Curvature Radiation from a quantum-electrodynamics point of view

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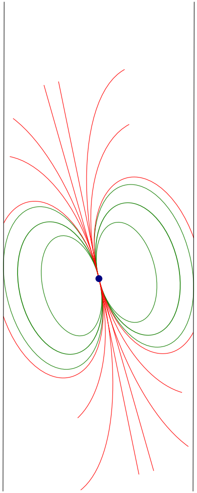
Part I

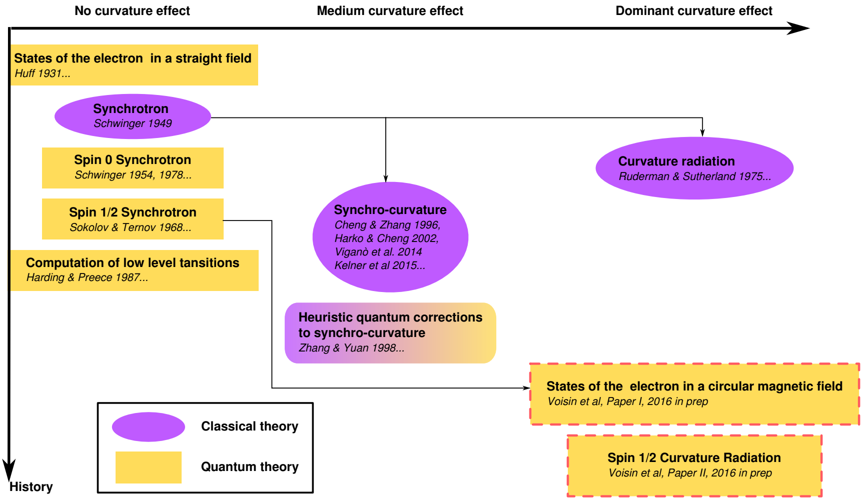
High energy radiation mechanisms in pulsar magnetospheres

A pair production factory



Radiation processes , a QED zoo :

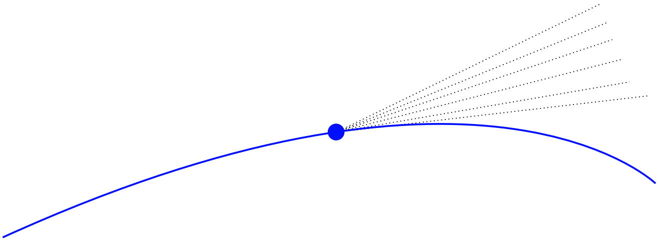
- Synchrotron
- Curvature radiation
- Quantum transitions (LMXBs, Maser?...)
- Compton and Inverse Compton scatterings
- $\gamma + \gamma \to e_+ + e_-$
- $\gamma + \vec{B} \to e_+ + e_-$
- ...


No curvature effect
Medium curvature effect
Dominant curvature effect
States of the electron in a straight field
Huff 1931...
Synchrotron
Schwinger 1949
Spin 0 Synchrotron
Schwinger 1954, 1978...
Spin 1/2 Synchrotron
Sokolov & Ternov 1968...
Computation of low level tansitions
Harding & Preece 1987...
Curvature radiation
Ruderman & Sutherland 1975...
Synchro-curvature
Cheng & Zhang 1996,
Harko & Cheng 2002,
Viganò et al. 2014
Kelner et al 2015...
Heuristic quantum corrections
to synchro-curvature
Zhang & Yuan 1998...
States of the electron in a circular magnetic field
Voisin et al, Paper I, 2016 in prep
Spin 1/2 Curvature Radiation
Voisin et al, Paper II, 2016 in prep
Classical theory
Quantum theory
History

Curvature radiation: classical picture

Proposed by Ruderman and Sutherland (1975).

- Classical curvature peak : $E_{\rm max} = \gamma^3 \hbar c/\rho \sim 20\gamma_7^3$ GeV
- Total power radiated : $W = \frac{e^2 c}{6\pi\epsilon_0\rho^2}\gamma^4 \sim 2\cdot 10^8 \gamma_7^4$ GeV/s
- Emission time scale $\tau_e \sim E_{\rm max}/W = 10^{-7}$s
- **But unphysical trajectory : $\vec{\mathbf{v}} \| \vec{\mathbf{B}} \Rightarrow \vec{\mathbf{v}} \wedge \vec{\mathbf{B}} = 0$!**



Parameters : $\gamma \sim 10^7$, $B \sim 10^8$ T, $\rho \sim 10^4$m

Synchro-curvature radiation

This regime has been studied by several authors (Cheng and Zhang (1996), Harko and Cheng (2002),Kelner et al. (2015) , Viganò et al. (2014)). The last gives :

The maximum Lorentz factor :

$$\gamma_{\max} = \left(\frac{3}{2} \frac{E_{\parallel} \rho}{e} \right)^{1/4} \quad (1)$$

The pitch angle :

$$\sin \alpha = \sin \alpha_0 \exp \left(-\frac{t}{\tau_\alpha} \right) \quad (2)$$

With :

$$\tau_\alpha = \frac{\gamma_{\max} m_e c}{e E_{\parallel}} \quad (3)$$

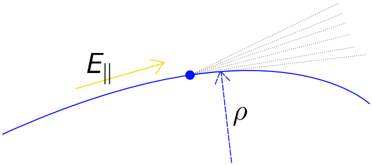


Figure : A particle pushed by an electric field $E_{\parallel}$, along a magnetic field line of curvature radius ρ.

Synchro-curvature radiation

This regime has been studied by several authors (Cheng and Zhang (1996), Harko and Cheng (2002),Kelner et al. (2015) , Viganò et al. (2014)). The last gives :

The maximum Lorentz factor :

$$\gamma_{\max} \simeq 2 \cdot 10^7 E_{\parallel 12}^{1/4} \rho_4^{1/4} \qquad (4)$$

The pitch angle :

$$\sin\alpha = \sin\alpha_0 \exp\left(-\frac{t}{\tau_\alpha}\right) \qquad (5)$$

With :

$$\tau_\alpha = 2 \cdot 10^{-8} \gamma_{\max 7} E_{\parallel 12}^{-1} \text{ s} \qquad (6)$$

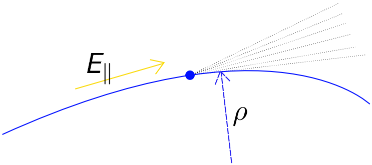


Figure : A particle pushed by an electric field $E_\parallel$, along a magnetic field line of curvature radius ρ.

But... Energy levels become quantized

Energy states of an electron in a straight magnetic field :

$$E = \sqrt{m^2c^4 + \underbrace{\hbar\omega_c mc^2 n}_{\text{Perpendicular momentum}} + \underbrace{(cp_{\|})^2}_{\text{Parallel momentum}}} \qquad (7)$$

"Pitch angle" corresponding to the first Landau level :

$$\sin\alpha_1 \sim 2\cdot 10^{-9} B_8 \gamma_7^{-1} \qquad (8)$$

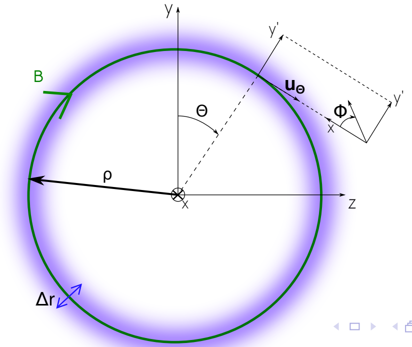
The first Landau state is reached in 10 meters !

Part II

State of an electron in a circular magnetic field (Voisin et al., Paper I, 2016, in prep.)

Symmetries of an electron in a circular magnetic field

- Circular static magnetic field.
- Locally homogeneous on a length scale :
  $\lambda = \left(\frac{2\hbar}{eB}\right)^{1/2} \simeq 10^{-12} B_8^{-1/2}\mathrm{m}$
- Invariance by rotation around the axis of the circle and the magnetic field.



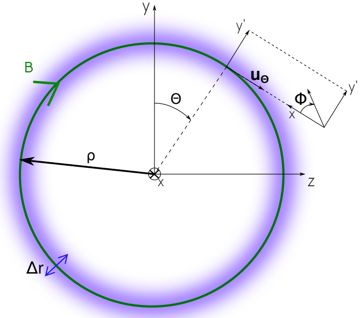
Equation and Approximation

Find proper states of the following set of operators :

$$\hat{J}_x \text{ and } \hat{J}_\theta$$
$$\hat{H}/c = \alpha^r(-i\hbar\partial_r) + \alpha^\theta(-i\hbar\partial_\theta) + \alpha^\phi(-i\hbar\partial_\phi + eA_\phi(r)) + \beta mc \tag{9}$$

We seek solutions at zeroth order in ϵ :

$$\epsilon = \frac{\lambda}{\rho} \simeq 10^{-16} B_8^{-1/2} \rho_4^{-1} \tag{10}$$



States of an electron in a circular magnetic field :

$$\Psi_{n,l_\perp,l_\parallel,a}(r,\theta,\phi) \propto e^{-\frac{r^2}{2\lambda^2}} e^{i\theta l_\parallel} e^{i\phi l_\perp} \left[e^{i\phi} L_{n-l_\perp}^{l_\perp}\left(\frac{r^2}{\lambda^2}\right) \chi_\uparrow^a(\theta) + L_{n-l_\perp}^{l_\perp+1}\left(\frac{r^2}{\lambda^2}\right) \chi_\downarrow^a(\theta) \right] \tag{11}$$

$$E = \sqrt{m^2c^4 + \hbar\omega_c mc^2 n + (\hbar\Omega)^2 l_\parallel^2} \tag{12}$$

With the pulsation around the circle : $\Omega = c/\rho$

- n : main perpendicular quantum number.
- $l_\perp + 1/2$: angular momentum around the magnetic field ($\hat{J}_\theta$).
- $l_\parallel$: angular momentum around the axis of the circle ($\hat{J}_x$).
- a : spin orientation

Part III

Radiation of an ultra-relativistic electron in a circular magnetic field (Voisin et al., Paper II, 2016, in prep.)

Defining curvature radiation from a quantum point of view

Proposition of definition

Curvature radiation results of transitions between the most localized quantum states that allow all spin orientations.

States concerned

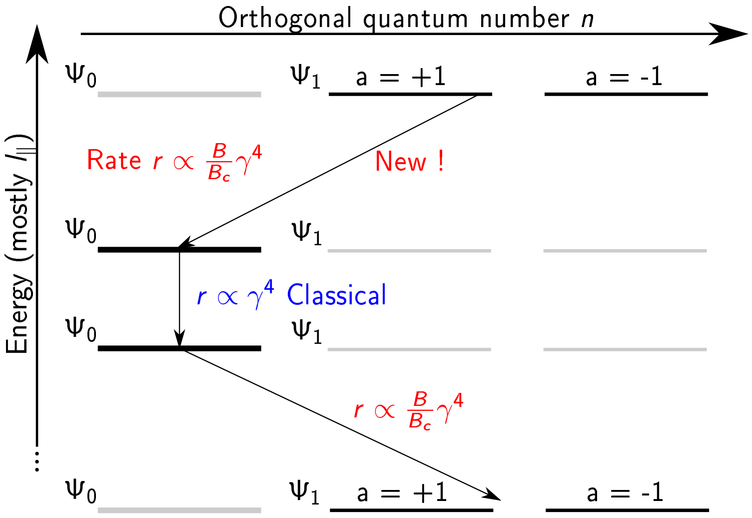
- Fundamental state : $l_{||}, n = 0, l_{\perp} = 0, a = 1$, spin anti-aligned.

$$\Psi_0 = \Psi_0^{\downarrow} \tag{13}$$

- First excited state : $l_{||}, n = 1, l_{\perp} = 0, a = \pm 1$, mixed spins.

$$\cos(\xi)\Psi_{1,a=1} + \sin(\xi)\Psi_{1,a=-1} = \underbrace{\Psi_1^{\uparrow}}_{l_{\perp}+1}(\xi) + \underbrace{\Psi_1^{\downarrow}}_{l_{\perp}}(\xi) \tag{14}$$

Allowed transitions and main rates

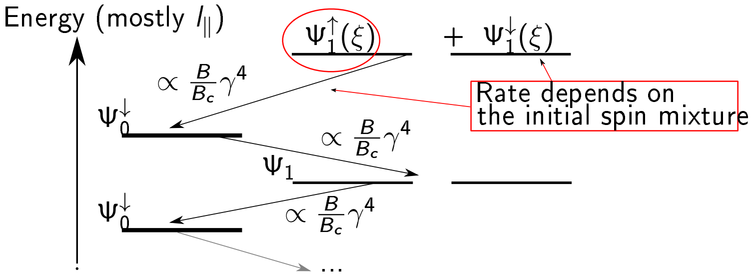


Spin loves flip-flops if $B \geq B_c = 4.4 \cdot 10^9\mathrm{T}$

The transition rates read :

$$\mathrm{d}w_{fi} = \|M_{fi}\|^2 2\pi\hbar\delta\left(E_f + \hbar\omega - E_i\right)\frac{\omega^2 \mathrm{d}o\mathrm{d}\omega}{c^3(2\pi)^3/V} \tag{15}$$

$$\|M_{fi}\|^2 \propto \left\|\int \overline{\Psi}_f\gamma^\mu e_\mu \Psi_i e^{-\imath\vec{k}\cdot\vec{x}}\mathrm{d}^3x\right\|^2 \tag{16}$$



Spectrum expressions

Classical curvature radiation is identical to its classical version up to high energy corrections. **Spin-flip expressions are very similar.**

$$
\begin{aligned}
\frac{\mathrm{d}^2 I^{\parallel}_{\downarrow\downarrow}}{\mathrm{d}o\,\mathrm{d}\omega} &= \frac{1}{2\pi\Omega}\frac{e^2\omega^2}{12\pi^3\epsilon_0 c}\left(\kappa^2+\frac{1}{\tilde{\gamma}^2}\right)^2 K^2_{2/3}(\xi)\\
\frac{\mathrm{d}^2 I^{\perp}_{\downarrow\downarrow}}{\mathrm{d}o\,\mathrm{d}\omega} &= \frac{1}{2\pi\Omega}\frac{e^2\omega^2}{12\pi^3\epsilon_0 c}\kappa^2\left(\kappa^2+\frac{1}{\tilde{\gamma}^2}\right) K^2_{1/3}(\xi)\\
\frac{\mathrm{d}^2 I^{\parallel}_{\uparrow\downarrow}}{\mathrm{d}o\,\mathrm{d}\omega} &= \frac{1}{2\pi\Omega}\frac{e^2\omega^2}{12\pi^3\epsilon_0 c}\left(\kappa^2+\frac{1}{\tilde{\gamma}^2}\right)\frac{1}{2\tilde{\gamma}^2}\frac{B}{B_c} K^2_{1/3}(\xi)\\
\frac{\mathrm{d}^2 I^{\perp}_{\uparrow\downarrow}}{\mathrm{d}o\,\mathrm{d}\omega} &= \frac{1}{2\pi\Omega}\frac{e^2\omega^2}{12\pi^3\epsilon_0 c}\left(\kappa^2+\frac{1}{\tilde{\gamma}^2}\right)\frac{1}{2\tilde{\gamma}^2}\frac{B}{B_c} K^2_{1/3}(\xi)
\end{aligned}
$$

Polarizations and symbols above are within the conventions of Jackson (1998)

Total radiated power

Spin-flip curvature radiation is mostly unpolarized :

$$I^{\parallel}_{\downarrow\downarrow} = \frac{1}{\epsilon_0 c}\left(\frac{\Omega e}{16\pi}\right)^2 \frac{28}{3}\tilde{\gamma}^4 \quad (17)$$

$$I^{\perp}_{\downarrow\downarrow} = \frac{1}{\epsilon_0 c}\left(\frac{\Omega e}{16\pi}\right)^2 \frac{4}{3}\tilde{\gamma}^4 \quad (18)$$

$$I^{\parallel}_{\uparrow\downarrow} = \frac{1}{\epsilon_0 c}\left(\frac{\Omega e}{16\pi}\right)^2 \frac{16}{3}\frac{B}{B_c}\tilde{\gamma}^4 \quad (19)$$

$$I^{\perp}_{\uparrow\downarrow} = \frac{1}{\epsilon_0 c}\left(\frac{\Omega e}{16\pi}\right)^2 \frac{16}{3}\frac{B}{B_c}\tilde{\gamma}^4 \quad (20)$$

Polarizations and symbols above are within the conventions of Jackson (1998)

Corrections at high energies

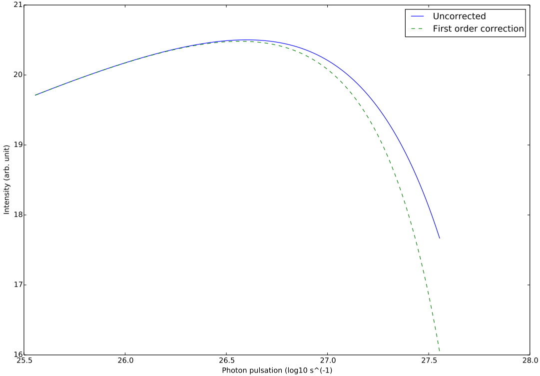


Figure : Effect of quantum corrections at high energies obtained by the replacement $\hbar\omega \rightarrow \hbar\omega\left(1 + \frac{\hbar\omega}{E}\right)$. Here Lorentz factor $= 2 \cdot 10^7$

Conclusion

- We prove that **curvature radiation can be derived in a fully consistent way** using quantum electrodynamics in a high-magnetic-field, ultra-relativistic regime.
- We show that **spin-flip transitions amount to 10% in "normal"** 10^8 **Teslas pulsars and dominate for magnetic fields** $\gtrsim B_c = 4.4 \cdot 10^9$ **Teslas.** : **spin-flip curvature radiation should not be neglected in young pulsars (Crab-like), and becomes dominant in magnetars.**
- For both we worked out **high energy quantum corrections** .
- **The spectra are very similar in shape** to the usual curvature/ synchrotron but differ in intensity (for spin-flip).
- Contrary to constant-spin (classical) radiation, **spin-flip curvature radiation is mostly unpolarized.**
- In the longer term, **"quantum synchro-curvature radiation"** should also be investigated.

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