Figures of lecture 3

The Kerr black hole

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https://relativite.obspm.fr/blackholes/paris23/

PSL graduate programs in Physics and in Astrophysics
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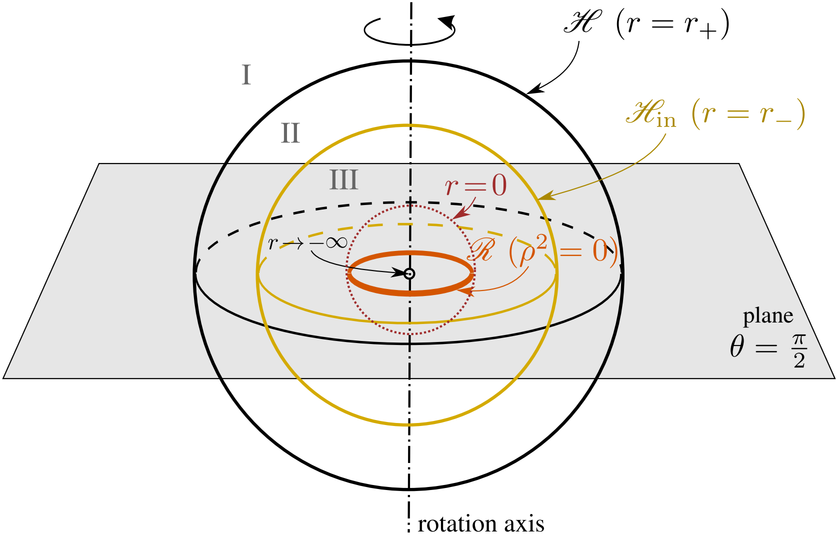
Home page for the lectures

https://relativite.obspm.fr/blackholes/paris23

includes

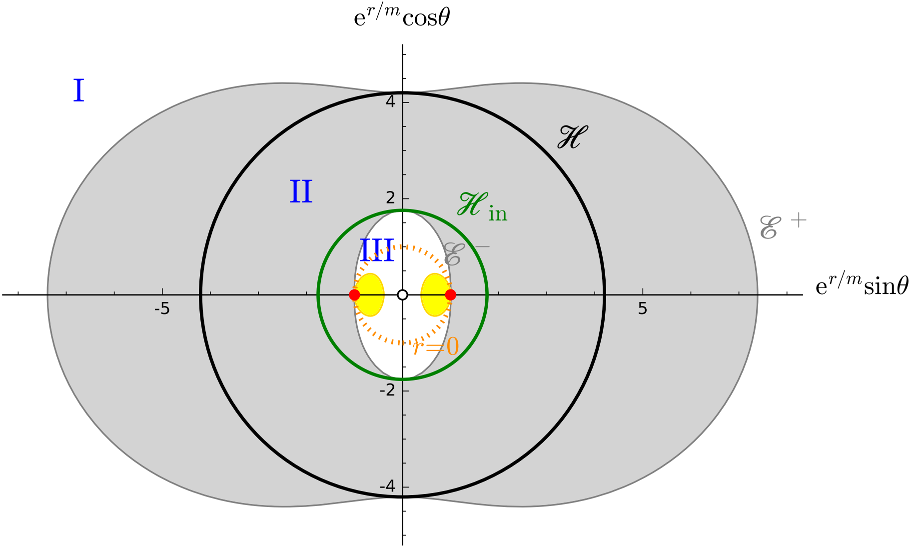
- the lecture notes (draft)
- some SageMath notebooks
- these slides

Section of constant Boyer-Lindquist time coordinate t

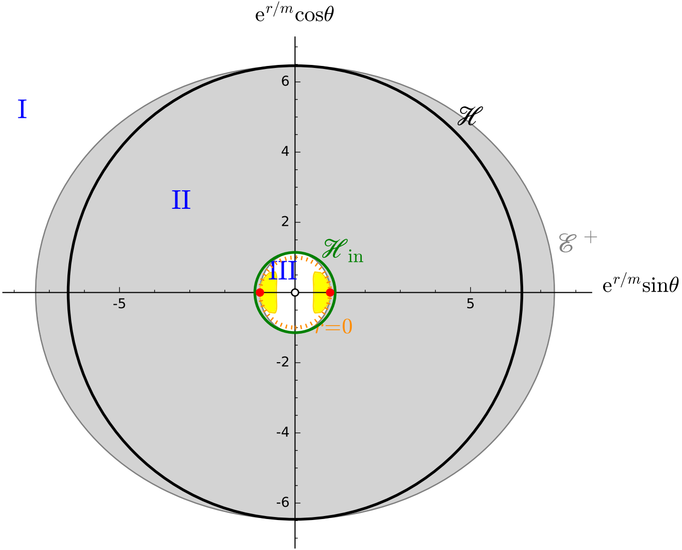


View of a section $t = \mathrm{const}$ in O'Neill coord. (R, θ, φ) with $R := \mathrm{e}^r$

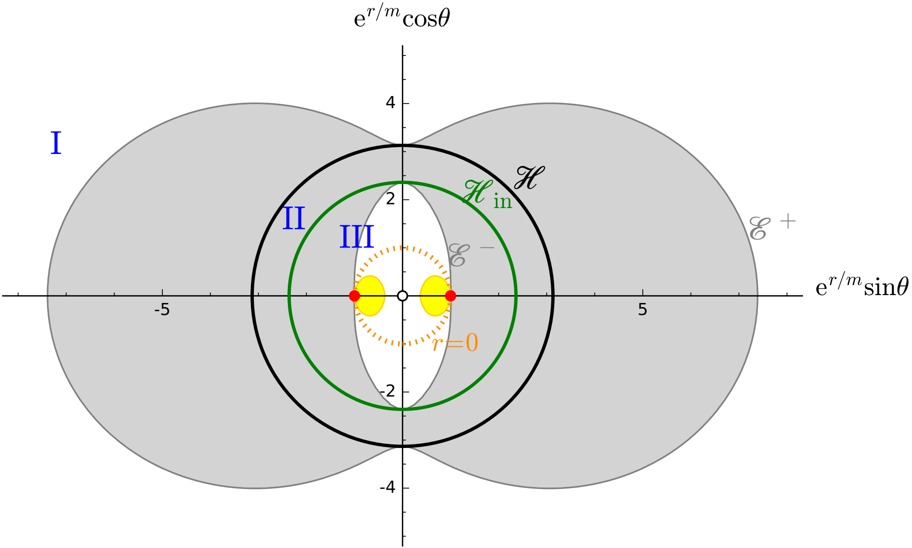
Meridional view of a $t = \text{const}$ slice $(a/m = 0.9)$



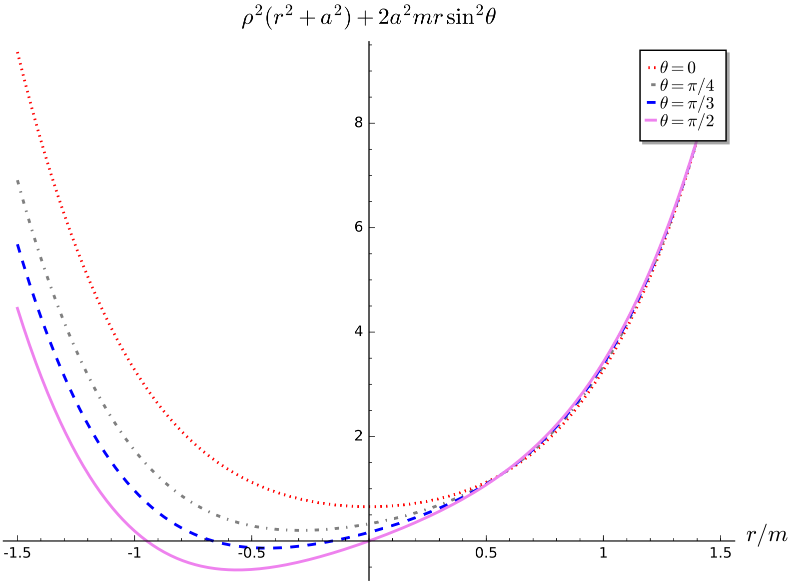
Meridional view of a $t = \mathrm{const}$ slice $(a/m = 0.5)$

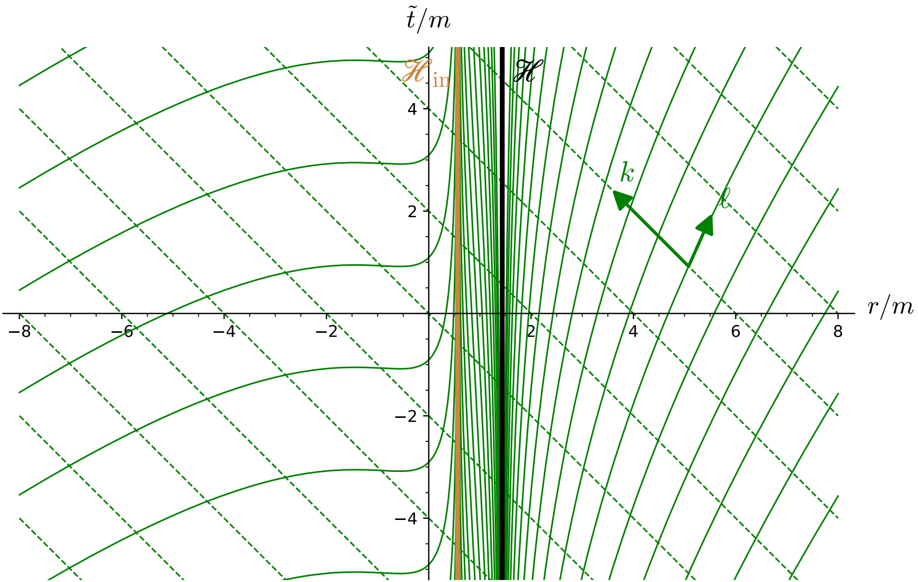


Meridional view of a $t = \text{const}$ slice ($a/m = 0.99$)

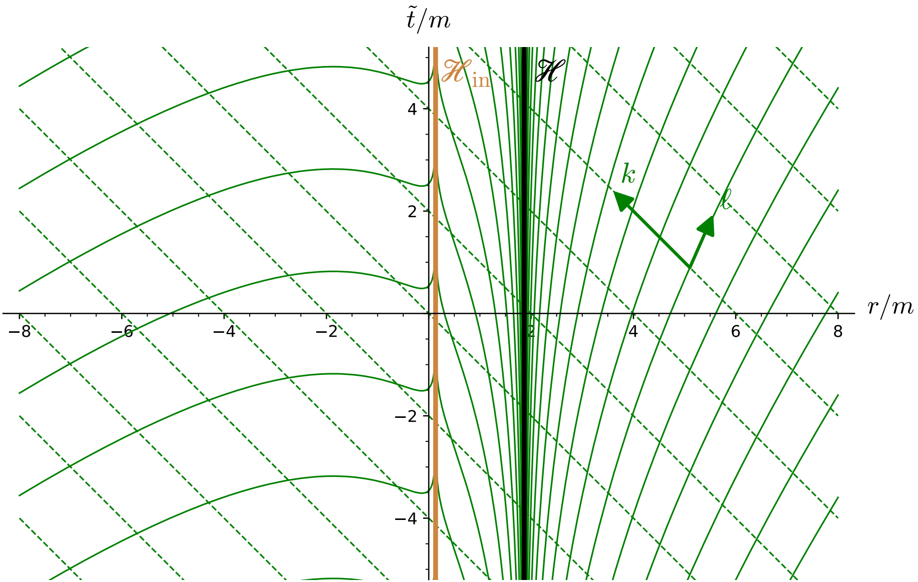


Sign of $g_{\varphi\varphi}$ for $a = 0.9\, m$

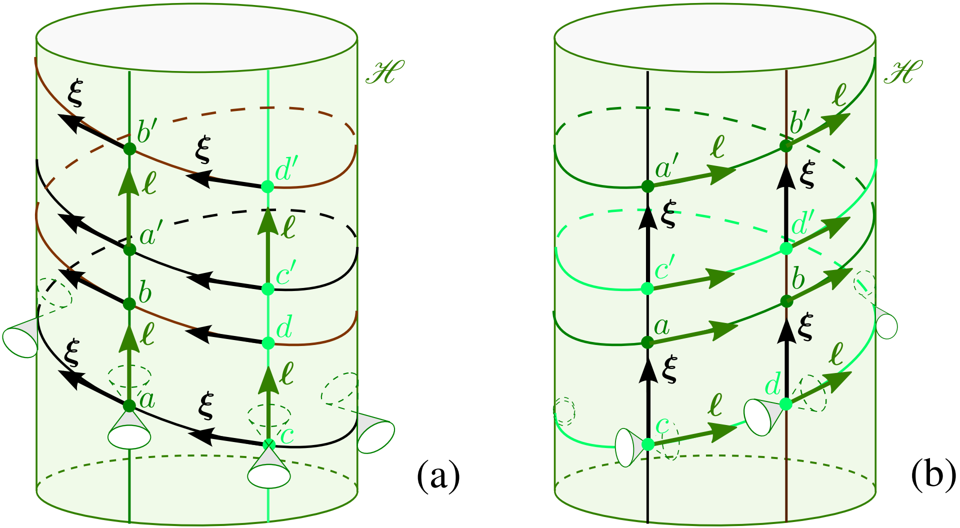



$\tilde{t}/m$
$\mathscr{H}_{\rm in}$
$\mathscr{H}$
k
ℓ
r/m
4
2
−2
−4
−8
−6
−4
−2
2
4
6
8

Principal null geodesics ($a/m = 0.5$)

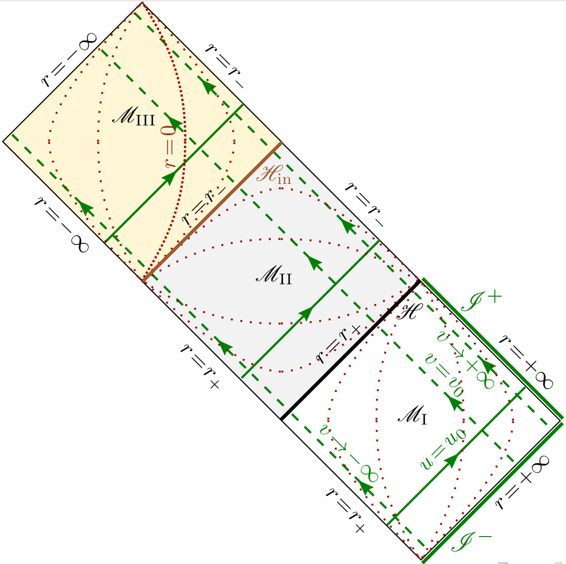


The event horizon and its null generators

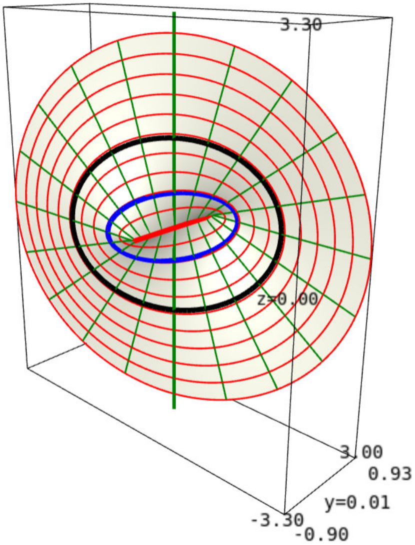


Conformal diagram of Kerr spacetime

with $\mathscr{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \mathscr{R}$



Meridional surface $\tilde{t} = \text{const}$ and $\tilde{\varphi} = 0$ or π

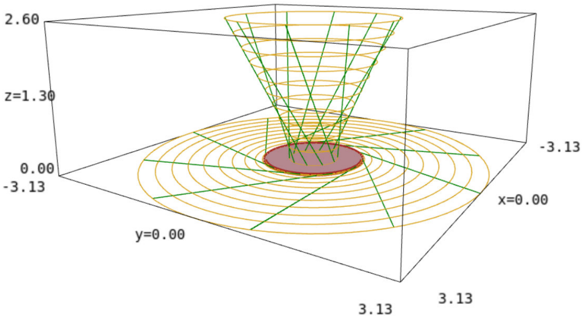


Part $r \geq 0$ viewed in Kerr-Schild coordinates (x, y, z)

- green lines: ingoing principal null geodesics
- black curve: event horizon $\mathscr{H}$
- blue curve: inner horizon $\mathscr{H}_{\rm in}$
- red segment: disk $r = 0$

For an interactive 3D view, see the SageMath notebook
https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Kerr_Schild.ipynb

Surfaces $\tilde{t} = \text{const}$ and $\theta = \text{const}$



Part $r \geq 0$ of the surfaces $(\tilde{t}, \theta) = \text{const}$ drawn in terms of the Kerr-Schild coordinates (x, y, z) for 2 values of θ: $\theta = \pi/2$ $(z = 0)$ and $\theta = \pi/6$

- green lines: ingoing principal null geodesics
- pink disk: $r = 0$
- red circle: curvature singularity

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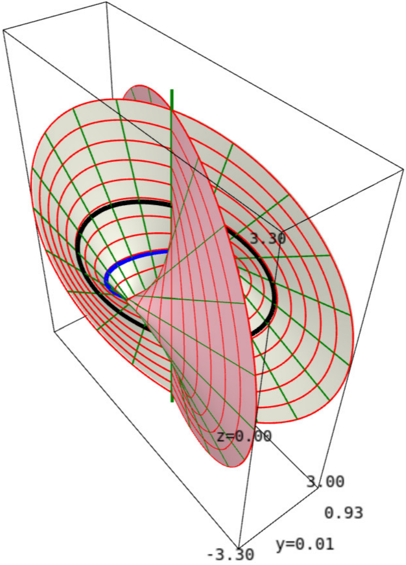
Meridional surface $\tilde{t} = \text{const}$ and $\tilde{\varphi} = 0$ or π

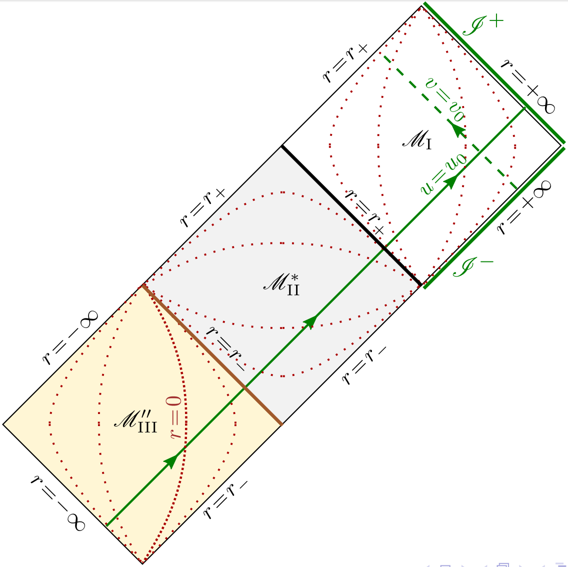


- Part $r \geq 0$ (grey) viewed in Kerr-Schild coordinates (x, y, z)
- Part $r \leq 0$ (pink) viewed in Kerr-Schild coordinates (x', y', z')

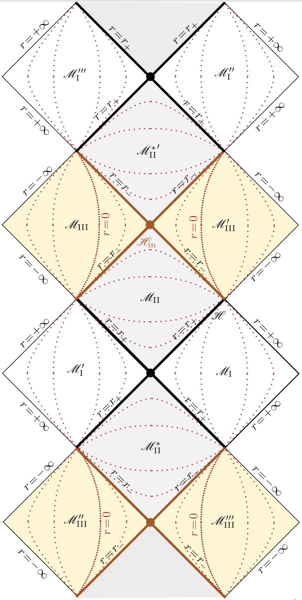
For an interactive 3D view, see the SageMath notebook
https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Kerr_Schild.ipynb

Minimal extension of $\mathscr{M}_{\rm I}$

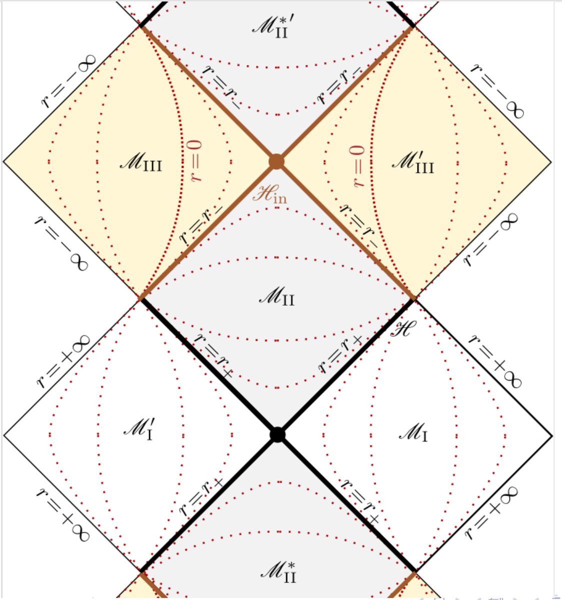
to ensure complete outgoing principal null geodesics



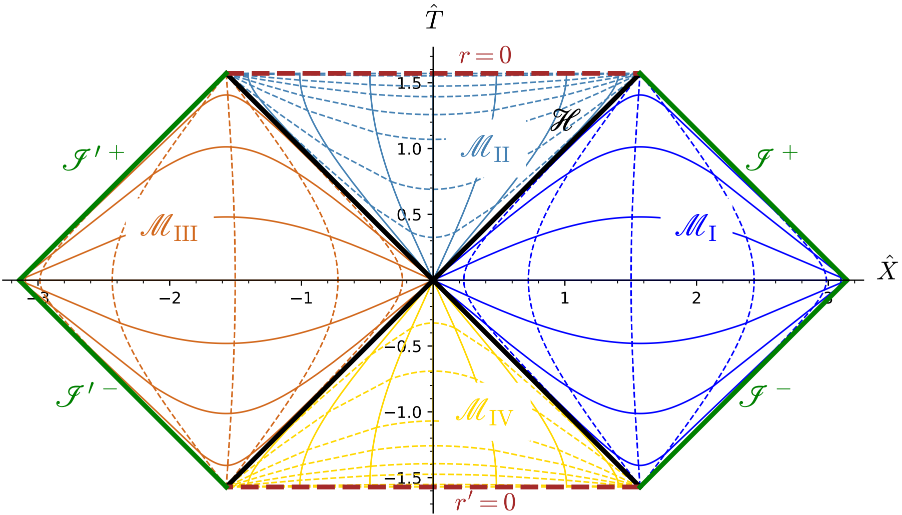
Carter-Penrose diagram of the maximal analytic extension



Carter-Penrose diagram of the maximal analytic extension



Carter-Penrose diagram of Schwarzschild spacetime



Cauchy horizon