

Figures of lecture 3

The Kerr black hole

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<https://relativite.obspm.fr/blackholes/paris25/>

PSL graduate program in Physics / Doctoral School PIF

ENS, Paris, France

14 May 2025

Home page for the lectures

<https://relativite.obspm.fr/blackholes/paris25/>

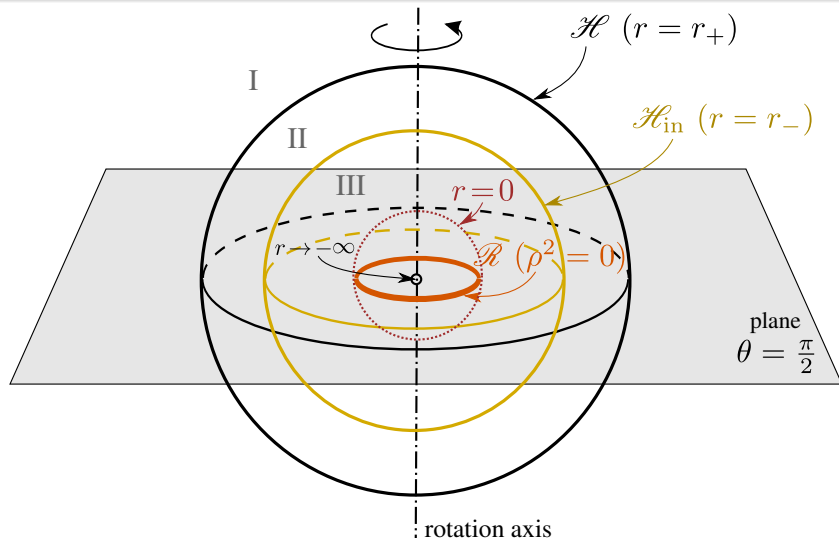
includes

- these slides
- the lecture notes (draft)
- some SageMath notebooks

Prerequisite

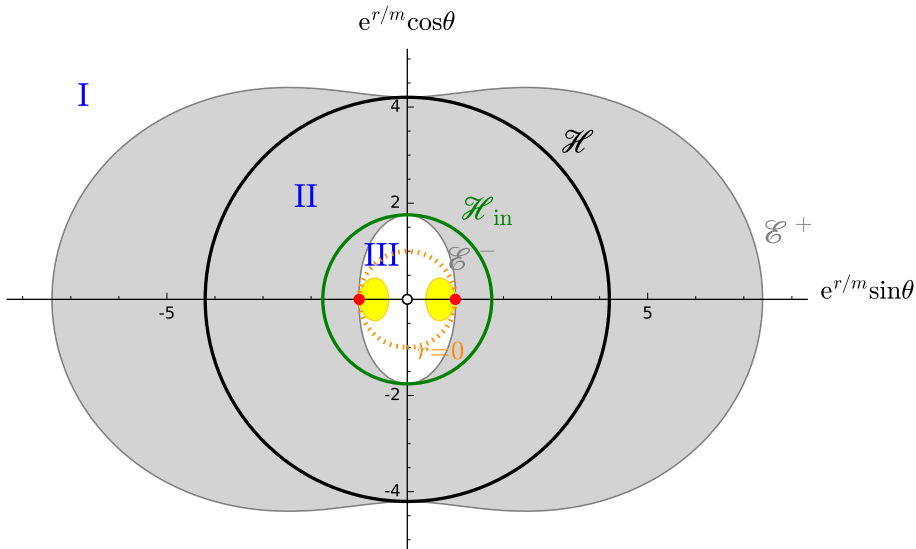
An introductory course on general relativity

Section of constant Boyer-Lindquist time coordinate t

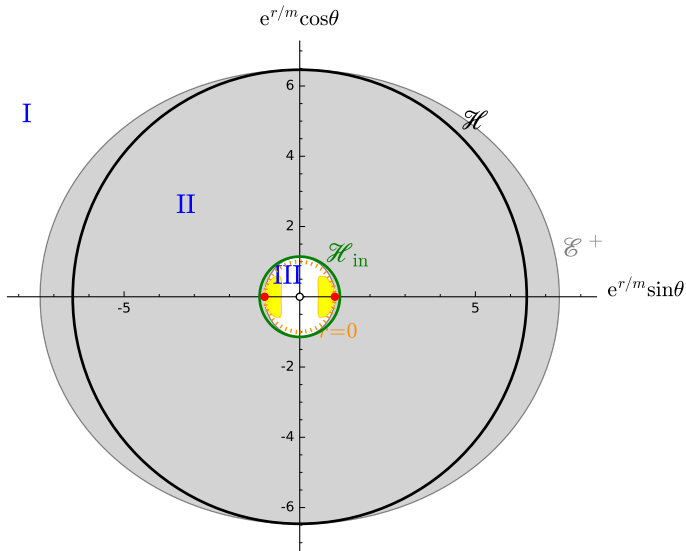


View of a section $t = \text{const}$ in O'Neill coord. (R, θ, φ) with $R := e^r$

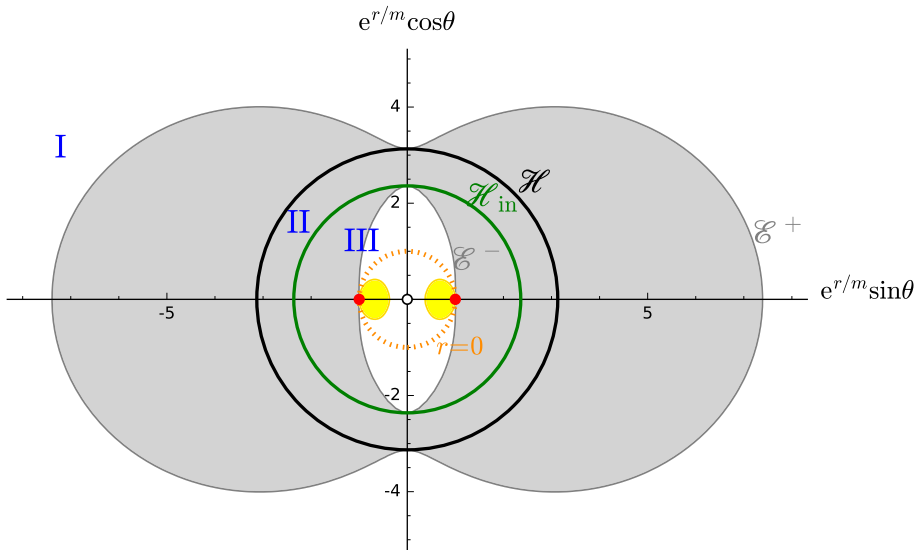
Meridional view of a $t = \text{const}$ slice ($a/m = 0.9$)



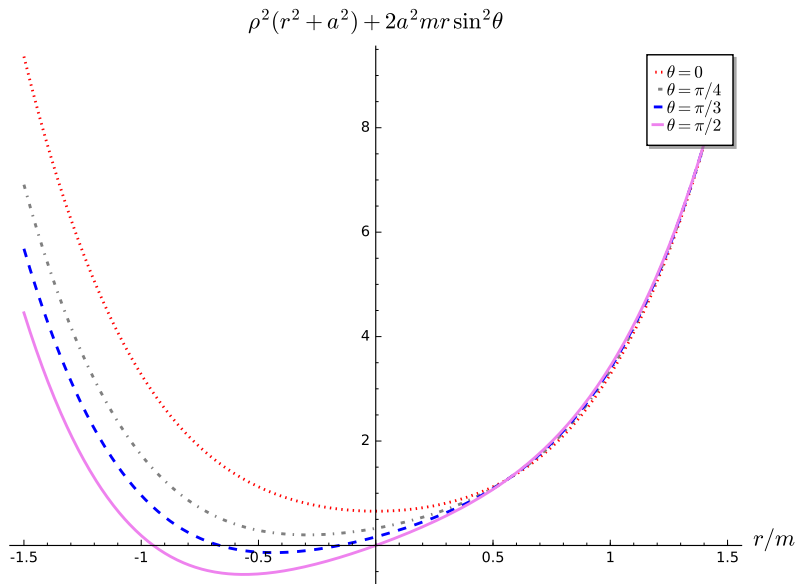
Meridional view of a $t = \text{const}$ slice ($a/m = 0.5$)



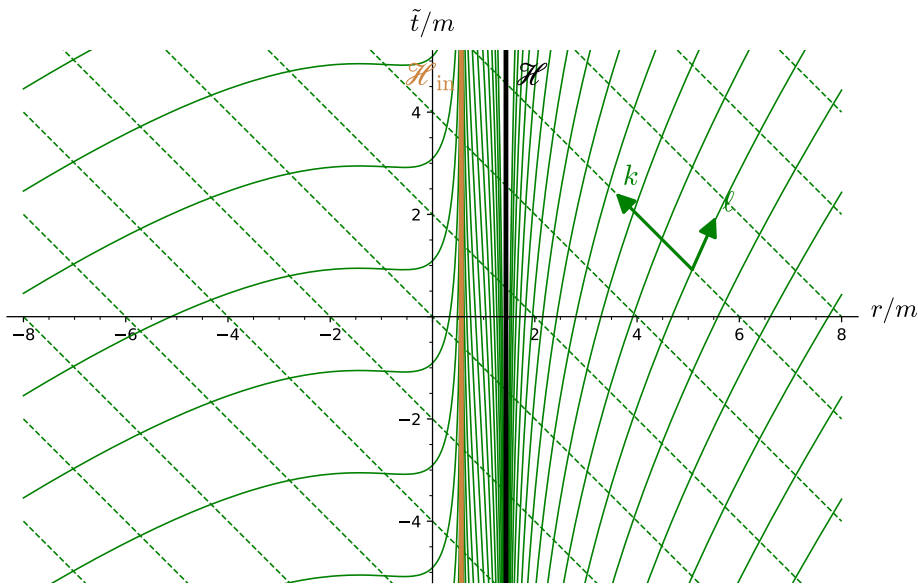
Meridional view of a $t = \text{const}$ slice ($a/m = 0.99$)



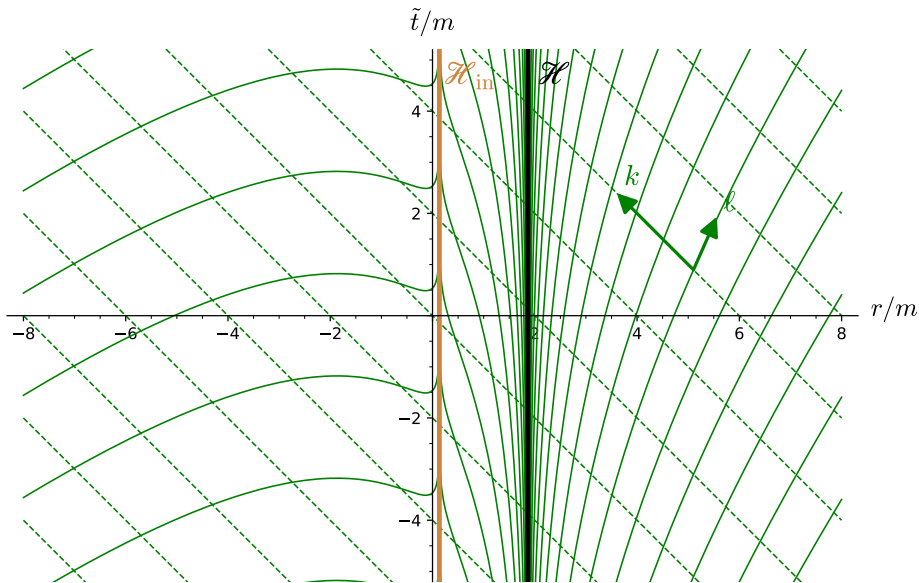
Sign of $g_{\varphi\varphi}$ for $a = 0.9m$



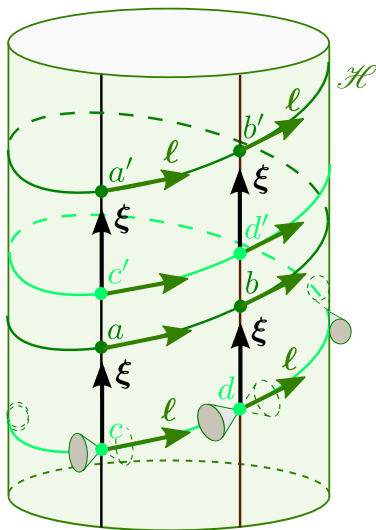
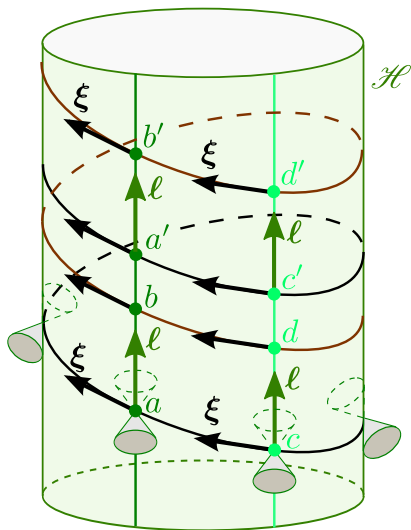
Principal null geodesics ($a/m = 0.9$)



Principal null geodesics ($a/m = 0.5$)

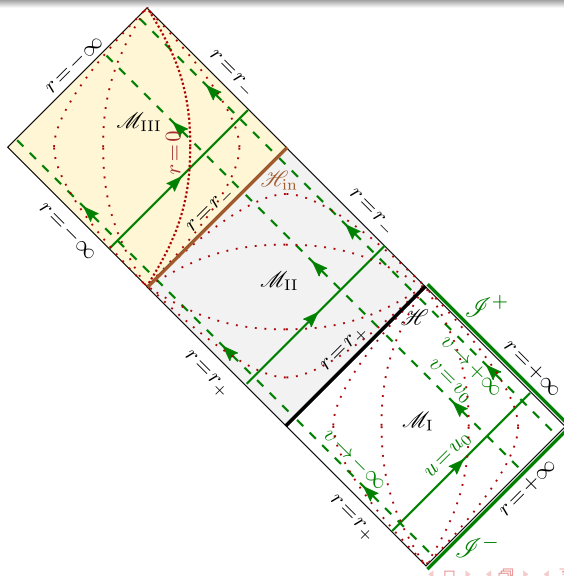


The event horizon and its null generators

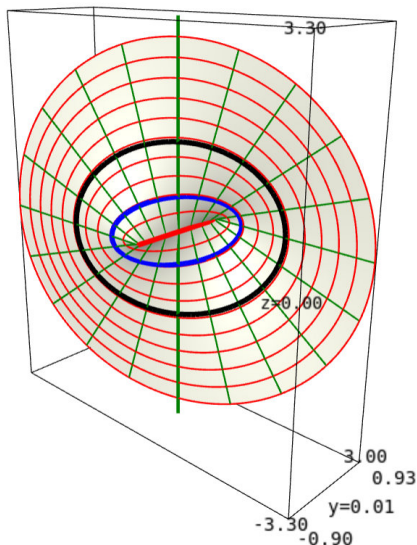


Conformal diagram of Kerr spacetime

with $\mathcal{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \mathcal{R}$



Meridional surface $\tilde{t} = \text{const}$ and $\tilde{\varphi} = 0$ or π



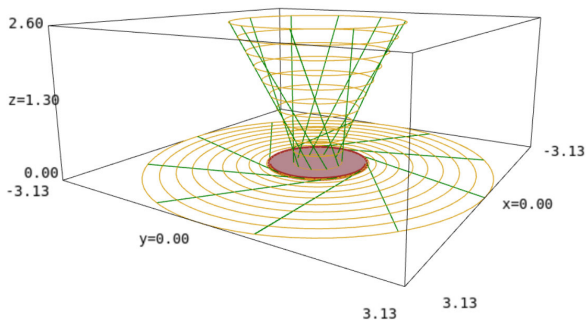
Part $r \geq 0$ viewed in Kerr-Schild coordinates (x, y, z)

- green lines: ingoing principal null geodesics
- black curve: event horizon $\mathcal{H}$
- blue curve: inner horizon $\mathcal{H}_{\text{in}}$
- red segment: disk $r = 0$

For an interactive 3D view, see the SageMath notebook

https://nbviewer.org/github/egourgoulhon/BHlectures/blob/master/sage/Kerr_Schild.ipynb

Surfaces $\tilde{t} = \text{const}$ and $\theta = \text{const}$



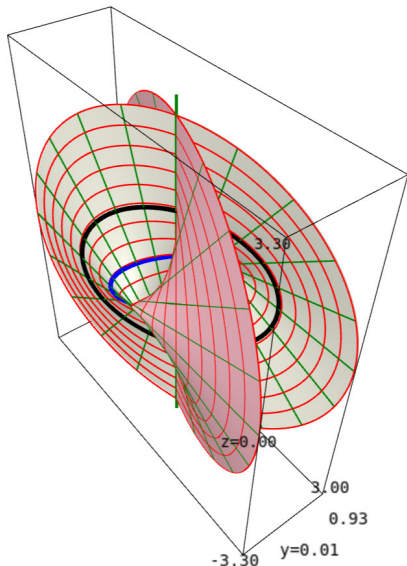
Part $r \geq 0$ of the surfaces $(\tilde{t}, \theta) = \text{const}$ drawn in terms of the Kerr-Schild coordinates (x, y, z) for 2 values of θ : $\theta = \pi/2$ ($z = 0$) and $\theta = \pi/6$

- green lines: ingoing principal null geodesics
- pink disk: $r = 0$
- red circle: curvature singularity

For an interactive 3D view, see the SageMath notebook

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Meridional surface $\tilde{t} = \text{const}$ and $\tilde{\varphi} = 0$ or π

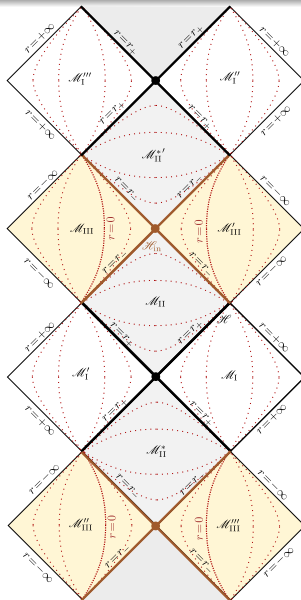


- Part $r \geq 0$ (grey) viewed in Kerr-Schild coordinates (x, y, z)
- Part $r \leq 0$ (pink) viewed in Kerr-Schild coordinates (x', y', z')

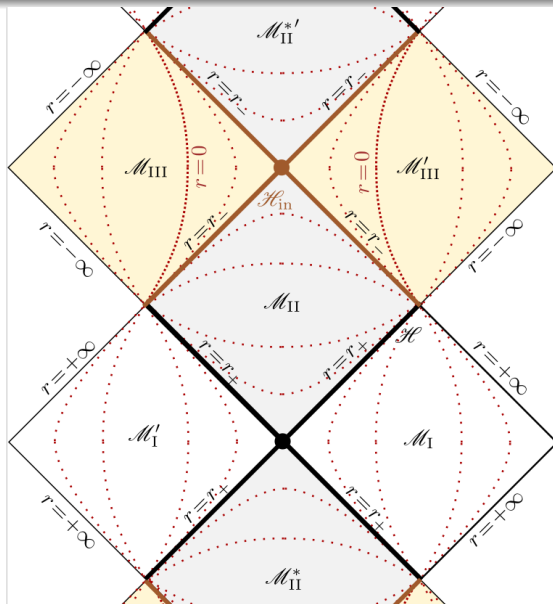
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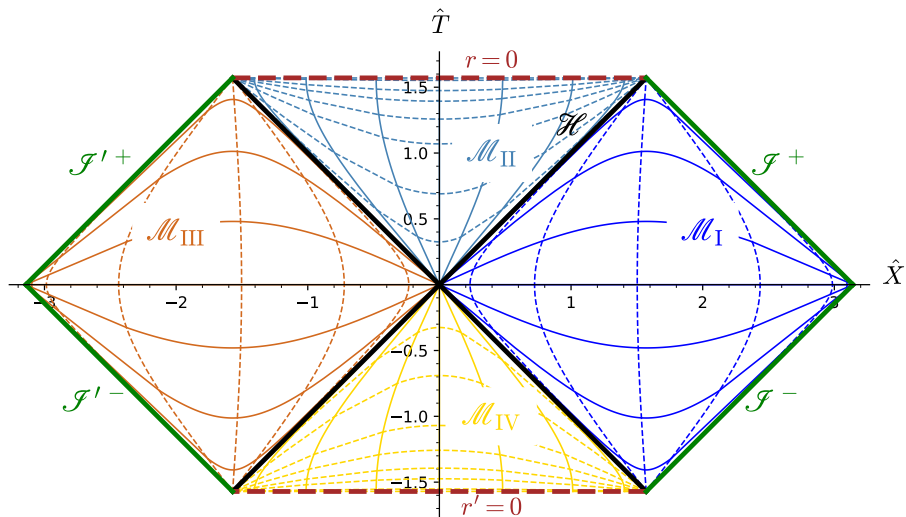
Carter-Penrose diagram of the maximal analytic extension



Carter-Penrose diagram of the maximal analytic extension



Carter-Penrose diagram of Schwarzschild spacetime



Cauchy horizon

