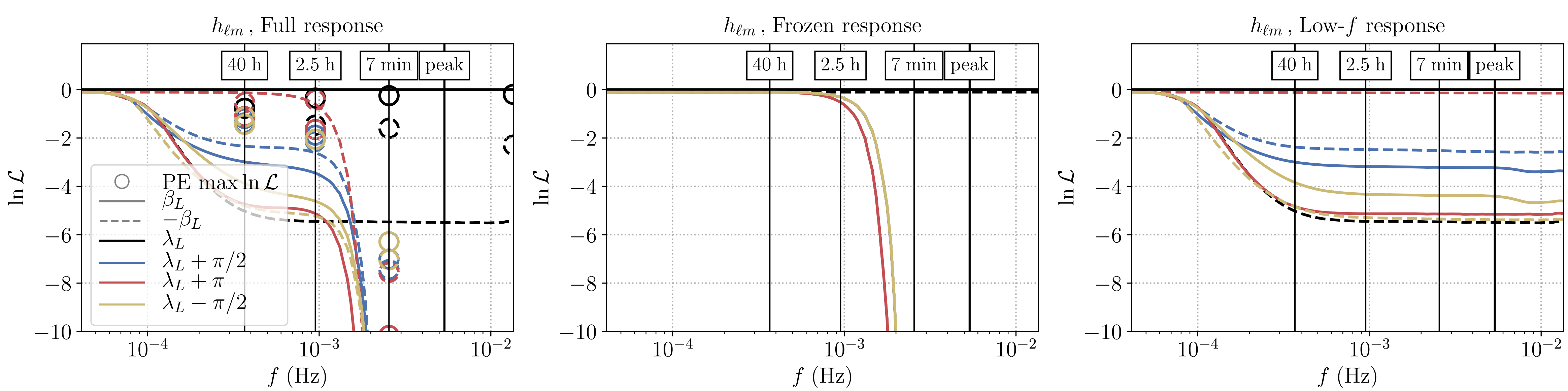
- 'Full': keep all terms
- 'Frozen': ignore LISA motion
- 'Low-f': ignore f-dependency
- 'Frozen Low-f': ignore both

High-f features crucial



Degeneracy breaking for 8 sky maxima

Log-likelihood values when frequency increases:

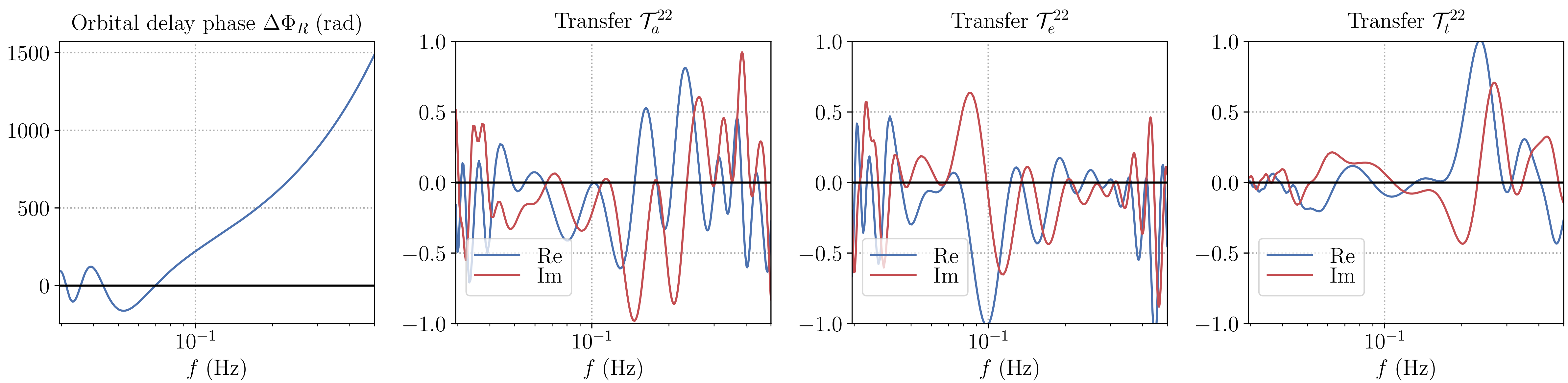


Outline

- Introduction and motivation
- The duration of Black Hole Binary signals in LISA
- The LISA response in the Fourier domain
- Methods for Bayesian parameter estimation
- Parameter estimation for Massive Black Hole Binaries
- **Parameter estimation for Stellar-mass Black Hole Binaries**
- Conclusions and outlook

SBHB analysis setting

Work led by Alexandre Toubiana at APC, in preparation



m_1 ($M_\odot$)	40	
m_2 ($M_\odot$)	30	
t_c (yrs)	8	
f_0 (mHz)	12.7215835397	
χ_1	0.6	
χ_2	0.4	
λ (rad)	1.9	
β (rad)	$\pi/3$	
ψ (rad)	1.2	
φ (rad)	0.7	
ι (rad)	$\pi/6$	
D_L (Mpc)	250	
T_{obs} (yrs)	4	10
SNR	13.5	21.5

From a fiducial system, vary:

- Initial frequency (earlier/later)
- Mass (heavy light)
- Mass ratio (q=3, q=8)
- Spin configuration
- Sky position (polar/equatorial)
- Inclination and distance

Time to coalescence: Tc = 8yrs
Tobs=4years: slowly chirping
Tobs=10yrs: merger during observations

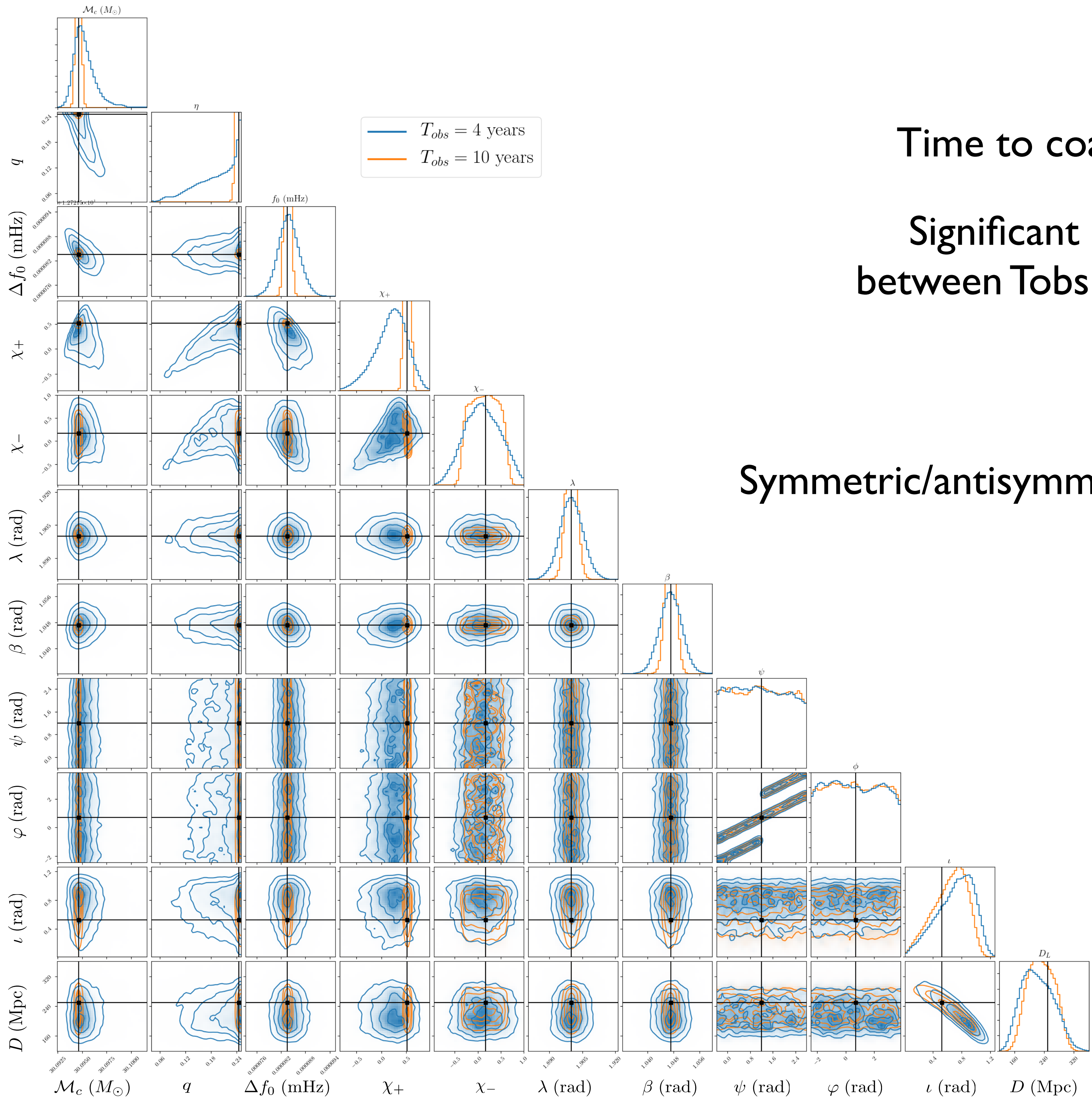
SNR

	T_{obs} = 4 years	T_{obs} = 10 years
Fiducial	13.5	21.1
Earlier	10.3	17.2
Later	11.8	/
Heavy	12.8	20.9
Light	14.1	21.1
q3	13.5	21.1
q8	13.5	21.1
Spinup	13.5	21.1
Spindown	13.5	21.1
Spinop12	13.5	21.1
Spinop21	13.5	21.1
Polar	12.8	20.1
Equatorial	14.9	23.1
Edgeon	/	14.7
Close	17.8	/
Far	/	15.1
Very Far	/	10.6

Note: SNR with 'Proposal' noise curve, not 'Requirement' (50% margin)

Detection might be a challenge [Moore&al]

SBHB parameter estimation results



Time to coalescence: Tc = 8yrs

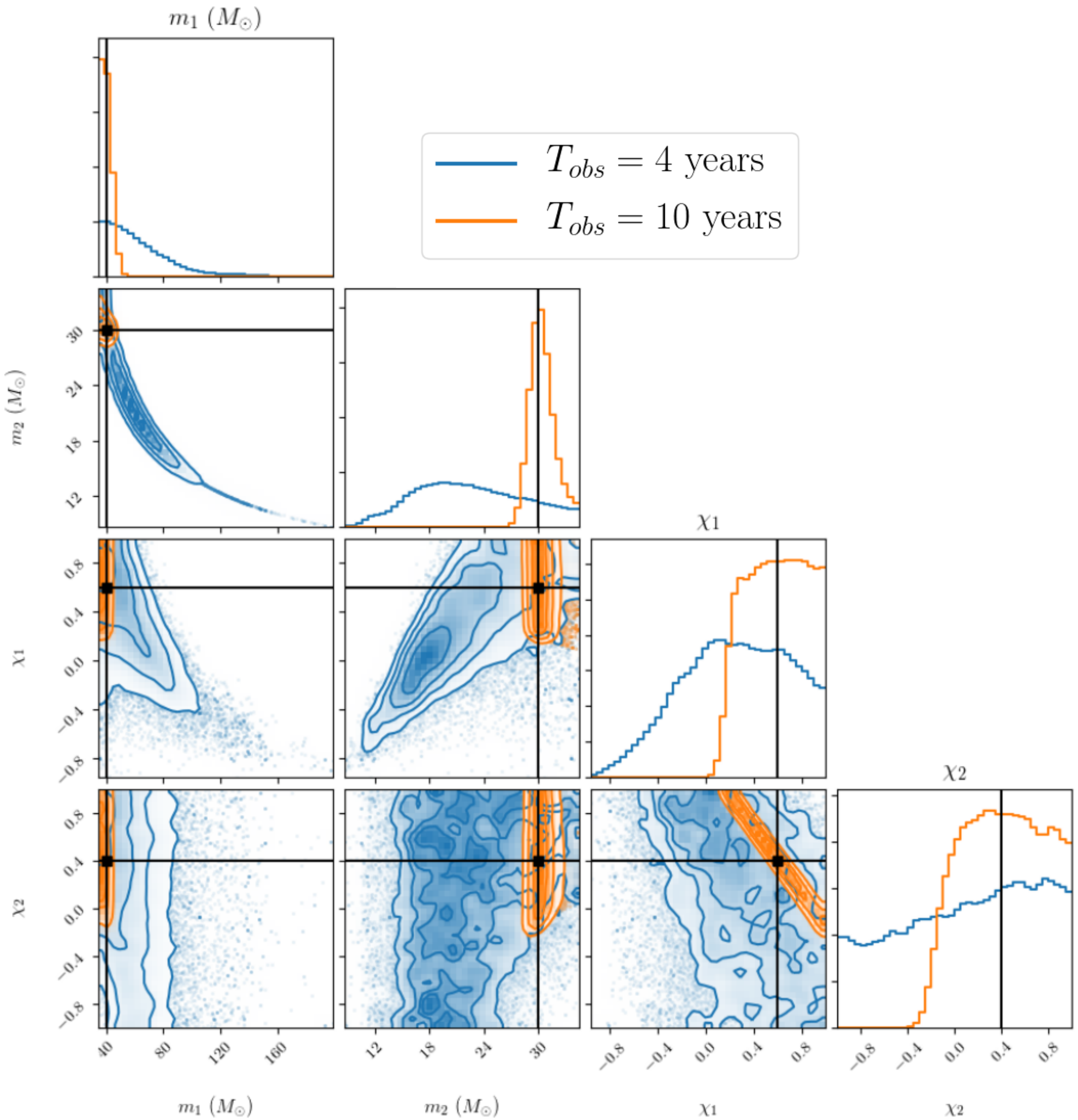
Significant qualitative differences between Tobs = 4yrs and Tobs = 10yrs

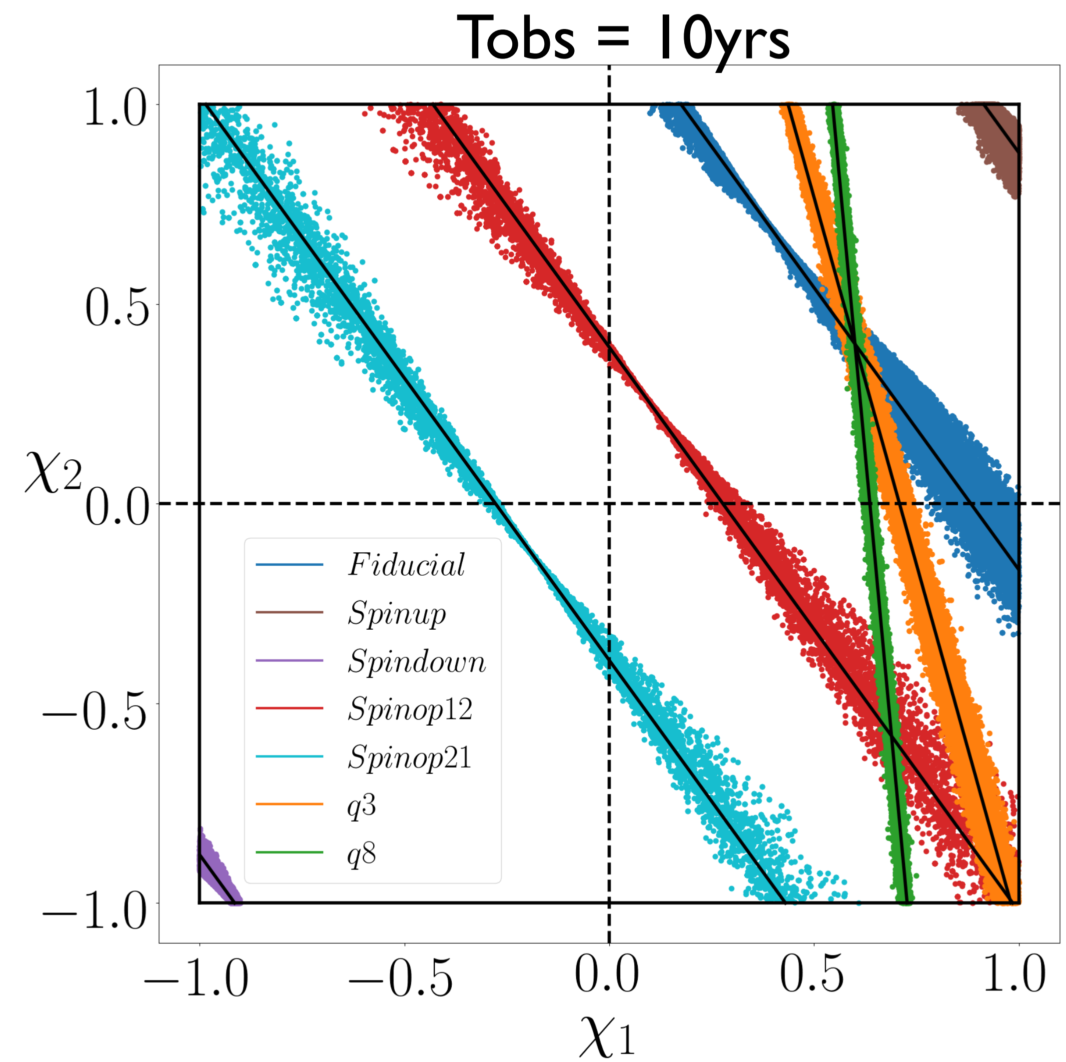
Symmetric/antisymmetric spin combinations:

$$\chi_+ = \frac{m_1\chi_1 + m_2\chi_2}{m_1 + m_2}$$

$$\chi_- = \frac{m_1\chi_1 - m_2\chi_2}{m_1 + m_2}$$

SBHB parameter estimation results: masses and spin





Leading spin-orbit combination in the phase at 1.5PN:

$$\chi_{PN} = \frac{1}{113}\left(94\chi_+ + 19\frac{q-1}{q+1}\chi_-\right)$$

Determining intrinsic parameters can depend strongly on the duration of observations (chirp/no chirp)

The relevant spin combination observed is χ_{PN}

SBHB parameter estimation results: sky position

As a consequence of the length of the signal and of the LISA motion:

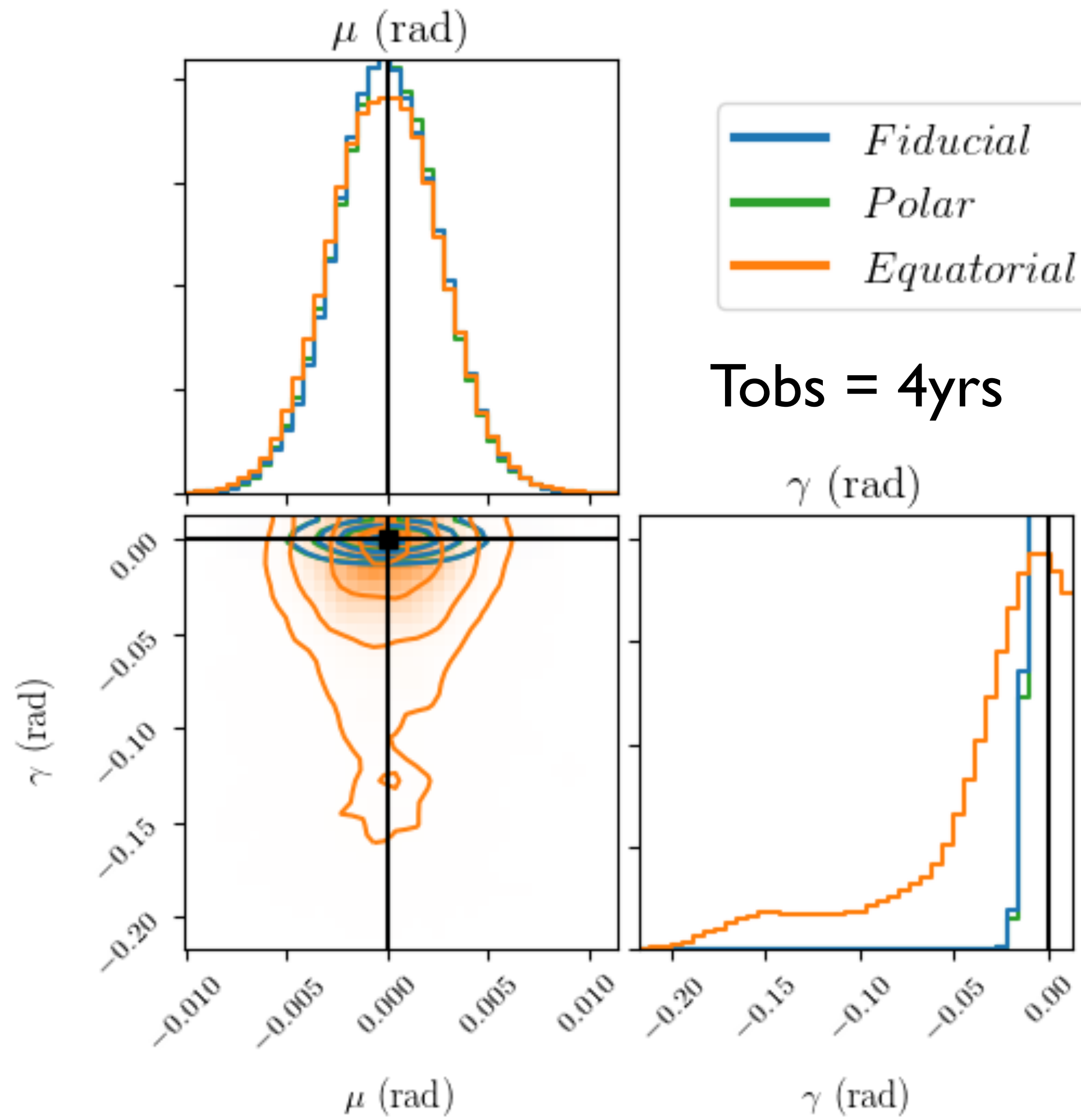
The sky localization is generally very good and very Gaussian

Signals near the ecliptic plane can show degeneracies in their localization

Example:

- Fiducial (SNR=13.5) $\beta = \pi/3$
- Polar (SNR=12.8) $\beta = \pi/2 - 0.09$
- Equatorial (SNR=14.9) $\beta = 0.09$

Offset spherical angles, centered on injected signal: eliminate coordinate effects near the pole



SBHB parameter estimation results: SNR and high-f

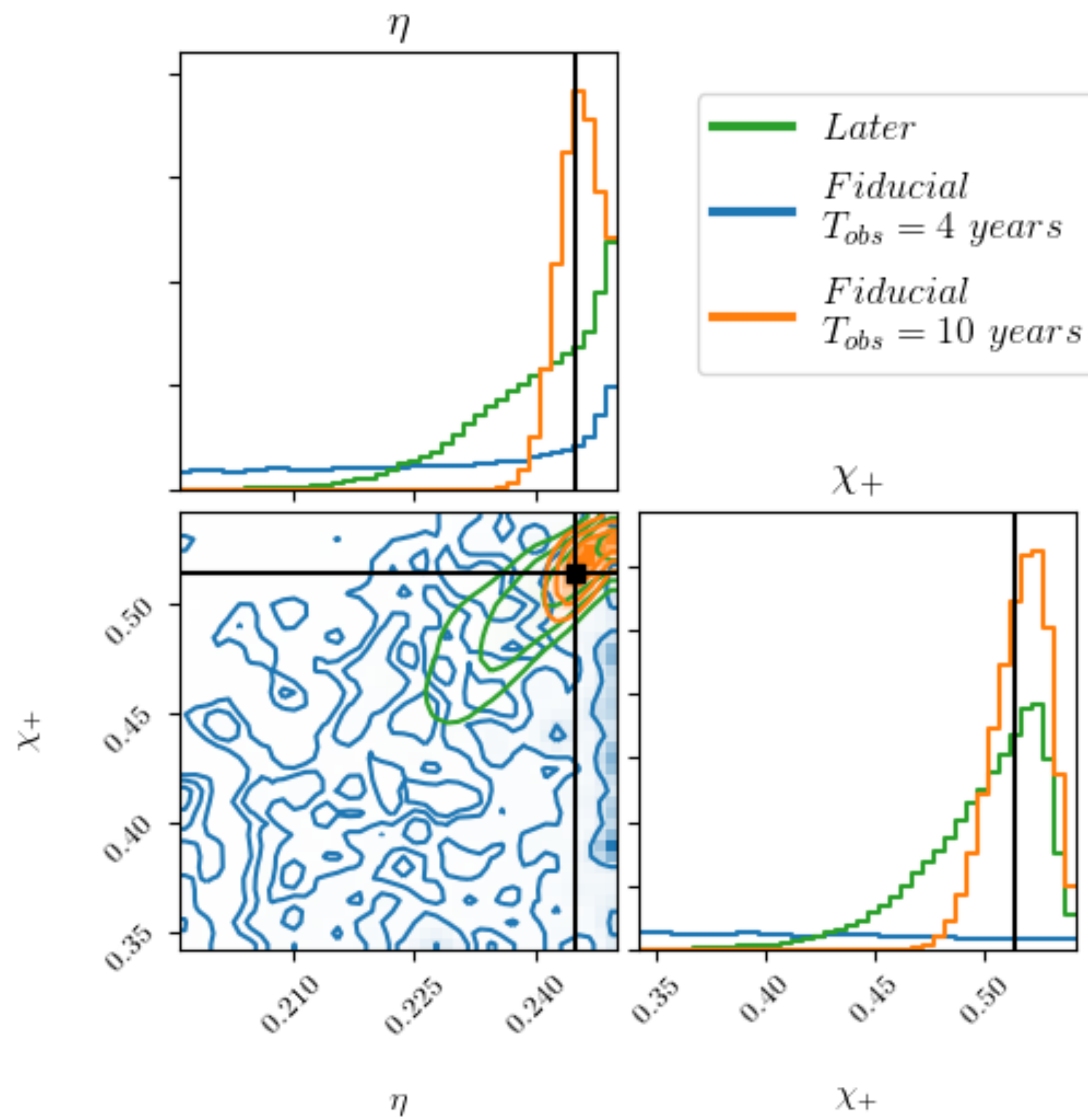
Determination of intrinsic parameters differs strongly for Tobs=4yrs and Tobs=10yrs

Is it due to the SNR increase, or to the signal reaching higher frequencies (more affected by subdominant PN terms) ?

Example:

- Fiducial Tobs=4yrs (SNR=13.5)
  $f = 12.1 - 16.5$ mHz
- Fiducial Tobs=10yrs (SNR=21.1)
  $f = 12.1$ mHz $\to$ merger
- 'Later' Tc=2yrs (SNR=11.8)
  $f = 21.4$ mHz $\to$ merger

Observing high frequencies matters in measuring intrinsic parameters



Conclusions and outlook

Highlights

- Developed a generic approach to the Fourier domain response of LISA
- Developed fast likelihood enabling zero-noise Bayesian explorations of high-SNR or long signals
- Explored the LISA parameter recovery of MBHB signals
- Analytic understanding of degeneracies in the MBHB likelihood when including only the dominant quadrupolar harmonic
- Shown the crucial role of higher modes in breaking degeneracies for MBHBs
- Shown that high-frequency effects in the response are crucial in breaking degeneracies when accumulating signal with time
- Explored the LISA parameter recovery of SBHB signals with aligned spins

Outlook

- Inclusion of spins and precession
- Optimize samplers for known degeneracies (MCMC jump proposals)
- Explore the parameter space (from most massive MBHB to IMBHB)
- Explore the effect of eccentricity
- Make the link to instrumental requirements
- Explore joint LISA/LIGO observations
- Assess waveform model requirements: how accurate need the waveforms to be ?
- Are these methods applicable to EMRIs ?

Higher-order corrections

Expansion of f-dependent convolution

Quadratic phase term: $\mathcal{T}(f) = \sum \frac{1}{p!} \left(\frac{i}{8\pi^2} \frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2} \right)^p \partial_t^{2p} G(f, t_f) \longrightarrow$ Evaluation on a stencil cf SUA [Klein&al 2014]

Amplitude: $\mathcal{T}(f) = \sum \frac{1}{(2i\pi)^p p!} \frac{1}{A} \frac{\mathrm{d}^p A}{\mathrm{d}f^p} \partial_t^p G(f, t_f)$

Delays: $\mathcal{T}(f) = \sum \frac{1}{(2i\pi)^p p!} \partial_t^p \partial_f^p G(f, t_f) \longrightarrow$ Evaluation through a change of time variable

Timescales and error estimates

IMR 'radiation reaction' timescale:

$$T_f^2 = -\frac{1}{4\pi^2} \frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2}$$

When SPA applicable to h:

$$T_f = T_{\mathrm{RR}}^{\mathrm{SPA}} = 1/\sqrt{2\dot{\omega}}$$

Amplitude timescales: $T_{A1} = \frac{1}{2\pi A} \frac{\mathrm{d}A}{\mathrm{d}f}$

Error measures: estimates for the magnitude of corrections

$$\epsilon_{\Psi 2} \equiv \frac{1}{2} T_f^2 \left| \frac{1}{G} \partial_{tt} G \right| \sim (T_{\mathrm{RR}}/T_0)^2 (\times 2\pi f d?)$$

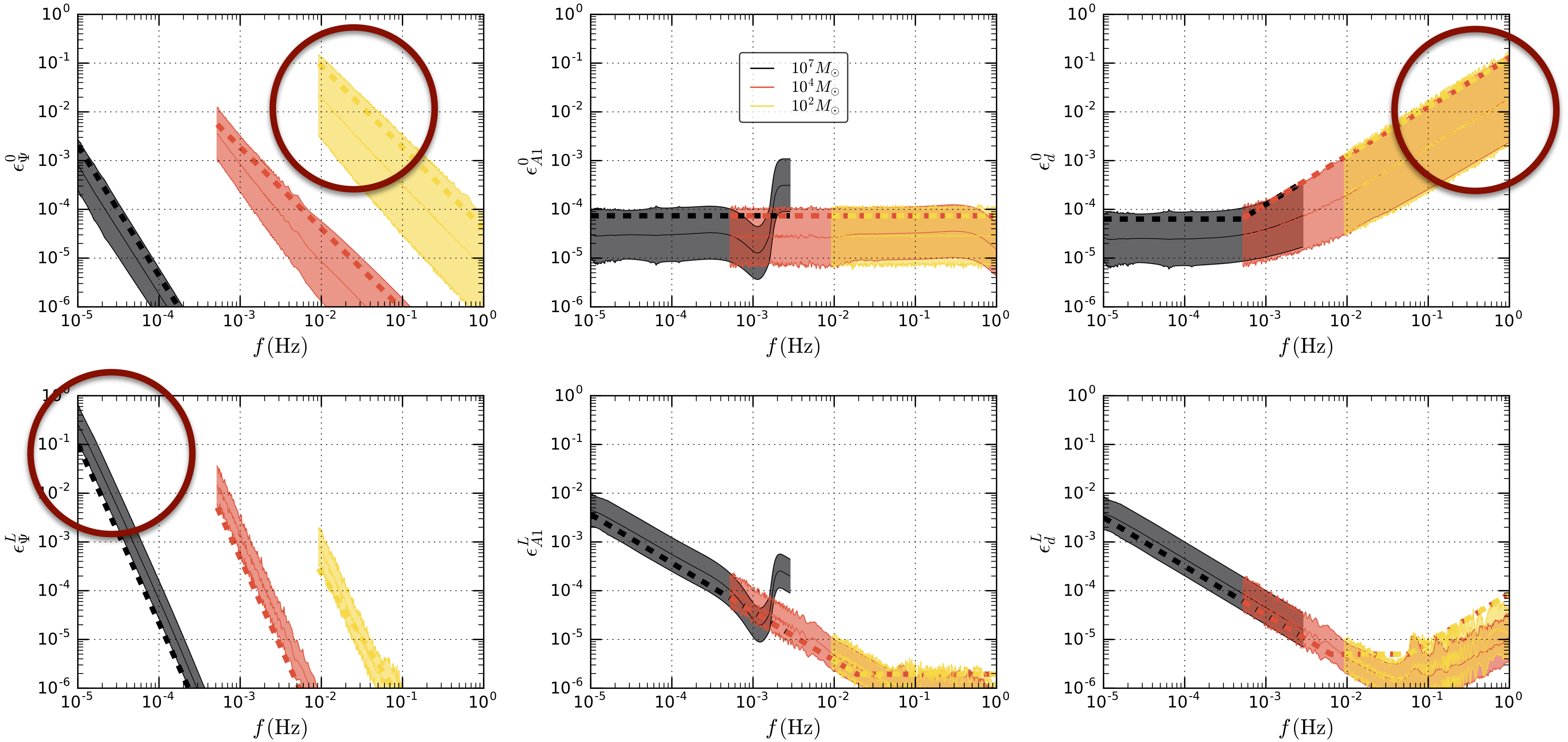
$$\epsilon_{A1} \equiv T_{A1} \left| \frac{1}{G} \partial_t G \right|$$

$$\epsilon_d \equiv \frac{1}{2\pi} \left| \frac{1}{G} \partial_{tf} G \right|$$

FD response error estimates - chirping

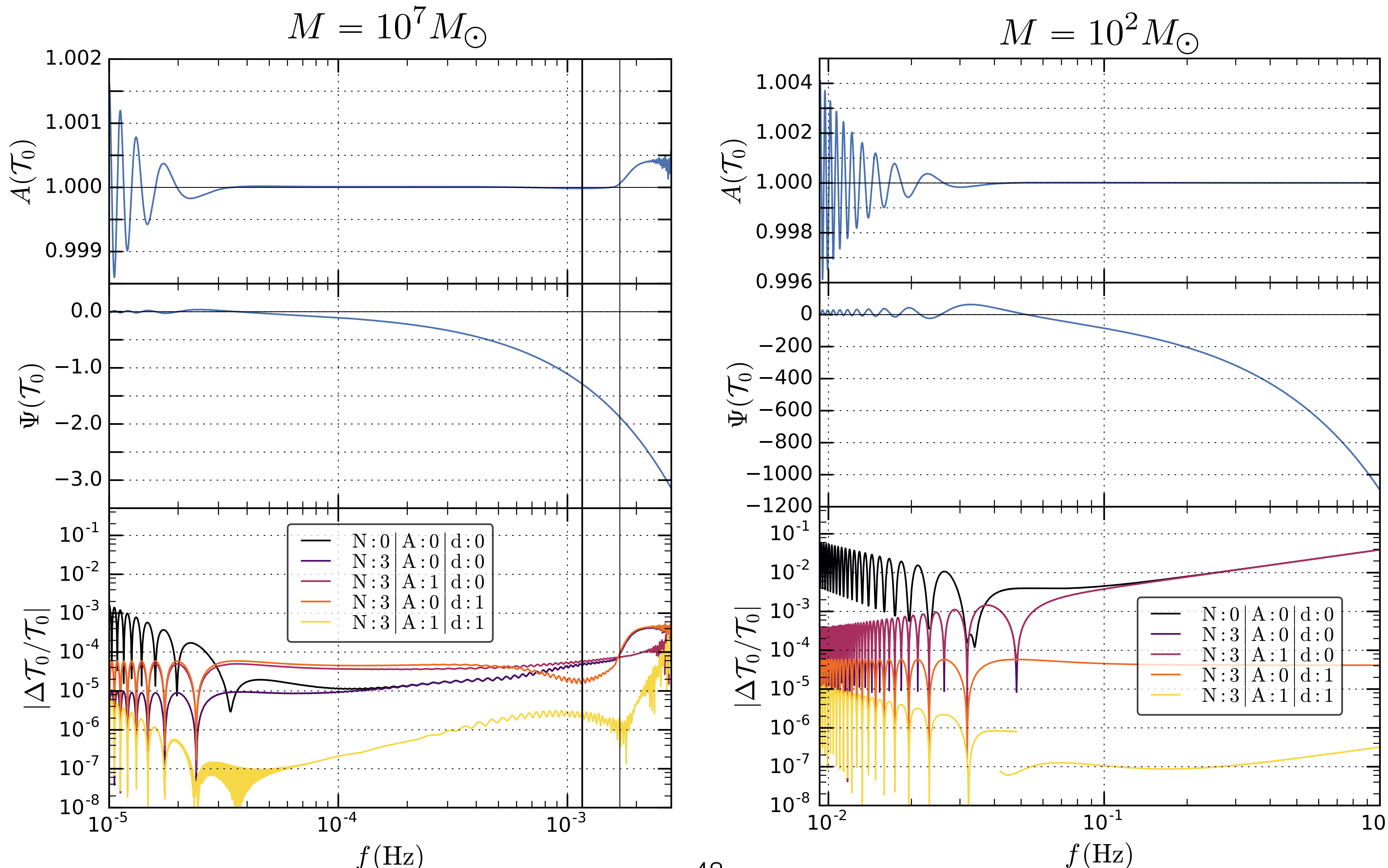
Error estimates: magnitude of the first term neglected in the perturbative series (averaged)

10 years signal



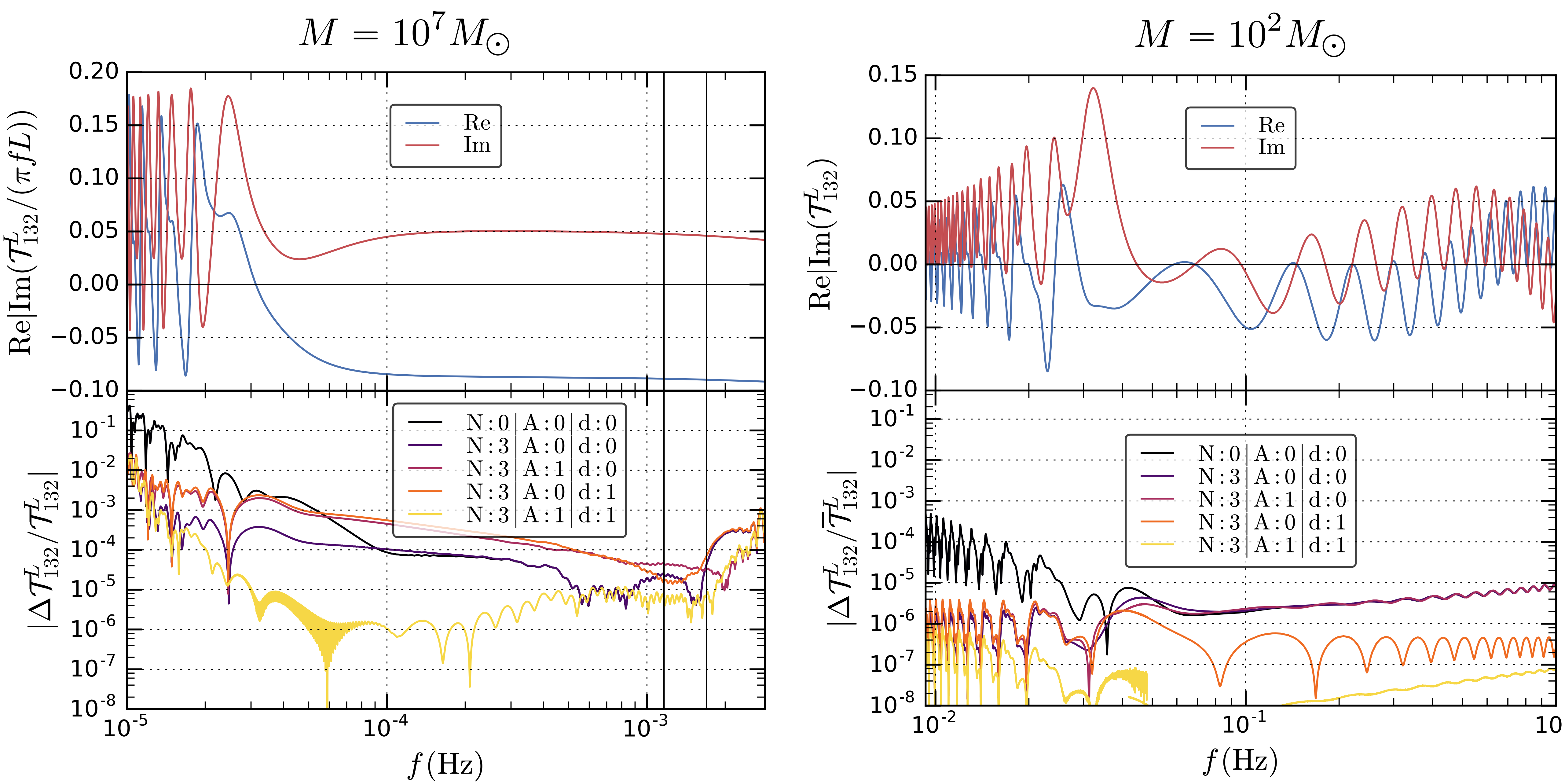
FD response errors - M1e7/M1e2 orbital

Errors: FDResponse[h] vs FFT[TDResponse[IFFT[h]]]



FD response errors - M1e7/M1e2 const.

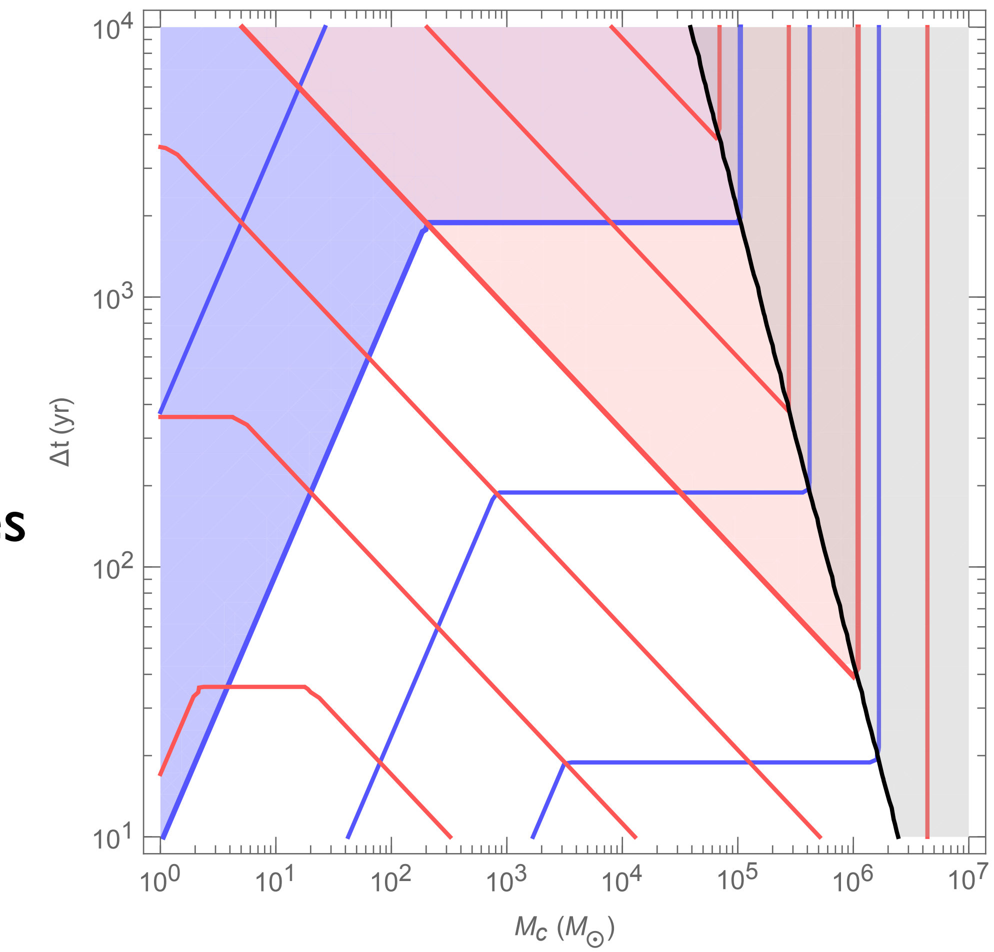
Errors: FDResponse[h] vs FFT[TDResponse[IFFT[h]]]



SOBH - slowly chirping systems

Slowly-chirping systems

- Some SOBHs will be >100-1000 years away from merger
- Quasi-monochromatic limit: breaks separation of timescale, in this limit analogous to galactic binaries

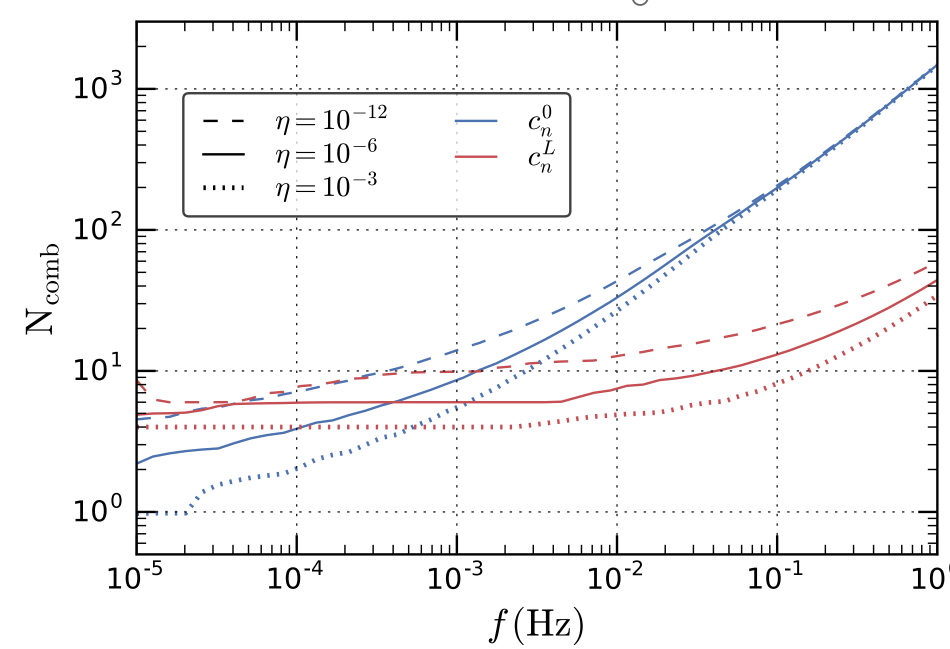


Handling the response

- Heterodyning (narrow frequency band)
- Response is periodic: convolution with a small frequency-dependent Fourier comb

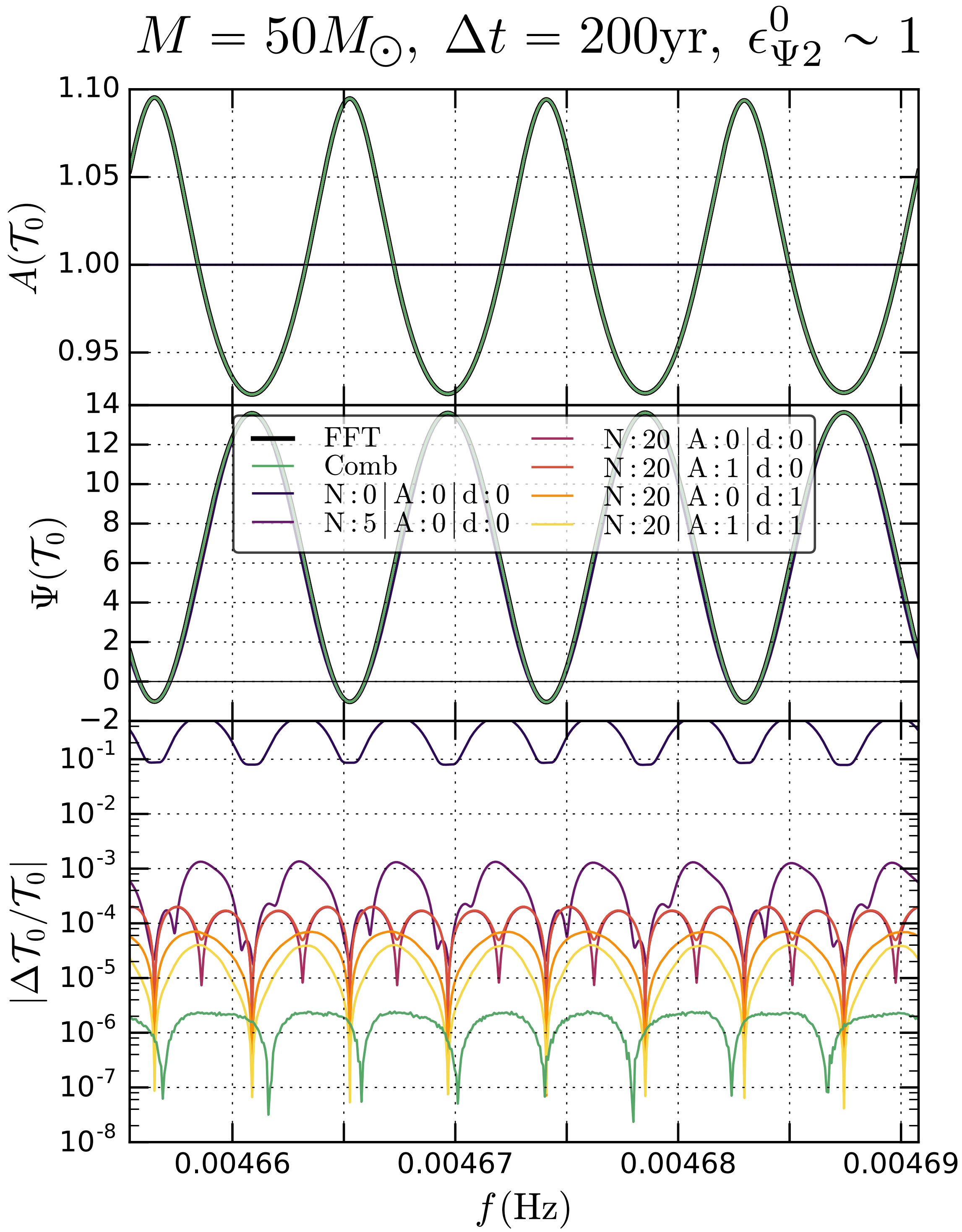
$$c_n(f) = \frac{\Omega_0}{2\pi} \int_0^{\frac{2\pi}{\Omega_0}} \mathrm{d}t \, e^{in\Omega_0 t} G(f,t)$$

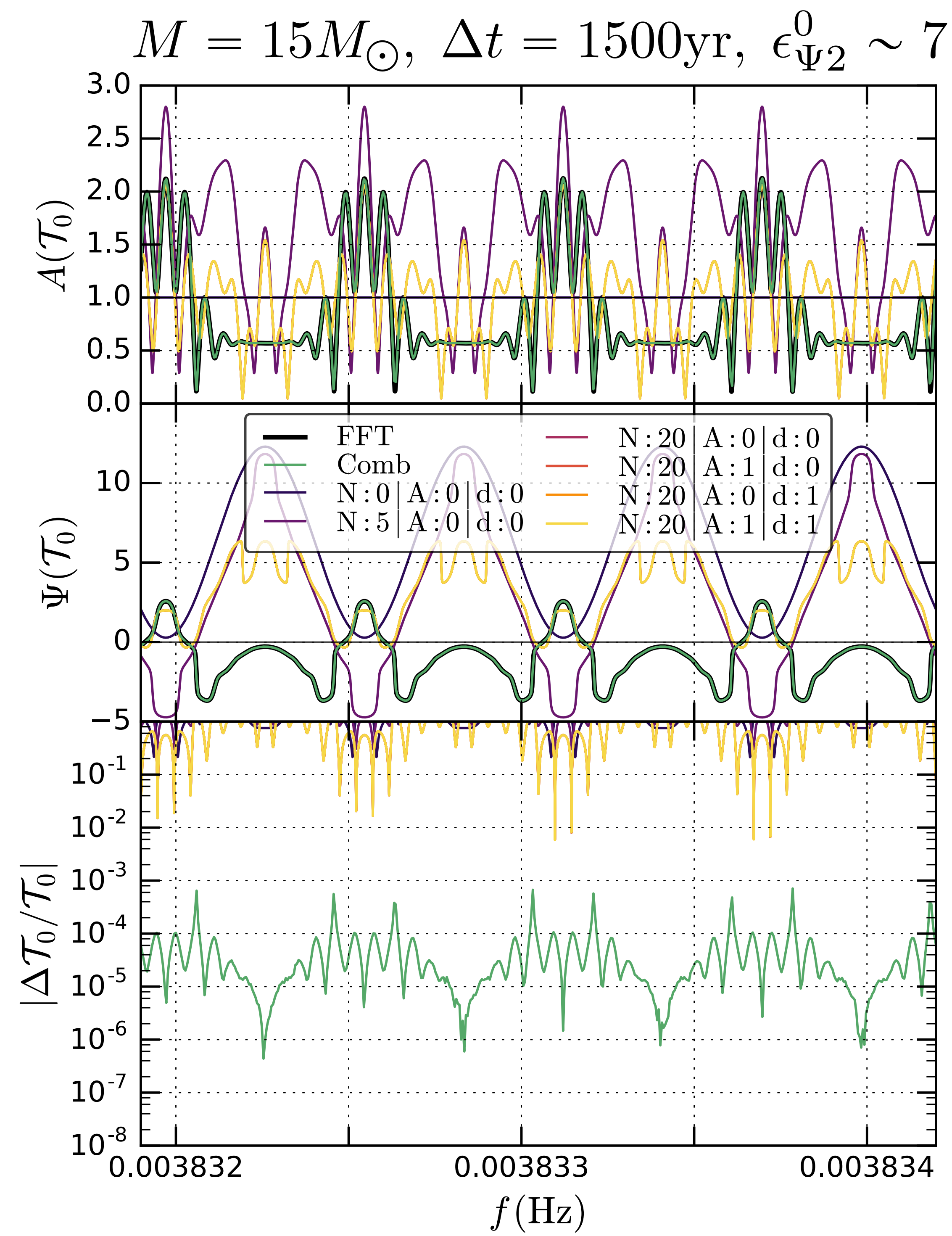
$$\tilde{s}(f) = \sum_{n \in \mathbb{Z}} c_n(f - nf_0) \tilde{h}(f - nf_0)$$



FD response errors - SOBH orb.

Errors: FDResponse[h] vs FFT[TDResponse[IFFT[h]]]

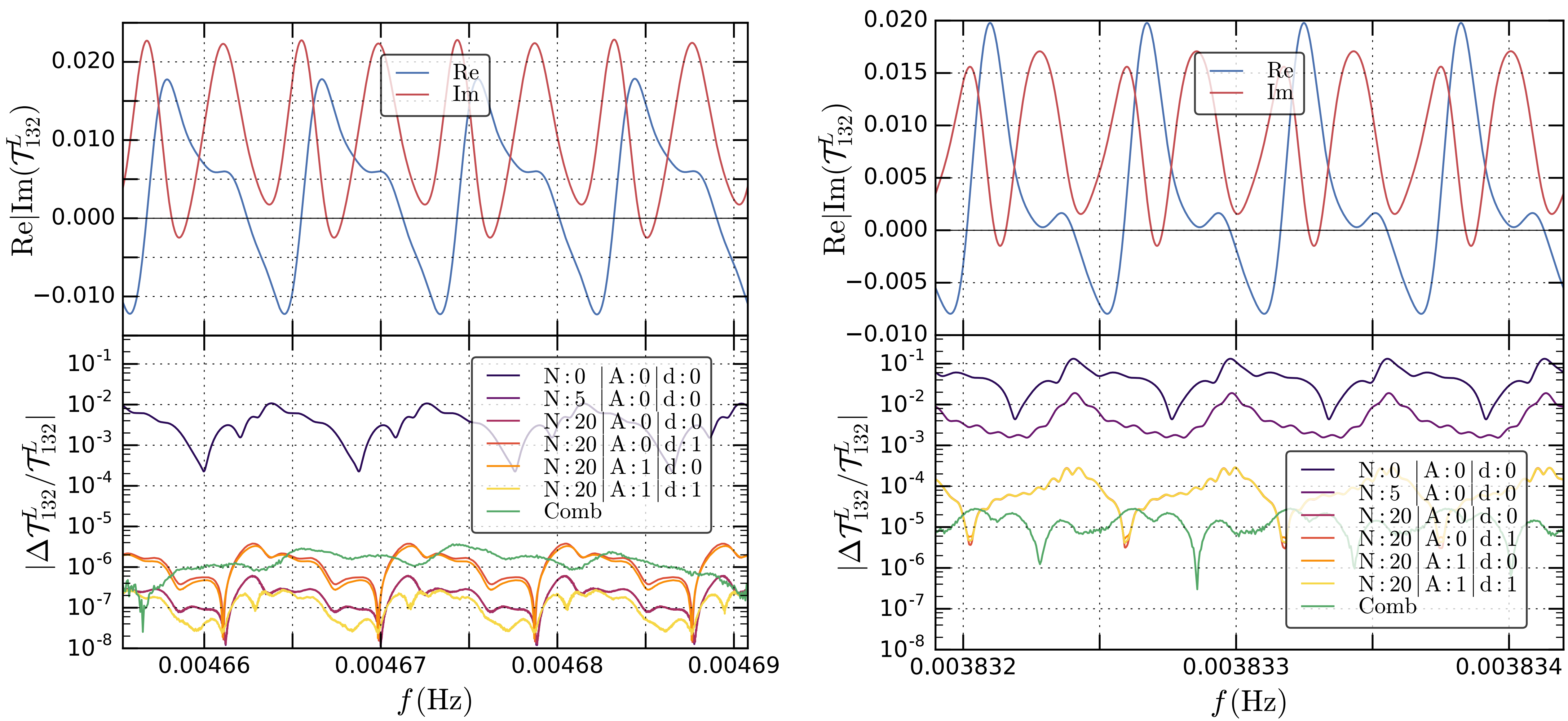




FD response errors - SOBH const.

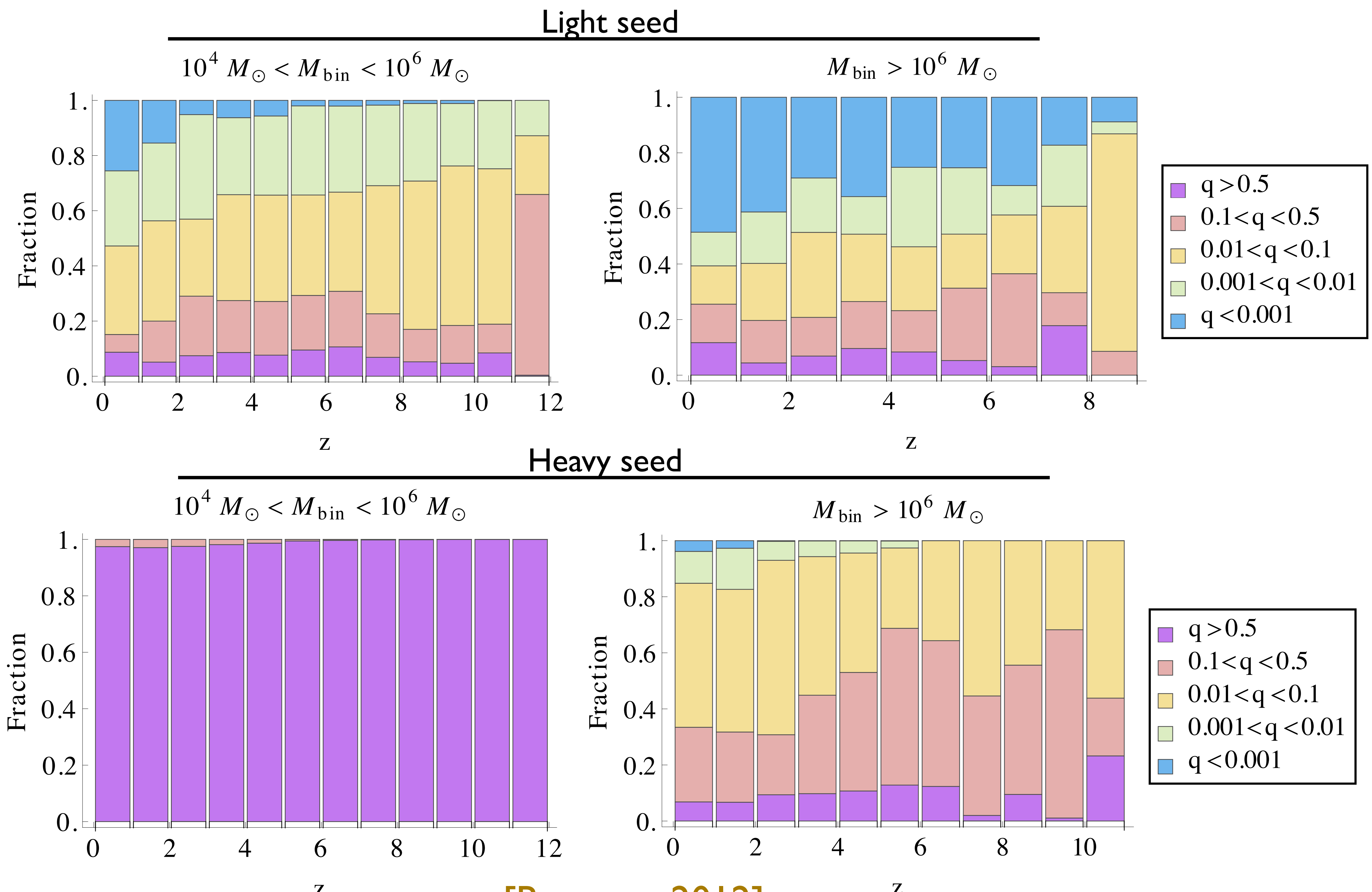
Errors: FDResponse[h] vs FFT[TDResponse[IFFT[h]]]

$M = 50M_\odot,\ \Delta t = 200\mathrm{yr},\ \epsilon^0_{\Psi 2} \sim 0.01$ $M = 15M_\odot,\ \Delta t = 1500\mathrm{yr},\ \epsilon^0_{\Psi 2} \sim 0.1$



LISA source properties: mass ratio

- Wide range of possible mass ratios



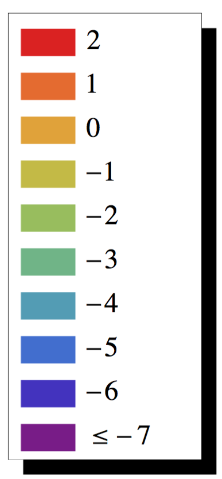
[Barausse 2012]

LISA source properties: spin

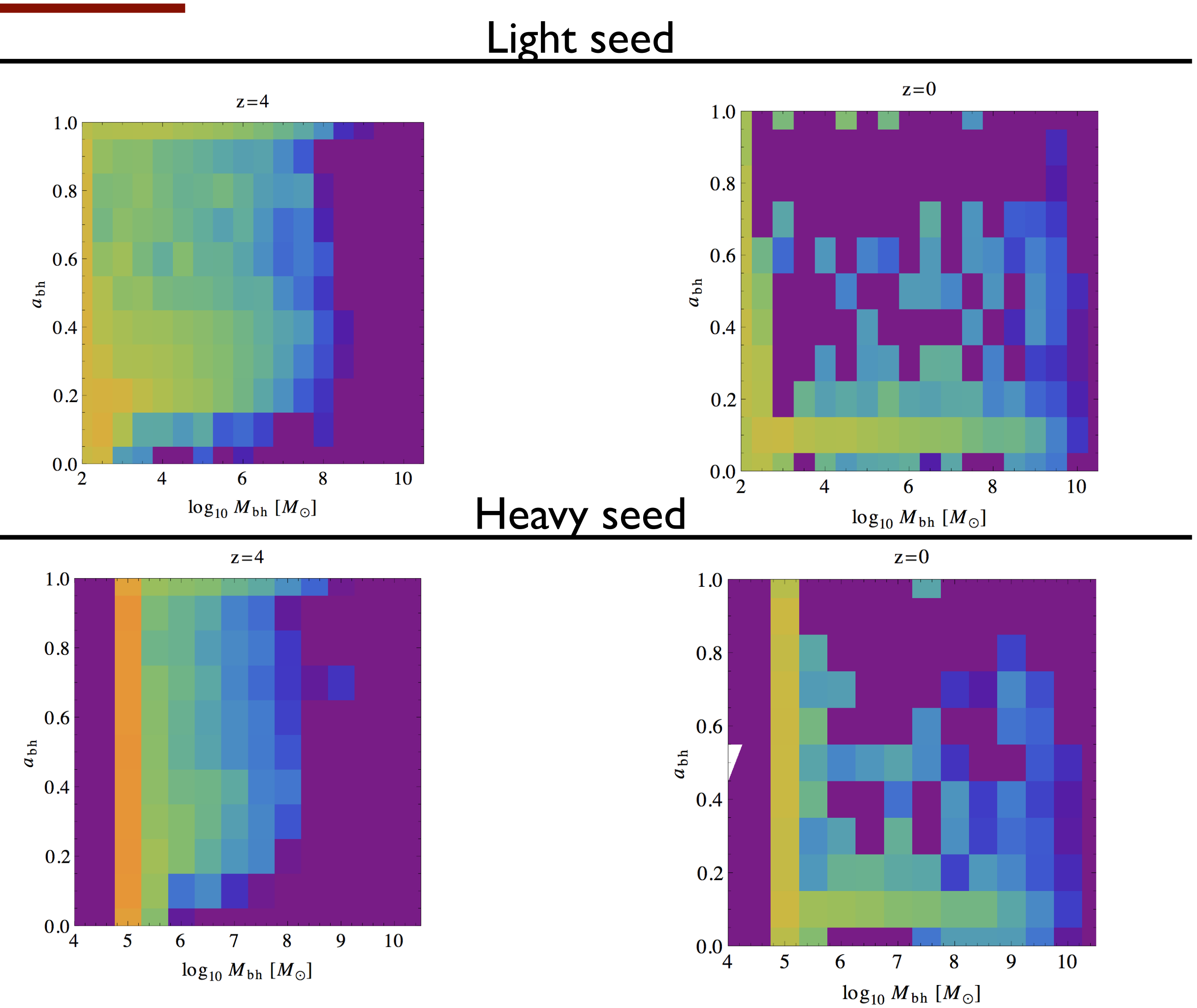
SMBH: spin magnitude

- Wide range of spin magnitude possible

$$\log_{10}\left(\frac{d^2 n_{\rm BH}[{\rm Mpc}^{-3}]}{da\, d\log_{10} M_{\rm BH}[M_\odot]}\right)$$



[Barausse 2012]



SMBH: spin alignment

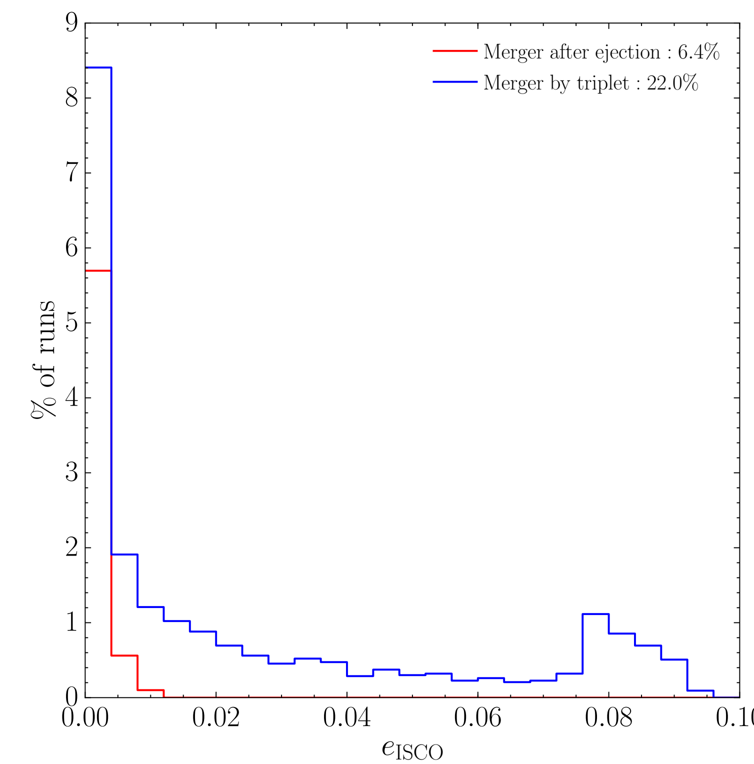
- High-z, gas-rich environment, massive circumbinary discs: tendency to align spins
- Low-z, gas-poor environment, small discs: generic spin orientation

LISA source properties: eccentricity

SMBH: triple systems

- Triple interactions could be common in hierarchical merging of SMBHs (up to 30%)
- Triplets can merge faster and have enhanced eccentricity

[Bonetti&al 2017]



SOBH: field vs cluster formation

- SOBHs seen by LIGO could have measurable eccentricity in the LISA band

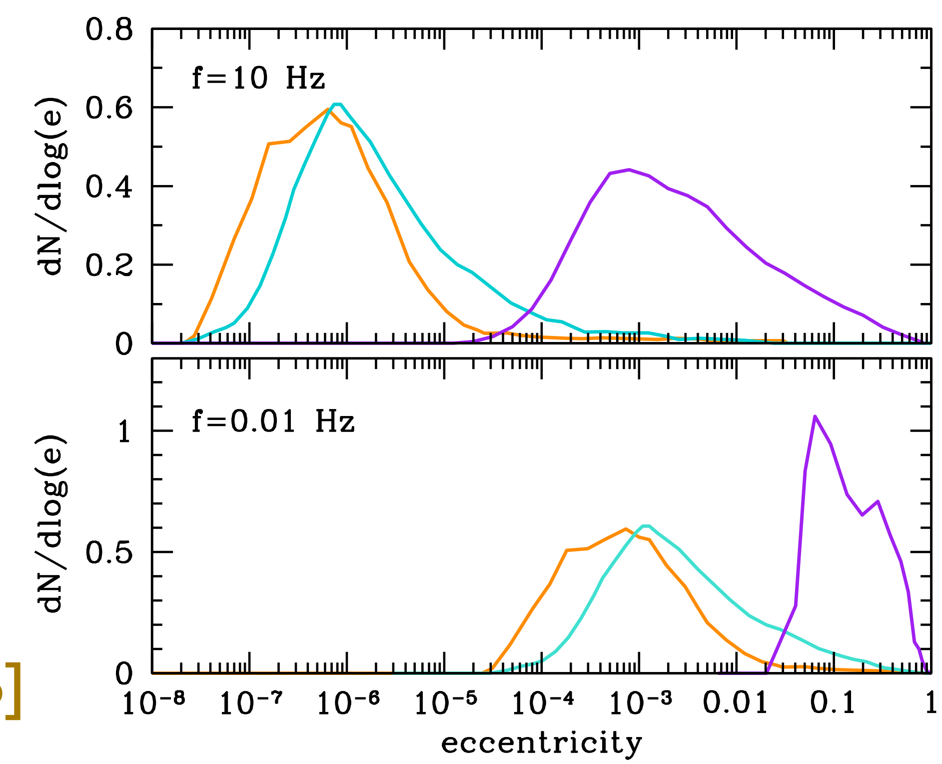


Figure 1. Eccentricity distributions predicted by the *field* (orange), *cluster* (turquoise) and *MBH* (purple) scenarios. The top panel show the distribution at the reference frequency $f_* = 10$Hz, while the bottom panel is the observable distribution $p(e_0)$ evolved "back in time" to $f_0 = 0.01$Hz.

[Nishizawa&al 2016]