Fluidization of collisionless plasma turbulence

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with

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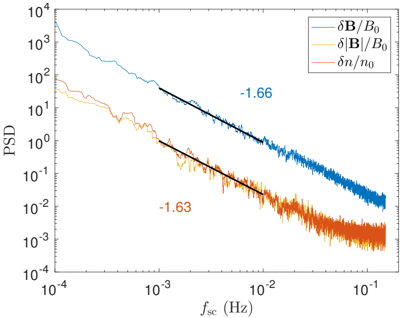
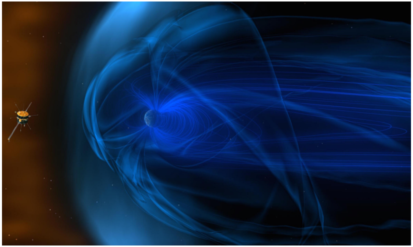
In a weakly collisional and $\beta \sim 1$ plasma like the solar wind, compressive fluctuations are ought to be subject to strong kinetic damping.

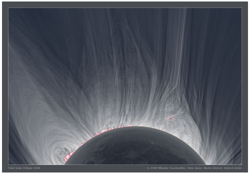


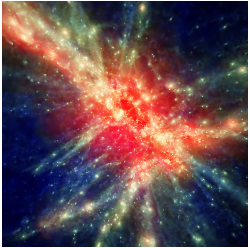


Why does spacecraft measurements of compressive fluctuations in the solar-wind turbulence show healthy Kolmogorov-like power-law spectra?

Collisionless plasma: Basic properties and examples

- Mean free path set by particle wave interactions.
- Dynamic strongly anisotropic with respect to the magnetic field.
  - $\perp$ essentially fluid-like $\Rightarrow$ turbulence
  - $\|$ kinetic $\Rightarrow$ Landau damping.
- Collisionless $\Rightarrow$ Kinetic theory (Vlasov) *a priori*.





Phase space turbulent cascade

In the inertial range, energy injected into perturbations can be thermalised (produce entropy) by:

- **Phase mixing** (Landau damping), producing fine scales in $v_\parallel$.
- **Turbulent mixing**, producing fine scale in real space.

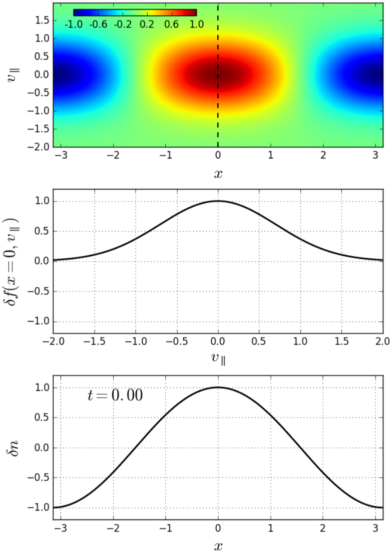
Which thermalisation route does the system favour?

Phase Mixing or Landau damping is related to the formation of infinitesimally fine scales in the distribution function, due to the streaming term $\mathbf{v} \cdot \nabla f$ in the kinetic equation.

$$\frac{\partial \delta f}{\partial t} + v_{\|} \frac{\partial \delta f}{\partial z} = 0$$

$$\delta f(z, v_{\|}, t) = \delta f(z - v_{\|} t, v_{\|}, 0)$$

The rapid oscillation imply that moments of the distribution function with respect to parallel velocity do decay.



Plasma Echo. (Gould et al. 1967)

VOLUME 19, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JULY 1967

PLASMA WAVE ECHO*

R. W. Gould
California Institute of Technology, Pasadena, California

and

T. M. O'Neil and J. H. Malmberg

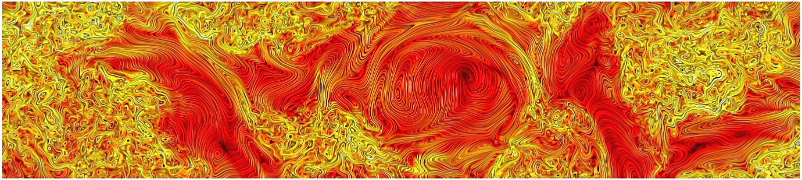
- $t = 0$ first pulse ; $e^{-ik_1x} \Rightarrow f_1(v)e^{(-ik_1x+ik_1vt)}$
- $t = \tau$ second pulse; $e^{ik_2x} \Rightarrow$
  $f_2(v)e^{(ik_2x-ik_2v(t-\tau))} + f_2(v)f_1(v)e^{(i(k_2-k_1)x-ik_2v\tau-i(k_2-k_1)vt)}$
- $t = \tau[k_2/(k_2 - k_1)] \Rightarrow$ Echo!

Turbulent spatial mixing

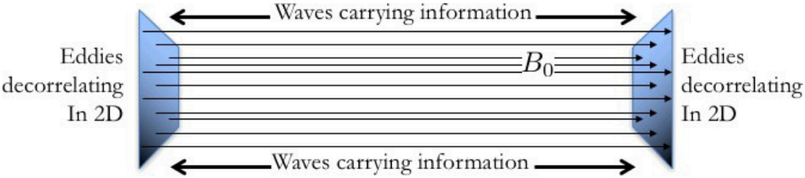
RMHD described fully nonlinear Alfvén waves travelling up and down the (strong) background magnetic field.

$$\partial_t \mathbf{z}_\perp^\pm \mp v_A \partial_z \mathbf{z}_\perp^\pm + \mathbf{z}_\perp^\mp \cdot \nabla_\perp \mathbf{z}_\perp^\pm = -\nabla_\perp p$$

Elsasser vector fields $\mathbf{z}_\perp^\pm = \mathbf{v}_\perp \pm \mathbf{b}_\perp$, $\mathbf{b}_\perp = \mathbf{B}_\perp / \sqrt{4\pi\rho}$, $v_A = B_0/\sqrt{4\pi\rho}$



Critical balance. (Goldreich and Sridhar 1995)



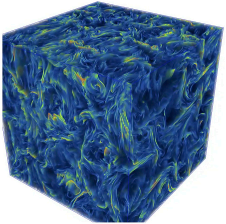
$$\partial_t \mathbf{z}_\perp^\pm \mp v_A \partial_z \mathbf{z}_\perp^\pm + \mathbf{z}_\perp^\mp \cdot \nabla_\perp \mathbf{z}_\perp^\pm = -\nabla_\perp p$$

$\tau_{nl} \sim \tau_A \Rightarrow$ turbulent eddies elongated along the local magnetic field.

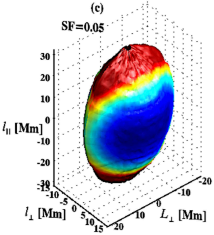
$$\partial_t \mathbf{z}_\perp^\pm \mp v_A \partial_z \mathbf{z}_\perp^\pm + \mathbf{z}_\perp^\mp \cdot \nabla_\perp \mathbf{z}_\perp^\pm = -\nabla_\perp p$$

- nonlinearity reduced by aligning the vector RMHD fields $\Rightarrow \tau_{nl} \sim \frac{\delta z \, sin\theta}{\lambda}$.
- aspect ratio of the fluctuations in the $\perp$ plane $\sim sin\theta$

sheetlike 3D anisotropic structures.

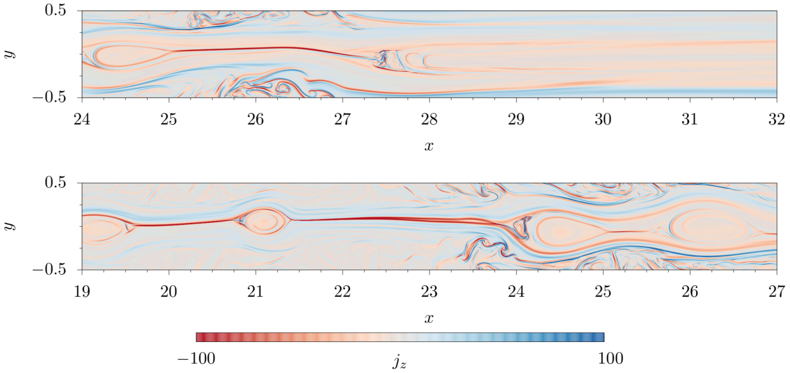


Current field $\nabla \times \mathbf{B}$



Tearing Disrupted Cascade. (Mallet et al. + Loueiro et al. 2017)

The tearing instability compete with the nonlinear evolution.



Walter et al. PRE 2018

Ordering

- Plasma near Maxwellian equilibrium: $f = F_0 + \delta f$
- Strong (uniform) magnetic field: $\omega \ll \Omega_i$, $k_{\parallel} \ll k_{\perp}$
- Long wavelength: $k_{\perp} \ll \rho_i$

KRMHD = hybrid fluid-kinetic description of magnetized weakly collisional plasma.

Fluid part = RMHD

Kinetic part:

$$\frac{dg^{(i)}}{dt} + v_{\parallel}\nabla_{\parallel}g^{(i)} + v_{\parallel}F_0\nabla_{\parallel}\phi^i$$

$$\phi^i = \alpha^i \int dv_{\parallel} g^i(v_{\parallel})$$

Phase space turbulent cascade

Energy injected into perturbations can be thermalised (produce entropy) by:

- **Phase mixing** (Landau damping), producing fine scales in $v_{\|}$ and thus making $C[g]$ finite even if the collisionality is small.

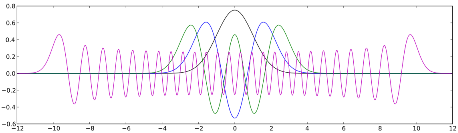
$$C[g] \sim \nu v_{th}^2 \frac{\partial^2 g}{\partial v_{\|}^2}$$

- **Turbulent mixing**, producing fine scale in real space.

$$\frac{\partial g^i}{\partial t} + \underbrace{\mathbf{u}_\perp \cdot \nabla_\perp g^i}_{\text{Turbulent Mixing}} + \underbrace{v_{\|} \nabla_{\|} g^{(i)} + v_{\|} F_0 \nabla_{\|} \phi^i}_{\text{Phase Mixing}} = C[g]$$

Which thermalisation route does the system favour?

Hermite formulation enables a spectral representation of velocity space. It is natural way to separate the 'fluid' part of the problem from the 'kinetic' one.



$$H_m(x) = (-1)^m e^{x^2} \frac{d^m}{dx^m} e^{-x^2} \Rightarrow$$

$$g(v_\parallel) = \sum_{m=0}^{\infty} \frac{H_m(v_\parallel / v_{th}) F_0}{\sqrt{2^m m!}} g_m$$

$$\begin{cases} H_0 = 1 \Rightarrow g_0 = \dfrac{\delta n}{n} \\ H_1 = 2x \Rightarrow g_1 = \sqrt{2}\dfrac{u_\parallel}{v_{th}} \\ H_2 = 4\left(x^2 - \dfrac{1}{2}\right) \Rightarrow g_2 = \dfrac{1}{\sqrt{2}} \dfrac{\delta T_\parallel}{T} \end{cases}$$

$m = 0, 1, 2$ corespond to 'fluid' moments.

Hermite Space Formulation

- makes the numerical scheme spectrally accurate in the $v_\parallel$ coordinate.
- provides an elegant analytical framework to study phase mixing.
- the integro-differential kinetic equations becomes a fluid-like hierarchy of equations.

$$\frac{dg_0^i}{dt} + v_{th}\nabla_\parallel \frac{g_1^i}{\sqrt{2}} = 0,$$

$$\frac{dg_1^i}{dt} + v_{th}\nabla_\parallel \left(g_2^i + \frac{(1 - 1/\Lambda^i)}{\sqrt{2}} g_0^i \right) = 0,$$

$$\frac{dg_m^i}{dt} + v_{th}\nabla_\parallel \left(\sqrt{\frac{m+1}{2}} g_{m+1}^i + \sqrt{\frac{m}{2}} g_{m-1}^i \right) = C[g_m^i], \quad m \geq 2.$$

Dynamics in Phase space

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla\right) g_m^i + v_{th}\nabla_\parallel \left(\sqrt{\frac{m+1}{2}} g_m^i + \sqrt{\frac{m}{2}} g_{m-1}^i\right) = -\eta m g_m^i$$

all moments are advected by the same velocity

higher moments couple to lower ones $\Rightarrow$cascade in hermite space

at large enough m, free energy is removed by collisions

- **Landau damping/phase mixing** is the transfer of free energy from low moments $(\delta n, u_\parallel, \delta T_\parallel)$ into higer one $(g_m^i \geq 3)$.
- **Turbulence** is the mixing of $(\delta n, u_\parallel, \delta T_\parallel)$ by $\mathbf{u}_\perp$ and $\mathbf{b}_\perp$ transferring their energy to small scales.

Hermite "Cascade"

How transfer of free energy to high m's occurs linearly?

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla_\perp\right) g_m + v_{th}\nabla_\parallel \left(\sqrt{\frac{m+1}{2}}g_m + \sqrt{\frac{m}{2}}g_{m-1}\right) = -\nu m g_m$$

Let Fourier transform in the z direction ($\frac{\partial}{\partial z} \to ik_\parallel$) and introduce:

$$\tilde{g}_m(k_\parallel) \equiv \left(i \mathrm{sgn} k_\parallel\right)^m g_m(k_\parallel)$$

$$\frac{\partial \tilde{g}_m}{\partial t} + \frac{|k_\parallel| v_{th}}{\sqrt{2}} \left(\sqrt{m+1}\tilde{g}_{m+1} - \sqrt{m}\tilde{g}_{m-1}\right) = -\nu m \tilde{g}_m$$

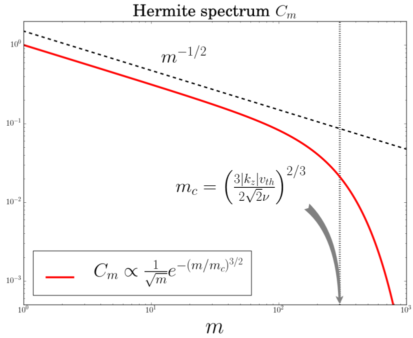
$$\frac{\partial \tilde{g}_m}{\partial t} + \frac{|k_\parallel| v_{th}}{\sqrt{2}} m^{1/4} \frac{\partial}{\partial m} m^{1/4} \tilde{g}_m = -\nu m \tilde{g}_m$$

How transfer of free energy to high m's occurs linearly?

$$C_m = \frac{1}{2}\left\langle|\tilde{g}_m|^2\right\rangle \text{ satisfies:}$$

$$\cancel{\frac{\partial C_m}{\partial t}} + |k_{\|}| v_{th} \frac{\partial}{\partial m}\sqrt{2m}\, C_m = -\nu m C_m$$

[Zocco & AAS PoP **18**, 102309 (2011)]



Linearly all the energy that is injected will dissipate at the rate $\sim |k_{\|}| v_{th}$

$$\boxed{\tilde{g}_m = \tilde{g}_m^+ + (-1)^m \tilde{g}_m^-}$$

$$\frac{\partial \tilde{g}_m^\pm}{\partial t} \pm \sqrt{2}|k_\parallel| v_{th} m^{1/4} \frac{\partial}{\partial m} m^{1/4} \tilde{g}_m^\pm = -\nu m \tilde{g}_m^\pm$$

Phase Mixing

$\tilde{g}_m^+ = \dfrac{\tilde{g}_m + \tilde{g}_{m+1}}{2}$, propagate from low to high m.

Un-Phase-Mixing

$\tilde{g}_m^- = (-1)^m \dfrac{\tilde{g}_m - \tilde{g}_{m+1}}{2}$, propagate from high to low m.

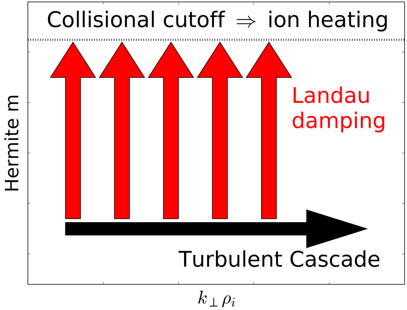
In energy terms: $C_m = C_m^+ + C_m^-$ satisfies:

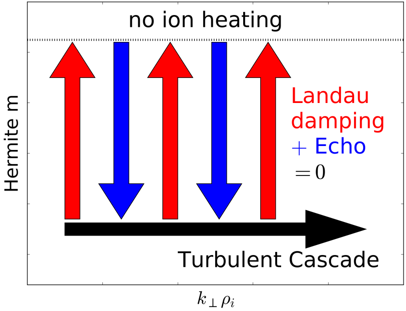
$$\frac{\partial C_m}{\partial t} + \frac{\partial}{\partial m} |k_\parallel| v_{th} \sqrt{2m} \left(C_m^+ - C_m^- \right) = -2\nu m C_m$$

Hermite flux to high m can be cancelled by the '−' modes

$$\left(\frac{\partial \tilde{g}_m^{s\pm}}{\partial t}\right)_{nl} = - \sum_{p_\parallel + q_\parallel = k_\parallel} \mathbf{u}_\perp(p_\parallel) \cdot \nabla_\perp \left[\delta^{+}_{k_\parallel, q_\parallel} \tilde{g}_m^{s\pm}(q_\parallel) + \delta^{-}_{k_\parallel, q_\parallel} \tilde{g}_m^{s\mp}(q_\parallel)\right],$$

Hermite flux to high m can be cancelled by the '-'modes





Numerical Code

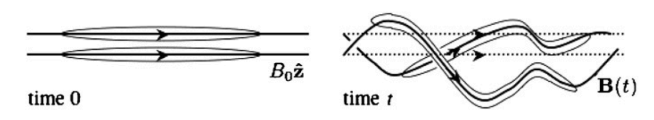
```
8888888b.                                         888    888
888   Y88b                                        888    888
888    888                                        888    888
888   d88P 888d888 .d88b.  88888b.d88b.   .d88b.  888888 88888b.   .d88b.  888  888 .d8888b
8888888P"  888P"  d88""88b 888 "888 "88b d8P  Y8b 888    888 "88b d8P  Y8b 888  888 88K
888        888    888  888 888  888  888 88888888 888    888  888 88888888 888  888 "Y8888b.
888        888    Y88..88P 888  888  888 Y8b.     Y88b.  888  888 Y8b.     Y88b 888      X88
888        888     "Y88P"  888  888  888  "Y8888   "Y888 888  888  "Y8888   "Y88888  88888P'
```

A Fourier-Hermite pseudo-spectral code for strongly magnetised fluid-kinetic plasma dynamics

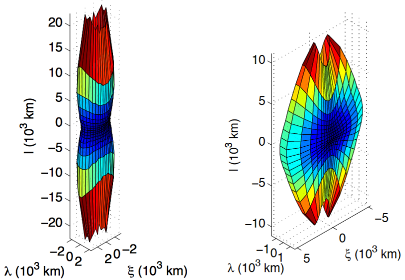
- Extension of an incompressible MHD Open Source solver (http://aqua.ulb.ac.be/turbo)
- FFTW
- Runge-Kutta RK4
- Parallelized in one direction using MPI
- Shift dealiasing

Parallel cascade of compressible fluctuations

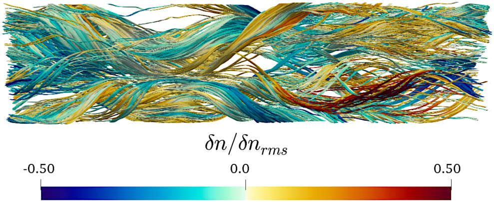
If $k_{||}$ remains small, compressive fluctuations are not strongly damped.

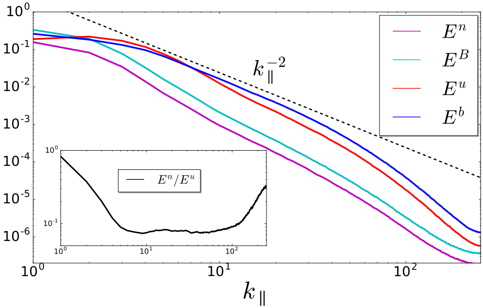


Schekochihin et al. APJS 2009

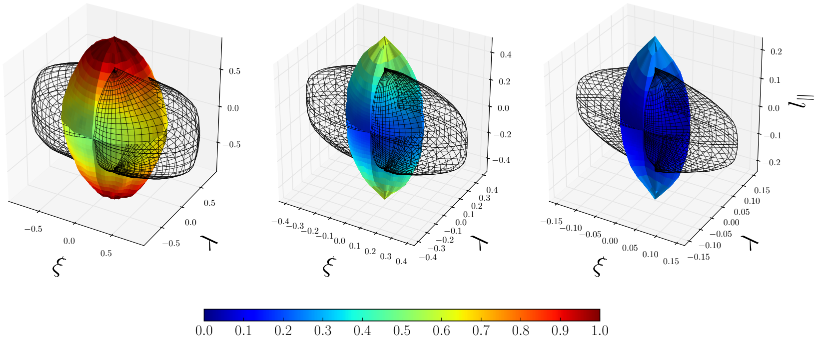


Parallel cascade

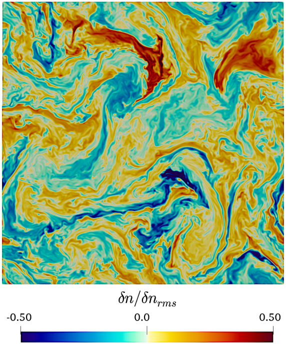


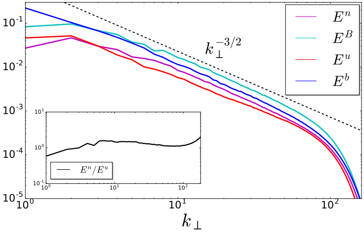


Parallel cascade



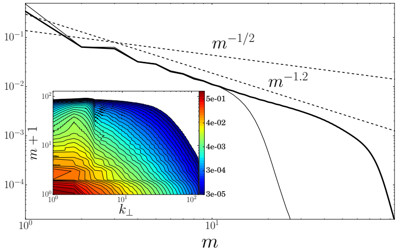
Perpendicular cascade

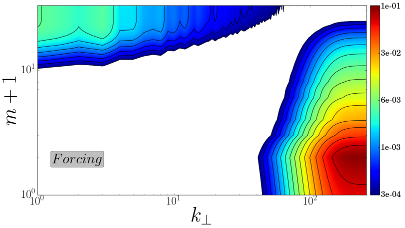




Hermite-Fourier spectra

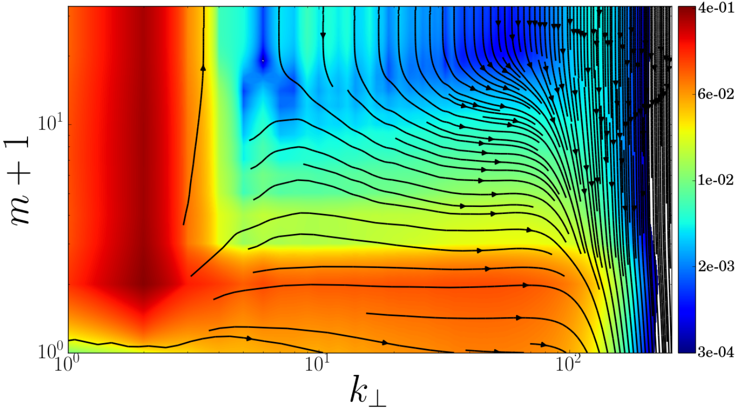
The steepening of the Hermite spectrum gives rise to free-energy dissipation which vanish as $\nu_{ei} \to 0$.





Hermite-Fourier flux

$$\frac{\partial |g_m(k_\perp)|^2}{\partial t} = -\frac{\partial \Pi(k_\perp, m)}{\partial k_\perp} - \frac{\partial \Gamma(k_\perp, m)}{\partial m}$$



Conclusions

- Nonlinear cascade scatters energy in the phase space so as to generate a stochastic version of the plasma echo.
- Stochastic echo impede Landau damping by reducing the net flux to small velocity space scales.
- **Collisionless plasma turbulence in the solar wind behave in a more "fluid-like" fashion than expected**.