Hairy black holes in scalar tensor theories

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Gravitation and scalar fields: LUTH

Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973.
- contain or are limits of other modified gravity theories. $f(R)$, massive gravity etc.
- Are there non trivial black hole solutions in Horndeski theory?
- No hair paradigm

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973], [Deffayet et.al.]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= K(\phi, X),\\
L_3 &= -G_3(\phi, X)\Box\phi,\\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],\\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]

Horndeski theory includes,

- R, $f(R)$ theories, Brans Dicke theory with arbitrary potential
- Scalar-tensor interaction terms: $G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, $P^{\mu\rho\nu\sigma}\nabla_\mu\nabla_\nu\phi\nabla_\rho\phi\nabla_\sigma\phi$, $V(\phi)\hat{G}$ (Fab 4)
- higher order Galileons : $\Box\phi(\nabla\phi)^2(DGP), (\nabla\phi)^4$ (ghost condensate)
- Higher order terms originate form KK reduction of Lovelock theory ([Van Acoleyen et.al. arXiv:1102.0487 [gr-qc]], [CC, Goutéraux and Kiritsis])
- Gallileons in flat spacetime have Gallilean symmetry [Nicolis et.al.: arXiv:0811.2197 [hep-th]]
- Horndeski theories appear at "decoupling limit" of DGP and massive gravity theories

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What about black holes in scalar-tensor theories?

Black holes have no hair [recent review Herdeiro and Radu 2015]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges
and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...
For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

non minimally coupled scalars and static spacetimes [Babichev and CC], Gauss-Bonnet term [Sotiriou and Zhou] minimally coupled complex scalar and stationary spacetimes [Herdeiro and Radu]

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Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi] = \int_{\mathcal{M}} \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right) \mathrm{d}^4x + S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \\ \phi \mapsto \tilde{\phi} = \Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
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- Static and spherically symmetric solution

$$ds^2 = -\left(1 - \frac{m}{r}\right)^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1 - \frac{m}{r}\right)^2} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}} \frac{m}{r - m}}$$

- Geometry is that of an extremal RN.
Problem: The scalar field is **unbounded** at $(r = m)$.

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Scalar-tensor theories and black holes

- In scalar tensor theories "regular" black hole solutions are GR black holes with a constant scalar field
- Is it possible to have non-trivial and regular scalar-tensor black holes for an asymptotically flat or $\Lambda > 0$ space-time?
- How can we evade no-hair theorems?
- We will consider:
  - Higher order gravity theory
  - Translational symmetry for the scalar
  - A scalar field that does not have the same symmetries as the spacetime metric

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An integrability theorem and no-hair, [Babichev, CC and Hassaine]

Consider $L = L(g_{\mu\nu}, \nabla\phi, \nabla\nabla\phi) \subset L_H$,

- theory has shift symmetry in $\phi \to \phi + c$
  $\mathcal{E}_{(\phi)} = \nabla_\mu J^\mu = 0$, J^μ is a conserved current associated to the symmetry
- Suppose now a static and spherically symmetric spacetime,
  $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- and $\phi = qt + \psi(r)$.
  Galileon does not acquire the symmetries of spacetime. Are the EoM compatible?

Under these hypotheses:

$-qJ_r = \mathcal{E}_{tr} g^{rr}$ where $\mathcal{E}_{tr}$ is the tr-metric equation

No time derivatives present in the field equations

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Example theory

Consider the action,

$$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- Metric field equations read,

$$\zeta G_{\mu\nu} - \eta\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right) + g_{\mu\nu}\Lambda$$
$$+\frac{\beta}{2}\left((\nabla\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi\right.$$
$$\left.+g_{\mu\alpha}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = 0,$$

- Scalar field has translational invariance :$\phi \to \phi$+const.,
- Scalar field equation,

$$\nabla_\mu J^\mu = 0, \quad J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right)\partial_\nu\phi.$$

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  Generically $\phi = \mathit{constant}$ everywhere [Hui and Nicolis] and we have again the appearance of a no-hair theorem...
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Consistency of field equations

- Hypotheses: $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- $J^r = \beta G^{rr} - \eta g^{rr} = 0$ and $\phi(t,r) = q\,t + \psi(r)$,
- Geometric constraint, $f = \frac{(\beta+\eta r^2)h}{\beta(rh)'}$, fixing spherically symmetric gauge.
- Scalar field eq and (tr)-eq satisfied
- Unknowns $\psi(r)$ and $h(r)$ and have two ODE's to solve, the (rr) and (tt). Hence hypotheses are consistent.
- The system is integrable for spherical symmetry boiling down to a single second order non-linear ODE for an arbitrary Shift symmetric theory!

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Solving the remaining EoM

- From (rr)-component get ψ'

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Asymptotically flat limit : $\Lambda = 0$, $\eta = 0$

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Algebraic equation to solve: $q^2\beta^3 - 2\zeta\beta k + C_0 k^{3/2} = 0 \rightarrow k = constant$!
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Schwarzschild geometry with a non-trivial scalar field. But is the scalar regular on the horizon?

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de Sitter black hole

- Consider $S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta \left(\partial\phi\right)^2 + \beta G^{\mu\nu} \partial_\mu\phi\partial_\nu\phi \right]$
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- $q^2\eta = \Lambda - \Lambda_{eff} > 0$
- Hence for any arbitrary $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
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Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi]=\int_{\mathcal{M}}\sqrt{-g}\left(\frac{R}{16\pi G}-\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi-\frac{1}{12}R\phi^2\right)\mathrm{d}^4x+S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta}\mapsto\tilde{g}_{\alpha\beta}=\Omega^2 g_{\alpha\beta} \\ \phi\mapsto\tilde{\phi}=\Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
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BBMB completion [CC, Kolyvaris, Papantonopoulos and Tsoukalas]

- We would like to combine the above properties in order to obtain a hairy black hole.
- Consider the following action, $S(g_{\mu\nu}, \phi, \psi) = S_0 + S_1$ where
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Black hole with primary hair

- Solve as before assuming linear time dependence for Ψ
- Scalar ϕ has an additional branch regular at the "horizon"
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- A second solution reads,

$$h(r) = 1 - \frac{m}{r}, \qquad f(r) = (1 - \frac{m}{r})\left(1 - \frac{\gamma c_0^2}{12\beta r^2}\right)$$

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Conclusions

- Hairy black holes: non minimally coupled scalars and static spacetimes [Babichev and CC]
  minimally coupled complex scalar and stationary spacetimes [Herdeiro and Radu]: in both cases scalars have not the same symmetry as spacetime
- For a theory with Shift symmetry and higher order terms
- Scalar field with linear time dependence: EoM compatible. System is integrable
- Time dependence essential for regularity on the event horizon
- Higher order terms essential for novel branches of black holes
- Method can be applied in differing Gallileon context [Kobayashi and Tanahashi], in higher dimensions, including gauge fields.
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- Higher order terms essential for novel branches of black holes
- Method can be applied in differing Gallileon context [Kobayashi and Tanahashi], in higher dimensions, including gauge fields.
- Is there a way to find observable for q? Is there a distinction possible?
- Thermodynamics and stability.
- Can we go beyond spherical symmetry?

Conclusions

- Hairy black holes: non minimally coupled scalars and static spacetimes [Babichev and CC]
  minimally coupled complex scalar and stationary spacetimes [Herdeiro and Radu]: in both cases scalars have not the same symmetry as spacetime
- For a theory with Shift symmetry and higher order terms
- Scalar field with linear time dependence: EoM compatible. System is integrable
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