Scalar breathers with anti-de Sitter asymptotics

Gyula Fodor

Observatoire de Paris, Meudon
Wigner Research Centre for Physics, Budapest

Péter Forgács (Wigner Research Centre Budapest, Tours University)
Philippe Grandclément (Observatoire de Paris, Meudon)

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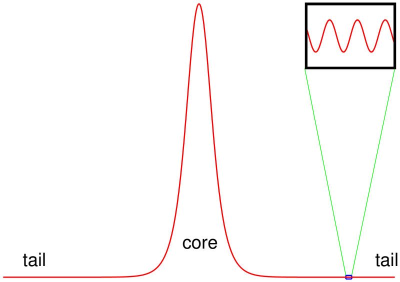
Zero cosmological constant case

Self-gravitating massive real scalar field can form spherically symmetric star-like objects – named oscillatons



extremely long living and stable, but the mass decreases very slowly because of a tiny scalar field radiation

Small amplitude oscillatons

G. Fodor, P. Forgács and M. Mezei, *Phys. Rev. D*, **81,** 064029 (2010)

- amplitude $\sim \varepsilon^2$
- size $\sim \frac{1}{m\varepsilon}$
- mass $M = \frac{1}{m}\left[1.753\,\varepsilon - 2.117\,\varepsilon^3\right]$
- mass loss rate

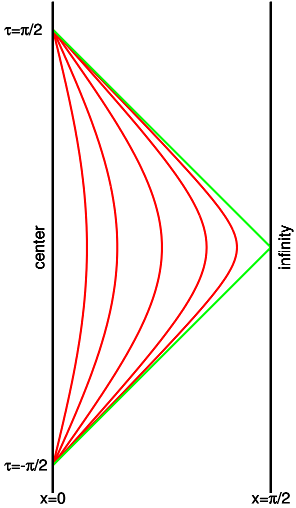
$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{30.0}{\varepsilon^2}\exp\left(-\frac{22.4993}{\varepsilon}\right)$$

extension of the mode equations to the complex plane, study the behavior near the pole, Borel summation

For large amplitude oscillatons the radiative tail can be calculated numerically by spectral methods

P. Grandclément, G. Fodor and P. Forgács, *Phys. Rev. D*, **84,** 065037 (2011)

Negative cosmological constant



$\Lambda < 0$ provides an effective attractive force
– formation of localized solutions is easier

Anti-de Sitter spacetime

$$ds^2 = \frac{1}{k^2 \cos^2 x} \left(-d\tau^2 + dx^2 + \sin^2 x \, d\Omega^2 \right)$$

A light ray can travel to infinity and back in a finite time

This is related to the instability of AdS
– a wave packet can bounce back many times to the center, and finally collapse to a black hole

(What about the boundary conditions?)

$\Lambda < 0$: asymptotically anti-de Sitter case

Negative cosmological constant acts as an effective attractive force

Exactly periodic solutions exist for real scalar fields

– oscillatons without radiative tail

– we call them AdS breathers
there is no energy loss, similarly to the sine-Gordon breather

There are breather solutions even for massless free scalar fields

– their size is determined by the cosmological constant $\sim 1/\sqrt{-\Lambda}$

– for massive fields the size is given by the scalar field mass

Rest of the talk: massless Klein-Gordon field, $U(\phi) = 0$, minimally coupled to Einstein's gravity

AdS breathers – massless minimally coupled real scalar

G.Fodor, P. Forgács and P. Grandclément,
Phys. Rev. D **92**, 025036 (2015), arXiv:1503.07746 [gr-qc]

We apply three methods:

- Spectral code for constructing time-periodic solutions
- Time-evolution code to study stability
- High-order small-amplitude expansion to get analytical results

Extension of the results of M. Maliborski and A. Rostworowski, *Phys. Rev. Lett.* **111,** 051102 (2013)
– methods that work well for $2n+1$ spacetime dimensions
– results presented only for $4+1$ dimensions

We give $3+1$ and $4+1$ results, can reach higher amplitudes, find maximal mass state, higher amplitude unstable states, and some resonance-like structures

$d+1$ dimensional Einstein's equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad , \qquad T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}$$

the contracted Bianchi identity gives the wave equation

$$\nabla^{\mu}\nabla_{\mu}\phi = 0$$

usually ϕ is rescaled to make $8\pi G = d-1$

We look for spherically symmetric solutions with metric

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2 x}\left(-Ae^{-2\delta}\mathrm{d}t^{\,2} + \frac{1}{A}\mathrm{d}x^2 + \sin^2 x\,\mathrm{d}\Omega^2_{d-1}\right)$$

where $L^2 = -\frac{d(d-1)}{2\Lambda}$, A and δ are functions of t and x

– anti-de Sitter corresponds to $A=1$ and $\delta=0$

Introducing the variables $\Phi = \frac{\partial\phi}{\partial x}$ and $\Pi = \frac{e^\delta}{A}\frac{\partial\phi}{\partial t}$

– there are evolution equations for the scalar field

$$\frac{\partial\Pi}{\partial t} = \frac{1}{\tan^{d-1}x}\,\frac{\partial}{\partial x}\left(\frac{A\tan^{d-1}x}{e^\delta}\Phi\right)$$
$$\frac{\partial\Phi}{\partial t} = \frac{\partial}{\partial x}\left(\frac{A}{e^\delta}\Pi\right)$$

– and constraints for the metric variables

$$\frac{\partial\delta}{\partial x} = -\frac{8\pi G}{d-1}\sin x\cos x\left(\Phi^2+\Pi^2\right)$$
$$\frac{\partial A}{\partial x} = A\frac{\partial\delta}{\partial x} + \left[d\tan x + (d-2)\cot x\right](1-A)$$

fourth order method of lines code for time-evolution

General asymptotic behavior

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2 x}\left(-Ae^{-2\delta}\mathrm{d}t^2 + \frac{1}{A}\mathrm{d}x^2 + \sin^2 x\,\mathrm{d}\Omega^2_{d-1}\right)$$

coordinate distance from infinity $y = \pi/2 - x$
the leading order behavior of ϕ for $3+1$ dimensions is y^0 or y^3

$$\begin{aligned}
\phi(t,y) &= \phi_0(t) - \frac{1}{2}\ddot{\phi}_0(t)y^2 + \phi_3(t)y^3 + \ldots \\
A(t,y) &= 1 - \left(\dot{\phi}_0(t)\right)^2 y^2 + A_3(t)y^3 + \ldots \\
\delta(t,y) &= \frac{1}{2}\left(\dot{\phi}_0(t)\right)^2 y^2 + \ldots
\end{aligned}$$

Schwarzschild-AdS : $A = 1 + my^3 + \ldots$
The mass is finite only if $\phi_0(t)$ is constant
– can be shifted to zero

Asymptotically AdS case

From $\phi_0(t) = 0$ it follows that $A_3(t) \equiv m$ constant

$$\begin{aligned}
\phi(t,y) &= \phi_3(t)y^3 + \phi_5(t)y^5 - \frac{m}{2}\phi_3(t)y^6 + \phi_7(t)y^7 + \ldots \\
A(t,y) &= 1 + my^3 + 3\left(\phi_3(t)\right)^2 y^6 + \ldots \\
\delta(t,y) &= \frac{3}{2}\left(\phi_3(t)\right)^2 y^6 + \ldots
\end{aligned}$$

ϕ_{2k+1} and their first time derivatives can be freely specified on the initial time slice $t = 0$

however, ϕ_{2k} are determined

– corner conditions on the initial data

Any single mode initial data have only odd coefficients

– corresponds to small amplitude zero mass configuration

– what happens when you numerically evolve such initial data?

Small-amplitude expansion

The scalar field and the metric functions are expanded in powers of a small parameter ε

$$\phi = \sum_{\substack{n=1\\ odd}}^{\infty} \phi^{(n)} \varepsilon^n \ , \quad A = 1 + \sum_{\substack{n=2\\ even}}^{\infty} A^{(n)} \varepsilon^n \ , \quad \delta = \sum_{\substack{n=2\\ even}}^{\infty} \delta^{(n)} \varepsilon^n$$

ε is chosen as the central amplitude of ϕ at the moment of $t \to -t$ time reversal symmetry

- for general d spatial dimensions we can give analytic expressions for $A^{(2)}$ and $\delta^{(2)}$
- for $d = 3$ we can also give $\phi^{(3)}$
- for even d it is possible to go up to ε^{20} order by some algebraic manipulation program
  - then all coefficients can be given as a sum of *finite number* of eigenfunctions of the linearized problem

Linear order

To first order in ε the metric remains AdS, and there are periodic solutions for the scalar field

$$\phi^{(1)} = p_n \cos(\omega_n t) \quad , \qquad n \geq 0 \text{ integer}$$

$$p_n = \frac{n!}{(d/2)_n} \cos^d x \; P_n^{(d/2-1,d/2)}(\cos(2x))$$

$P_n^{(\alpha,\beta)}$ is the Jacobi polynomial,
$(\alpha)_n = \alpha(\alpha+1)\ldots(\alpha+n-1)$ is the Pochhammer symbol

$$\text{frequency: } \omega_n = d + 2n$$

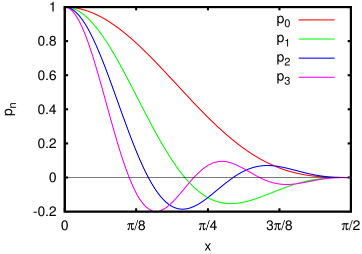
where d is the number of spatial dimensions

$\omega_n = d + 2n$

all frequencies are integers, so it is a fully resonant spectrum

$\rightsquigarrow$ turbulent instability $\rightsquigarrow$ black hole formation

For $d = 3$ spatial dimensions the p_n functions are



$$p_0 = \cos^3 x$$

$$p_1 = \frac{\cos^3 x}{3} \left[4\cos(2x) - 1\right]$$

$$p_2 = \frac{\cos^3 x}{3} \left[3\cos(4x) - 2\cos(2x) + 2\right]$$

$$p_3 = \frac{\cos^3 x}{15} \left[12\cos(6x) - 9\cos(4x) + 18\cos(2x) - 5\right]$$

n gives the number of nodes

Combination with arbitrary amplitudes and phases is a valid periodic solution of the linearized problem

$$\phi^{(1)} = \sum_{n=0}^{\infty} a_n \cos(\omega_n t + b_n) p_n \quad , \quad \omega_n = d + 2n$$

but to ε^3 order, there are $t \sin(\omega t)$ secular terms in $\phi^{(3)}$ if more then one a_n is nonzero

There is a one-parameter family of solutions emerging from each p_n linearized mode

We investigate the family emerging from the nodeless solution

Initial guess for numerical iteration: linearized solution $p_0 \cos(3t)$

KADATH library developed by Philippe Grandclément at Observatoire de Paris - Meudon

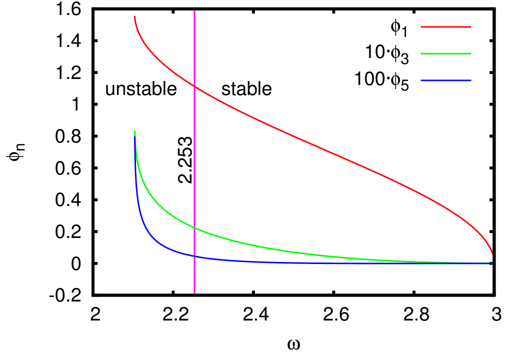
– multidomain spectral method

– radial direction: Chebyshev polynomials

– time direction: Fourier decomposition

$$\phi = \sum_{\substack{k=1 \\ odd}}^{\infty} \phi_k \cos(k\omega t) \ , \quad A = \sum_{\substack{k=0 \\ even}}^{\infty} A_k \cos(k\omega t) \ , \quad \delta = \sum_{\substack{k=0 \\ even}}^{\infty} \delta_k \cos(k\omega t)$$

Central value of the Fourier modes as function of oscillation frequency



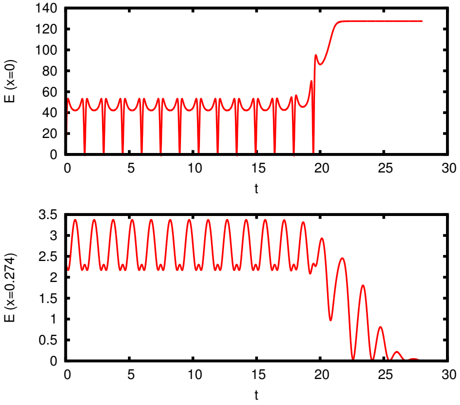
Using the solution as initial data for a time-evolution code:

AdS breathers with frequency $\omega < 2.253$ are unstable

Energy density as function of time for an unstable configuration

– at the center

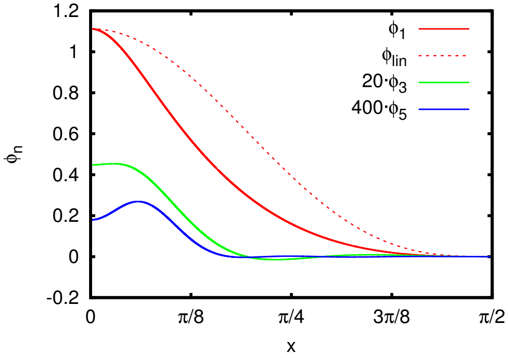
– at a radius larger than the one where horizon appears



energy density starts to increase, but time coordinate stalls before horizon

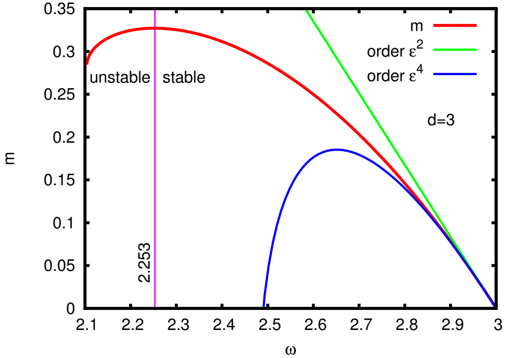
scalar field falls into the black hole

Radial profile for the first three modes of the scalar field for the largest mass stable AdS breather



– more compact than the linear solution, but similar shape

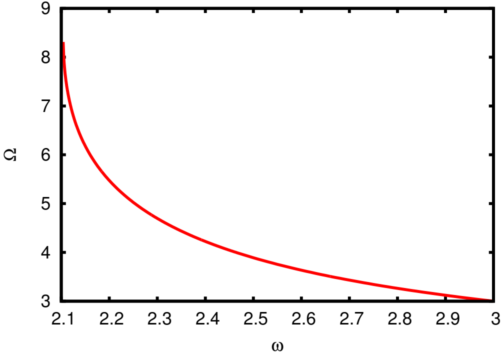
Mass as function of the oscillation frequency



AdS breather becomes unstable when the total mass starts to decrease with increasing central density

First two orders of the small-amplitude expansion is also plotted – in order to get ε^4 order results one has to calculate to ε^6 order

Central frequency Ω as function of asymptotic frequency ω

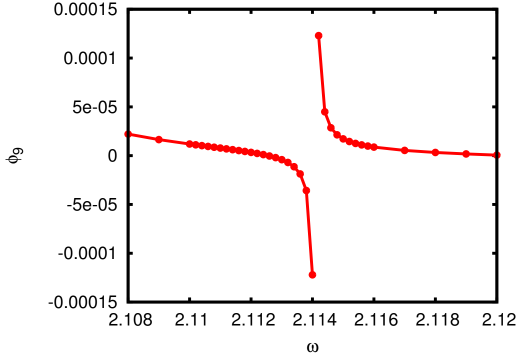


As the amplitude grows

the frequency observed outside the breather decreases

the frequency measured by a central observer grows

There are narrow resonance-like structures appearing in higher Fourier modes



– the increase is most apparent in one of the modes
– for $d = 4$ a similar peak was found in ϕ_5
– they are in the unstable domain

Concluding remarks

Periodic solutions, up to a certain amplitude, are on
"stability islands"
– general configurations collapse into black holes

AdS/CFT correspondence
– periodic solutions correspond to states that never thermalize

There are other asymptotically AdS localized regular configurations

- static axially symmetric electromagnetic states
  Herdeiro and Radu, *Phys. Lett. B* **749**, 393 (2015)
- vacuum gravitational wave geons
  Dias, Horowitz and Santos, *CQG* **29**, 194002 (2012)
  – helical symmetry