A history of black holes

from a physicist perspective

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Outline

1. The prehistory
2. Schwarzschild black hole
3. Kerr black hole
4. The Golden Age of black hole theory
5. Some recent developments
6. Testing general relativity with black holes

Outline

A two centuries-old prehistory...

$$\boxed{V_{\rm esc} > c} \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho > c^2 \iff \boxed{R > \sqrt{\frac{3c^2}{8\pi G\rho}}}$$

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John Michell (1784)

"If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight"

[Phil. Trans. R. Soc. Lond. **74**, 35 (1784)]

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Pierre Simon de Laplace (1796)

"Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles."

[Exposition du système du monde (1796)]

Limits of the Newtonian concept of a black hole

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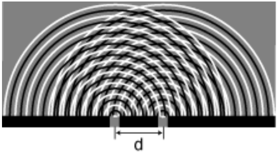
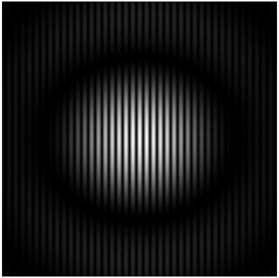
- $V_{\rm esc} \sim c \Longrightarrow$ gravitational potential energy $\sim$ mass energy Mc^2 $\Longrightarrow$ a *relativistic* theory of gravitation is necessary !
- No clear action of the gravitation field on electromagnetic *waves* in Newtonian gravity





[R. Taillet]

Outline

101 years ago : a relativistic theory of gravitation

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

$$\boldsymbol{R} - \frac{1}{2} R\, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}$$

[A. Einstein, Sitz. Preuss. Akad. Wissenschaften Berlin, 844 (1915)]

The Schwarzschild solution (1915)

Karl Schwarzschild (letter to Einstein 22 Dec. 1915 ; publ. submitted 13 Jan 1916)
Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, Sitz. Preuss. Akad. Wiss., Phys. Math. Kl. 1916, 189 (1916)

$\Longrightarrow$ First exact non-trivial solution of Einstein equation :

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right) c^2 \mathrm{d}t^2 + \left(1 - \frac{2m}{r}\right)^{-1} \mathrm{d}r^2 + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\varphi^2\right) \quad (1)$$

with

- coordinates [1] $(t, \bar{r}, \theta, \varphi)$
- "auxiliary quantity" : $r := (\bar{r}^3 + 8m^3)^{1/3}$
- parameter $m = GM/c^2$, with M gravitational mass of the "mass point"

1. Schwarzschild's notations : $r = \bar{r}$, $R = r$, $\alpha = 2m$

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The "center"

Origin of coordinates : $\bar{r} = 0 \iff r = 2m$

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Droste contribution (1916)

Johannes Droste (communication 27 May 1916)
The Field of a Single Centre in Einstein's Theory of Gravitation, and the Motion of a Particle in that Field, Kon. Neder. Akad. Weten. Proc. **19**, 197 (1917)

$\Longrightarrow$ derives the Schwarzschild solution (independently of Schwarzschild) via some coordinates (t, r', θ, φ) such that $g_{r'r'} = 1$; presents the result in the standard form (1) via a change of coordinates leading to the areal radius r
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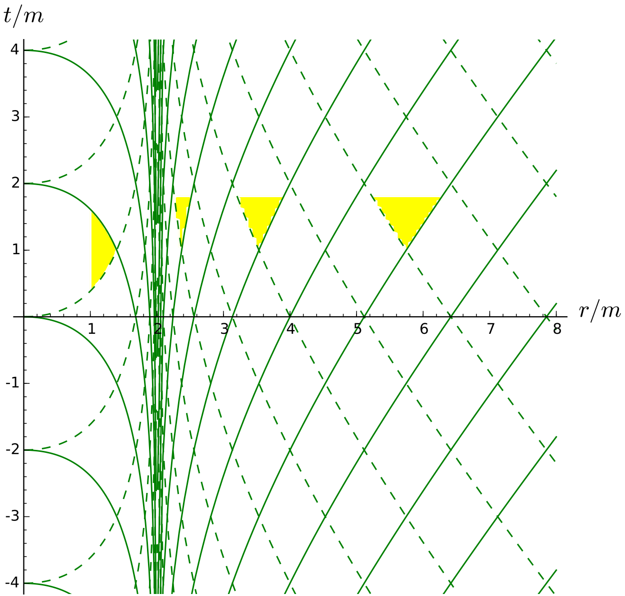
Apparent barrier at $r = 2m$

A particle falling from infinity never reaches $r = 2m$ within a finite amount of "time" t.

The *Schwarzschild radius* : $R_S := 2m = \dfrac{2GM}{c^2}$

The "barrier" at $r = R_S$



Radial null geodesics of Schwarzschild spacetime in term of Schwarzschild-Droste coordinates (t, r).
Solid (resp. dashed) lines correspond to outgoing (resp. ingoing) geodesics. The interiors of some future light cones are depicted in yellow.

The Schwarzschild solution : early discussions

- 1920 : Alexander Anderson : light cannot emerge from the region $r < R_S := 2m = \frac{2GM}{c^2}$ (region *"shrouded in darkness"*)

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- 1923 : George Birkhoff : outside any *spherical* body, the metric is Schwarzschild metric
- 1924 : Arthur Eddington introduced the coord. $t' := t - \frac{2m}{c} \ln\left(\frac{r}{2m} - 1\right)$, leading to

$$\mathrm{d}s^2 = -c^2\mathrm{d}t'^2 + \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) + \frac{2m}{r}\left(c\mathrm{d}t' - \mathrm{d}r\right)^2 \qquad (2)$$

but did not noticed that the metric components w.r.t. coordinates (t', r, θ, φ) are regular at $r = 2m$!
Actually, Eddington's aim was elsewhere : comparing Whitehead theory (1922) to general relativity

The Schwarzschild solution : Lemaître breakthrough

Georges Lemaître (1932)
L'univers en expansion, Publ. Lab. Astron. Géodésie Univ. Louvain **9**, 171 (1932) ; reprinted in Ann. Soc. Scient. Bruxelles A **53**, 51 (1933)

et la nouvelle forme du champ s'écrit sans singularité

(11.12) $$ds^2 = -2m\frac{d\chi^2}{r} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + dt^2,$$

où

(11.13) $$r = \left[\frac{3}{2}\sqrt{2m}(t-\chi)\right]^{\frac{2}{3}}.$$

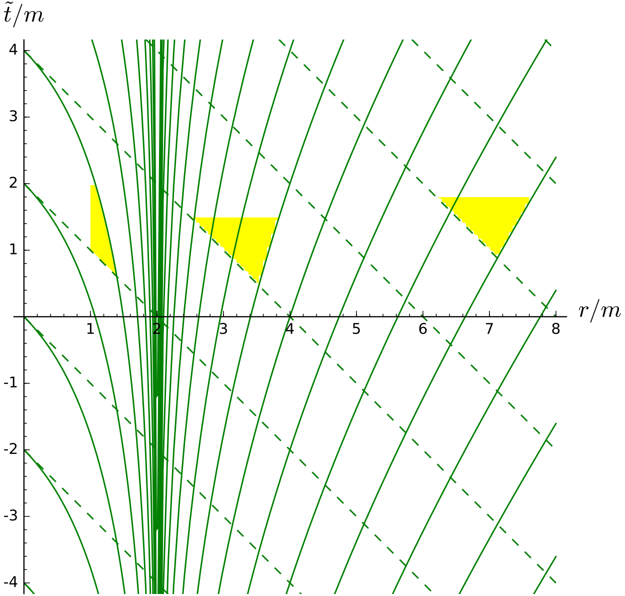
La singularité du champ de Schwarzschild est donc une singularité fictive, analogue à celle qui se présentait à l'horizon du centre dans la forme originale de l'univers de de Sitter.

The singularity at $r = R_S$ is a mere **coordinate singularity** : the metric components are regular in Lemaître coordinates $(\tau, \chi, \theta, \varphi)$:

$$\mathrm{d}s^2 = -c^2\mathrm{d}\tau^2 + \frac{R_S}{r}\mathrm{d}\chi^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) \quad (3)$$

$$r = r(\tau,\chi) := \left[\frac{3}{2}\sqrt{R_S}\,(c\tau - \chi)\right]^{2/3} \quad (4)$$

No longer any barrier at $r = R_S$

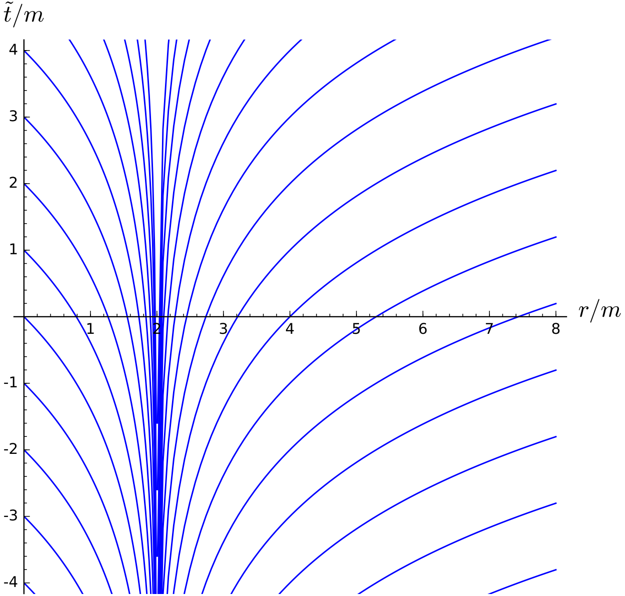


Radial null geodesics of Schwarzschild spacetime in term of ingoing Eddington-Finkelstein coordinates $(\tilde{t}, r)$

$$\tilde{t} = t + \frac{2m}{c} \ln \left| \frac{r}{2m} - 1 \right|$$

The ingoing null geodesics (dashed lines) do enter the region $r < R_S$.

Pathology of Schwarzschild-Droste coordinates



Hypersurfaces of constant Schwarzschild-Droste coordinate t in term of the ingoing Eddington-Finkelstein coordinates $(\tilde{t}, r)$

Gravitational collapse : Lemaître-Tolman solutions

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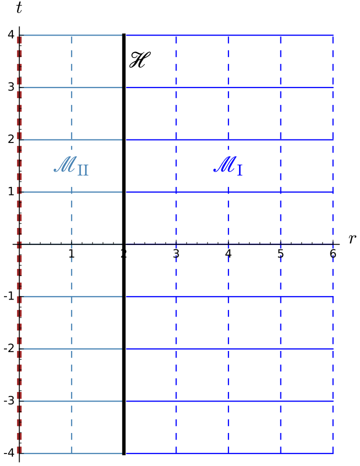
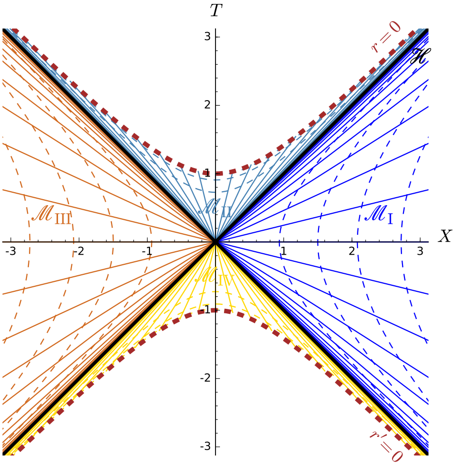
- 1939 : Robert Oppenheimer & Hartland Snyder : gravitational collapse of a homogeneous dust ball of radius R (special case of Lemaître's general solution)
  $\Longrightarrow$ for an external observer, $R \to R_{\rm S}$ as $t \to +\infty$
  $\Longrightarrow$ *"frozen star"*

The Schwarzschild solution : the complete picture

John L. Synge (1950), Martin Kruskal (1960), George Szekeres (1960) : complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times \mathbb{S}^2$ manifold)



Schwarzschild-Droste coordinates (t, r)



Carter-Penrose diagram of Schwarzschild spacetime

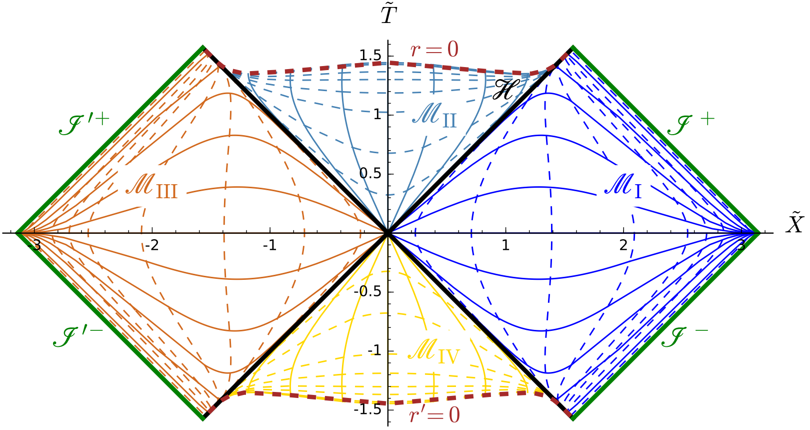
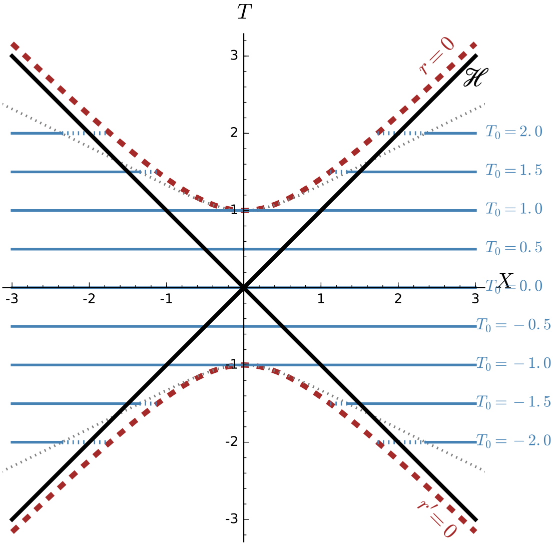


Figure drawn with `SageMath` : `http://sagemanifolds.obspm.fr`

Einstein-Rosen bridge

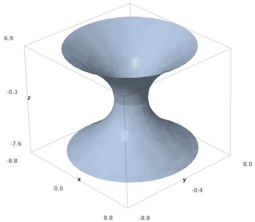


Connecting the asymptotically flat regions $\mathscr{M}_{\rm I}$ and $\mathscr{M}_{\rm III}$ by hypersurfaces $T = T_0 = \text{const}$ (blue horizontal lines).

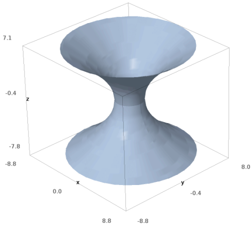
$\Longrightarrow$ isometric embedding of equatorial sections $(T = T_0, \theta = \pi/2)$ in the Euclidean 3-space

Rem : for $|T_0| > 1$, the dotted parts cannot be embedded isometrically in Euclidean space.

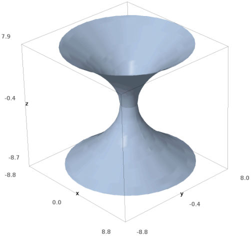
Evolving Einstein-Rosen bridge



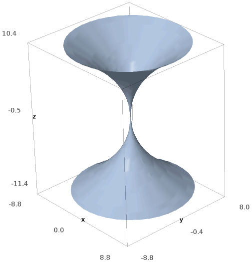
$T_0 = 0$ (Flamm paraboloid)



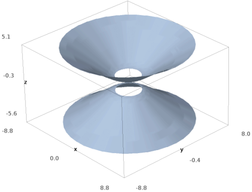
$T_0 = 0.5$



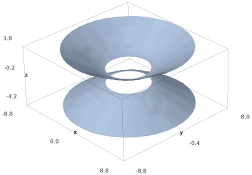
$T_0 = 0.9$



$T_0 = 1$



$T_0 = 1.5$



$T_0 = 2$

Outline

Rotation enters the game : the Kerr solution

Almost 50 years after Schwarzschild : Roy Kerr (1963)

$$\begin{aligned} \mathrm{d}s^2 = & -\left(1-\frac{2mr}{\rho^2}\right)\mathrm{d}v^2 + 2\mathrm{d}v\,\mathrm{d}r - \frac{4amr\sin^2\theta}{\rho^2}\,\mathrm{d}v\,\mathrm{d}\tilde\varphi \\ & -2a\sin^2\theta\,\mathrm{d}r\,\mathrm{d}\tilde\varphi + \rho^2\mathrm{d}\theta^2 + \left(r^2+a^2+\frac{2a^2mr\sin^2\theta}{\rho^2}\right)\sin^2\theta\,\mathrm{d}\tilde\varphi^2. \end{aligned}$$

Boyer & Lindquist (1967) coordinate change $(v,r,\theta,\tilde\varphi)\to(t,r,\theta,\varphi)$:

$$\begin{aligned} \mathrm{d}s^2 = & -\left(1-\frac{2mr}{\rho^2}\right)\mathrm{d}t^2 - \frac{4amr\sin^2\theta}{\rho^2}\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\rho^2}{\Delta}\,\mathrm{d}r^2 \\ & +\rho^2\mathrm{d}\theta^2 + \left(r^2+a^2+\frac{2a^2mr\sin^2\theta}{\rho^2}\right)\sin^2\theta\,\mathrm{d}\varphi^2, \end{aligned}$$

where $\rho^2 := r^2 + a^2\cos^2\theta$, $\Delta := r^2 - 2mr + a^2$ and $r\in(-\infty,\infty)$

→ spacetime manifold $\mathscr{M} = \mathbb{R}^2\times\mathbb{S}^2 \setminus \{r=0 \ \&\ \theta=\pi/2\}$

→ 2 parameters : $m = \dfrac{GM}{c^2}$ and $a = \dfrac{J}{cM}$; black hole $\iff$ $0\le a\le m$

→ Schwarzschild metric for $a=0$

Physical meaning of the parameters M and J

- mass M : *not* a measure of the "amount of matter" inside the black hole, but rather a *characteristic of the external gravitational field*
  $\rightarrow$ measurable from the orbital period of a test particle in far circular orbit around the black hole *(Kepler's third law)*

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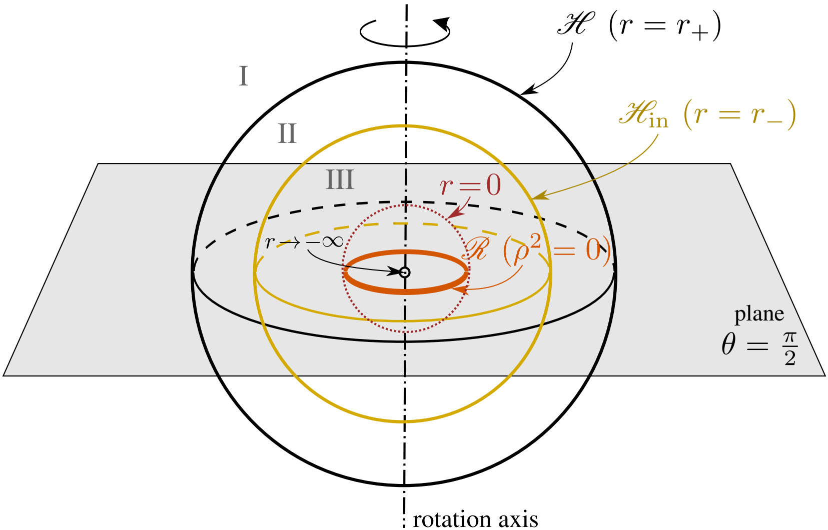
Remark : the radius of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole "centre" and the event horizon. A well defined quantity is the area of the event horizon, A.
The "radius" can be defined from it : for a Schwarzschild black hole :

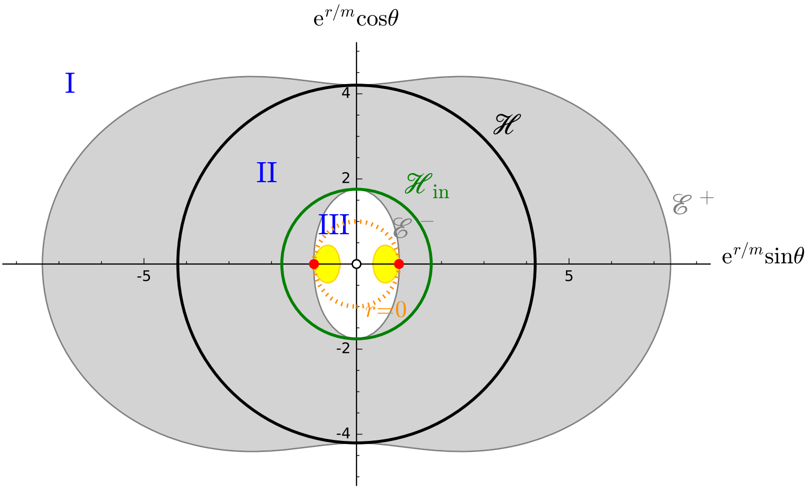
$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot}\right) \text{ km}$$

Kerr spacetime



Slice $t = \mathrm{const}$ of the Kerr spacetime viewed in O'Neill coordinates (R, θ, φ), with $R := \mathrm{e}^r$, $r \in (-\infty, +\infty)$

Kerr spacetime : ergoregion and Carter time machine



Meridional view of a section $t = \mathrm{const}$ of Kerr spacetime with $a/m = 0.90$

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The Golden Age of black hole theory

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- 1965-1972 : **the no-hair theorem**

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

The no-hair theorem

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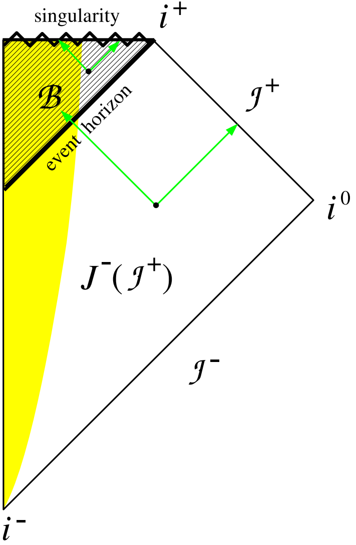
Astrophysical black holes have to be electrically neutral :

- $Q = 0$: Kerr solution (1963)

Other special cases :

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)

General definition of a black hole



The textbook definition

[Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$

where

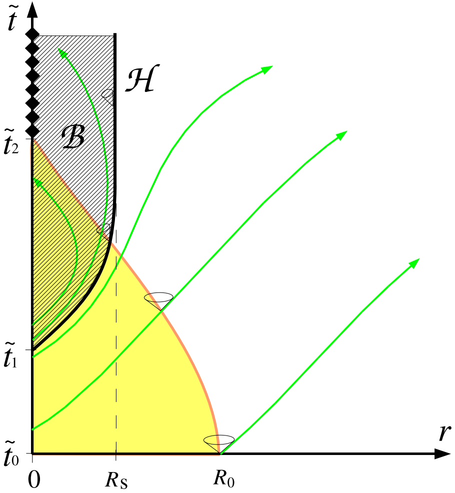
- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathscr{H} := \partial J^-(\mathscr{I}^+)$ (boundary of $J^-(\mathscr{I}^+)$)

$\mathscr{H}$ smooth $\Longrightarrow$ $\mathscr{H}$ null hypersurface

General definition of a black hole



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Main properties of black holes (1/2)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : the singularity *(NB : this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.

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- The singularity marks the limit of validity of general relativity : to describe it, a quantum theory of gravitation would be required.
- The event horizon $\mathscr{H}$ is a global structure of spacetime : no physical experiment whatsoever can detect the crossing of $\mathscr{H}$.

Main properties of black holes (2/2)

- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its "mean density" by $\bar{\rho} = M/(4/3\pi R^3)$, then
  - for the Galactic centre BH (Sgr A*) : $\bar{\rho} \sim 10^6 \ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-4}\ \rho_{\text{white dwarf}}$
  - for the BH at the centre of M87 : $\bar{\rho} \sim 2 \ \mathrm{kg\,m^{-3}} \sim 2\ 10^{-3}\ \rho_{\text{water}}$!

  $\Longrightarrow$ a black hole is a compact object : $\dfrac{M}{R}$ large, not $\dfrac{M}{R^3}$!
- Due to the non-linearity of general relativity, black holes can form in spacetimes without any matter, by collapse of gravitational wave packets.

Outline

The quasi-local approach : motivation

The standard definition of a black hole is highly non-local : determination of $\partial J^{-}(\mathscr{I}^{+})$ requires the knowledge of the entire future null infinity. Moreover this is *not locally linked with the notion of strong gravitational field.*

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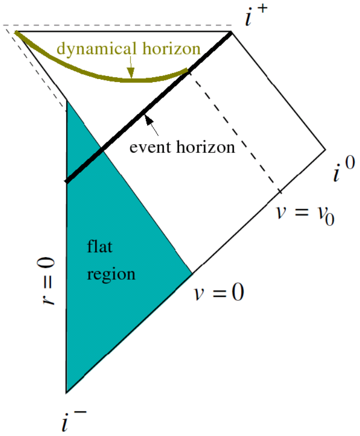

[Ashtekar & Krishnan, LRR **7**, 10 (2004)]

Example of event horizon in a **flat** region of spacetime : Vaidya metric, describing incoming radiation from infinity :

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv\, dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

with $m(v) = 0$ for $v < 0$

$dm/dv > 0$ for $0 \leq v \leq v_0$

$m(v) = M_0$ for $v > v_0$

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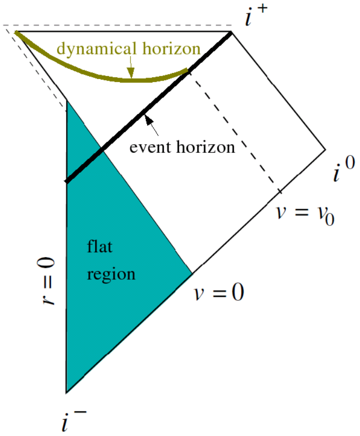

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$\Rightarrow$ no local physical experiment can locate the event horizon

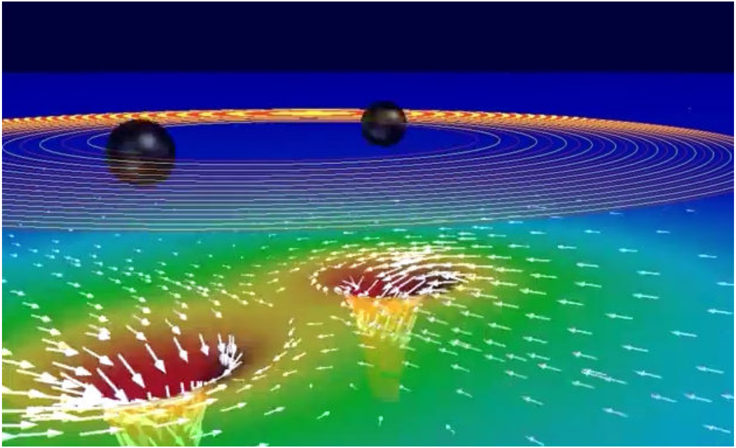
Quasi-local approaches to black holes

New paradigm for the theoretical approach to black holes, motivated by quantum gravity and numerical relativity, instead of *event horizons*, black holes are described by

- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)
- slowly evolving horizons (Booth and Fairhurst 2004)

All these concepts are **local** and are based on the notion of trapped surfaces

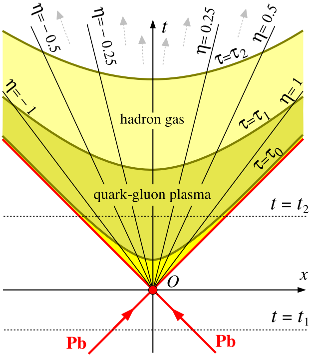
The 2000's : the triumph of numerical relativity



[Caltech/Cornell SXS]

[Scheel et al., PRD **79**, 024003 (2009)]

A recent hot topic : black holes and gauge/gravity duality



Spacetime diagram of a heavy-ion collision (LHC)
$\tau_0 \simeq 0.2\ \mathrm{fm}/c = 6\ 10^{-25}\ \mathrm{s}$
$\tau_1 \sim 10\tau_0$

Gauge/gravity duality ("holographic principle")

4D strongly-coupled gauge theory ≡ 5D gravitation
Example : AdS/CFT correspondence

Quark-gluon plasma (QGP) in heavy-ion collisions :
low-viscosity fluid with *anisotropic* pressure $(p_x < p_y)$

Thermalization of QGP ≡ 5D black hole formation

Gauge theory : QCD
Gravity : 5D Lifshitz-like spacetime (*anisotropic* generalization of AdS_5) with formation of a black brane (Vaidya-type collapse) ; new exact solutions with the help of SageManifolds
Results : faster thermalization in the transversal direction ; evolution of the entanglement entropy

[Aref'eva, Golubtsova & Gourgoulhon, J. High Ener. Phys. **09(2016)**, 142 (2016)]

Outline

Is general relativity unique ?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $\boldsymbol{g}$
- field equation involving only derivatives of $\boldsymbol{g}$ up to second order
- diffeomorphism invariance
- $\boldsymbol{\nabla} \cdot \boldsymbol{T} = 0$ ($\Longrightarrow$ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).

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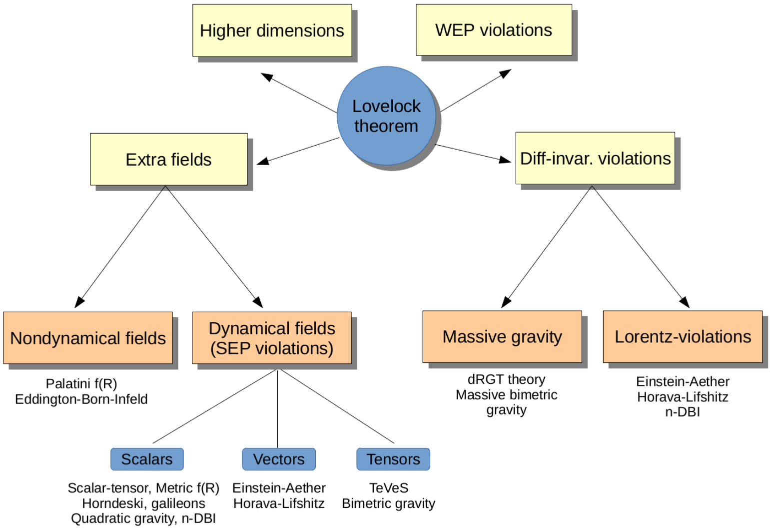
However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory :

- string theory
- loop quantum gravity
- ...

Extensions of general relativity



[Berti et al., CGQ **32**, 243001 (2015)]

Test : are astrophysical black holes Kerr black holes ?

- GR $\Longrightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\Longrightarrow$ BH may deviate from Kerr

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Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)

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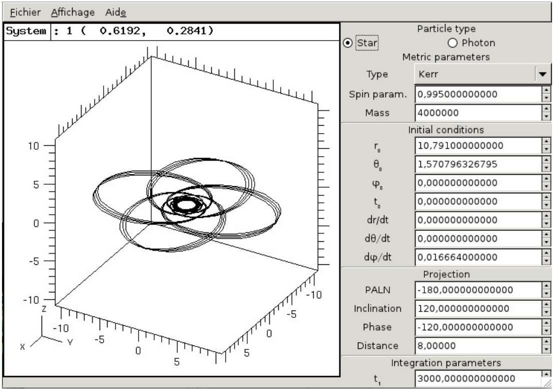
Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

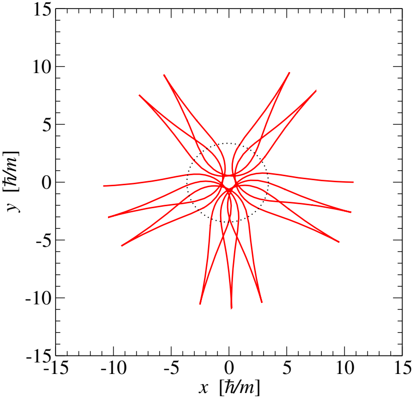
An example : rotating boson stars

Boson star = localized configurations of a self-gravitating massive complex scalar field Φ ≡ *"Klein-Gordon geons"*

[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the *Einstein-Klein-Gordon* system computed by means of Kadath
  [Grandclément, JCP **229**, 3334 (2010)]
- Timelike geodesics computed by means of Gyoto



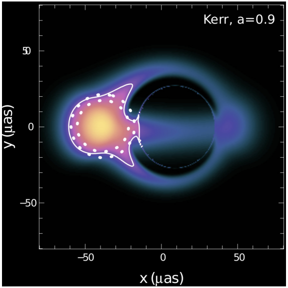
Zero-angular-momentum orbit around a rotating boson star based on a free scalar field $\Phi = \phi(r,\theta)e^{i(\omega t + 2\varphi)}$ with $\omega = 0.75\, m/\hbar$.

[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

Images of accretion torus around alternatives to the Kerr black hole

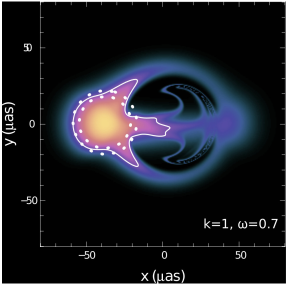
Kerr black hole

$a/M = 0.9$



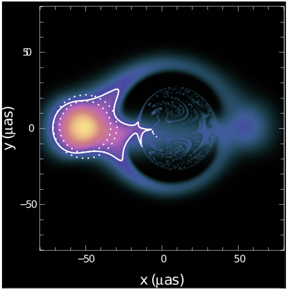
boson star [1]

$k=1,\ \omega=0.7\, m/\hbar$



hairy black hole [2]

$a/M = 0.9$



[1] Vincent, Meliani, Grandclément, Gourgoulhon & Straub, Class. Quantum Grav. **33**, 105015 (2016)
[2] Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D **94**, 084045 (2016)

Can we see a black hole from the Earth ?

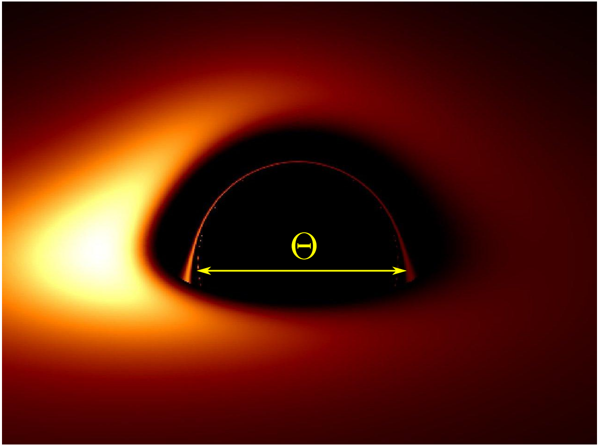


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_{\rm S}}{d}$$

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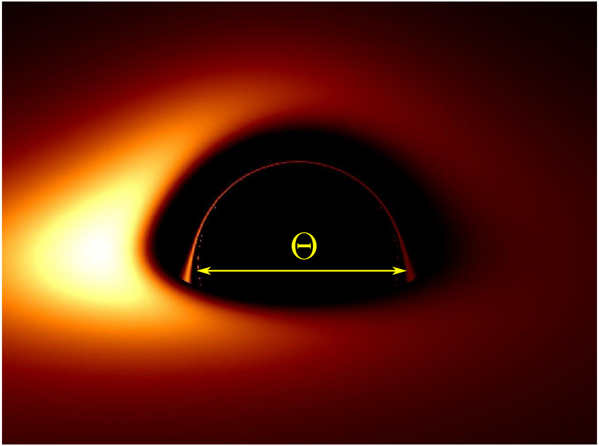

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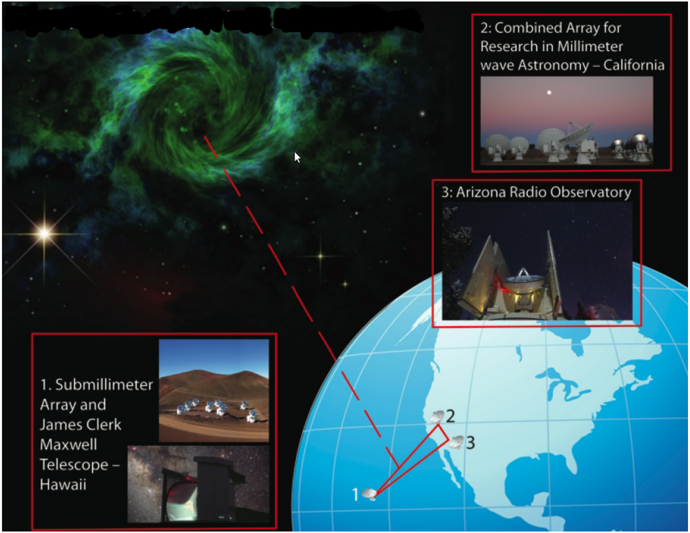
Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53\ \mu\mathrm{as}$
M87 : $\Theta = 21\ \mu\mathrm{as}$
M31 : $\Theta = 20\ \mu\mathrm{as}$

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

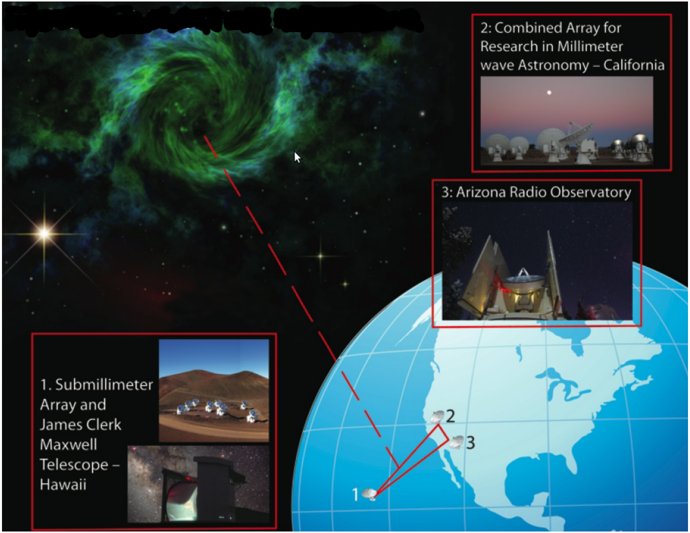
Reaching the μas resolution with VLBI



Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Reaching the μas resolution with VLBI



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Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu$as [Doeleman et al., Nature **455**, 78 (2008)]

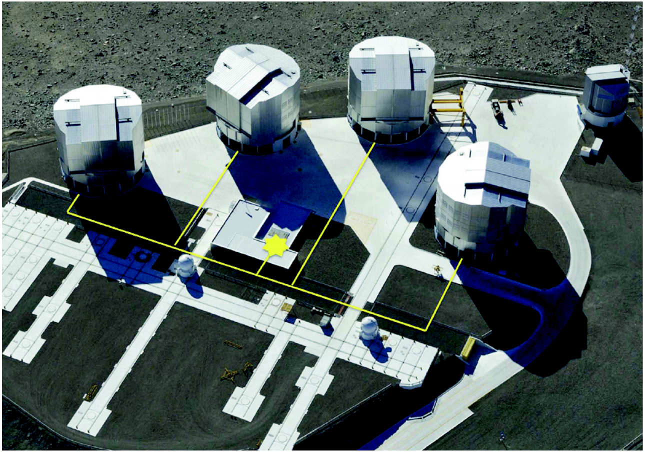
The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : $1.3\ \mathrm{mm} \to 0.8\ \mathrm{mm}$
- increase the number of stations ; in particular add ALMA

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020
August 2015 : VLBI observations involving ALMA and VLBA

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10\ \mu as$

Near-infrared optical interferometry : GRAVITY

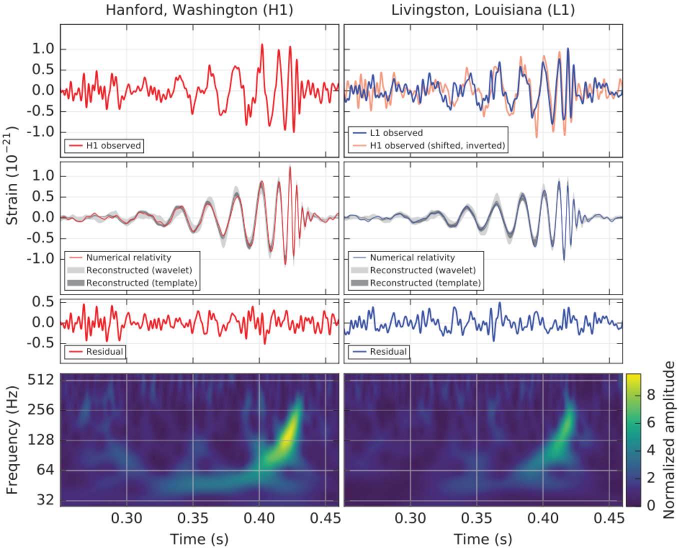
[MPE/GRAVITY team]

July 2015 : GRAVITY shipped to Chile and successfully assembled at the Paranal Observatory

Fall 2016 : observations have started !

Observing black holes via gravitational waves

A dream come true on September 14, 2015, 09:50:45 UTC



[Abbott et al., PRL **116**, 061102 (2016)]

Conclusions

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It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime.

The GW150914 event was both the first direct detection of gravitational waves and the first observation of the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.

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(historical part only)

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