Linear dilaton for asymptotically Lifshitz-like spacetimes

Anastasia Golubtsova[1]

based on a collabotation
with Irina Ia. Aref'eva and Eric Gourgoulhon

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Outline

1 Motivation

2 Asymptotycally Lifshitz backgrouds

3 Linear dilaton

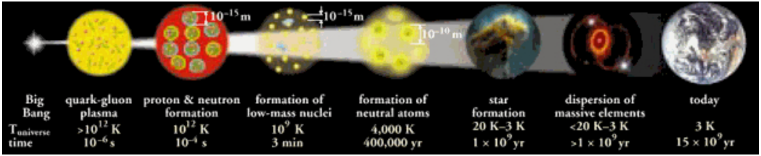
4 Out of equillibrium

5 Summary and Outlook

Motivation

Strongly coupled systems

- Ultra-cold atoms
- High temperature conductors
- Quantum liquids
- QUARK-GLUON PLASMA
- THE BIG BANG



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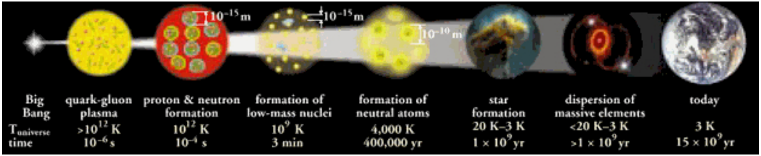

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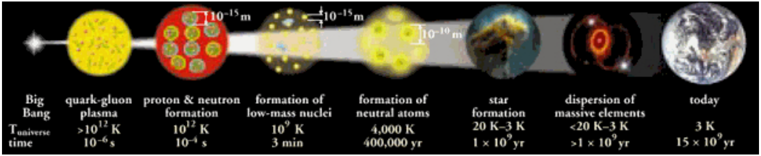

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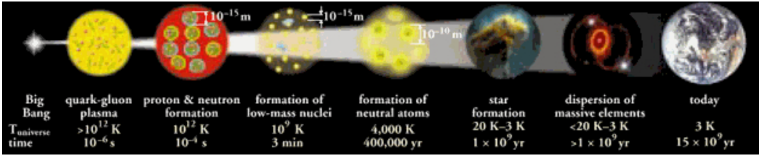
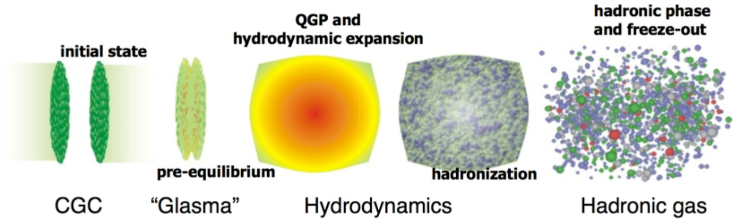


The quark-gluon plasma (2005)

Experiments on Heavy Ion Collisions at **RHIC** and **LHC**:

- A new state of matter: deconfined quarks, antiquarks, and gluons at high temperature.
- QGP does not behave like a weakly coupled gas of quarks and gluons, but a strongly coupled fluid.
- $\tau_{therm}(0.1fm/c) < \tau_{hydro} < \tau_{hard}(10fm/c) < \tau_f(20fm/c)$



Difficulties and solution

- Quantum field theories with large coupling constant: long distances, strong forces
- Perturbative methods are inapplicable
- No consistent quantum field theory at strong coupling

SOLUTION ?

GAUGE/GRAVITY DUALITY

A correspondence between the gauge theory in D Minkowski spacetime and supergravity in $(D+1)$ AAdS

't Hooft' 93, Susskind'94.

Example: The AdS/CFT correspondence

J.M. Maldacena, Adv.Theor.Math.Phys. 2, (1998).

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Holographic dictionary

- d gravity on AdS $=$ the $(d-1)$ strongly coupled theory
- $T=0$: AdS vacuum, $T \neq 0$: black-hole solutions in AdS.
- $4d$ Multiplicity in HIC $=$ BH entropy in AdS_5 Gubster et al.'08
- Thermalization time in $\mathcal{M}^{1,3}$ $=$ BH formation time in AdS^5
- Non-local observables: Wilson loops, Entarglement entropy, two point correlators.

PROFIT?

•Calculations in gravitational backgrounds with certain asymptotics

• Reduciton to classical mechanics

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Multiplicity: experimental data, theoretical estimation

- $D = 4$ **Multiplicity = Area of trapped surface in** $D = 5$

Experiment:
$S_{data} = \mathbf{s}_{NN}^{0.15}$
ALICE collaboration'10

Modified AdS:
$S_{data} = \mathbf{s}_{NN}^{0.12}$
Kiritis & Taliotis'11

Modified AdS+ ghosts:
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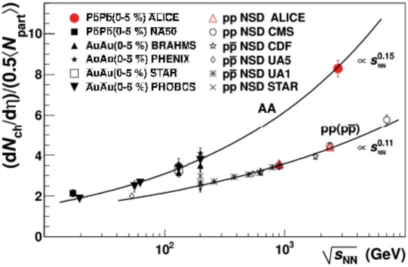

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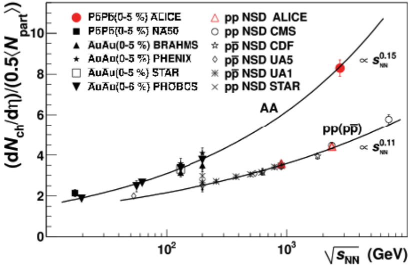

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Asymptotycally Lifshitz-like backgrouds and holography

Outline

1. **Spacetimes with Lifshitz scaling**
2. **Lifshitz-like backgrounds for holography**
   1. Lifshitz-like metrics
   2. Shock waves in Lishitz spacetimes
3. **Lifshitz-like backgrounds with spherical symmetry**
   1. Lifshitz black holes
   2. Lifshitz-Vadya solutions
   3. Boson stars in Lifshitz-like backgrounds

Lifshitz scaling

The AdS/CFT correspondence:

The Field Theory	The Gravitational Background
• the conformal group $SO(D,2)$	• the group of isometries
of a D-dimensional CFT	of AdS_{D+1}
$(t, x_i) \to (\lambda t, \lambda x_i)$, $i = 1, .., d-1$	$ds^2 = r^2\left(-dt^2 + d\vec{x}^2_{d-1}\right) + \frac{dr^2}{r^2}$

Generalizations?

Lifshitz scaling: $t \to \lambda^{\nu} t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r,$

where ν is the Lifshitz dynamical exponent

Lifshitz metric: $ds^2 = -r^{2\nu} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}^2_{d-1}$

Kachru, Liu, Millgan '08

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Lifshitz-like spacetimes for holography

- Lifshitz-like metrics

$$ds^2 = r^{2\nu}\left(-dt^2 + dx^2\right) + r^2 dy_1^2 + r^2 dy_2^2 + \frac{dr^2}{r^2},$$

$(t, x, y, r) \to (\lambda^\nu t, \lambda^\nu x, \lambda y_1, \lambda y_2, \frac{r}{\lambda})$, M. Taylor'08, Pal'09.
Gauge/gravity duality: theory with $T = 0$.

- Shock-waves in Lifshitz-spacetimes

$$ds^2 = \frac{\phi(y_1, y_2, z)\delta(u)}{z^2} du^2 - \frac{1}{z^2} du dv + \frac{1}{z^{2/\nu}}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{z^2},$$

$u = t - x$ and $v = t + x$, $z = 1/r^\nu$, I.Ya.Aref'eva, AG'14.
Gauge/gravity duality:
Multiplicity in HIC in $D = 4$ can be estimated by **the area of trapped surface** in AdS_5 formed in collision of shock waves.

Gubster, Pufu, Yarom'09

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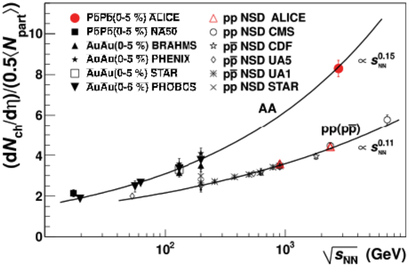

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5d Lifshitz spacetimes $\nu = 4$, $\nu = -2$
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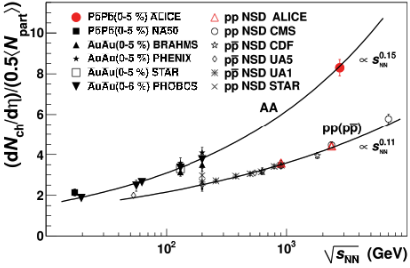

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Linear dilaton and asymptotycally Lifshitz-like metric

Possible models

A massive form field

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left(R + \Lambda - \frac{1}{6}\left(H_{(3)}^2 + m_0^2 B_{(2)}^2\right)\right),$$

with $H_{(3)} = dB_{(2)}$, Λ is negative cosmological constant

$$B_{(2)} = \sqrt{\frac{\nu-1}{\nu}}L^2 r^{2\nu} dt \wedge dx, \quad H_{(3)} = 2\nu\sqrt{\frac{\nu-1}{\nu}}L^2 r^{2\nu-1} dr \wedge dt \wedge dx$$

$$m_0 = \frac{\nu}{L^2}, \quad c^2 = \frac{(\nu+1)\nu}{16L^2}, \quad \Lambda = -\frac{4\nu^2+\nu+1}{2L^2}.$$

M. Taylor'08

- Lifshitz-metrics
- Shock waves and its collision
- No analytic black hole solutions

Lifshitz black holes, Lifshitz-Vaidya, etc.

- Black holes in Lifshitz background

$$ds^2 = r^{2\nu}\left(-f(r)dt^2 + dx^2\right) + r^2\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{r^2 f(r)},$$

where $f(r) = 1 - \frac{m}{r^{2\nu+2}}.$

Gravity/gauge duality: $T \neq 0$, non-local observables in equillibrium.

- Lifshitz-Vaidya metrics, a shell falling along $v = 0$.

$$ds^2 = -\frac{f(v,z)}{z^2}dv^2 - \frac{2dvdz}{z^2} + \frac{dx^2}{z^2} + \frac{(dy_1^2 + dy_3^2)}{z^{2/\nu}},$$

$f = 1 - m(v)z^{2\nu+2}, \quad m(v)$defines the thickness of the shell.

Gravity/gauge duality: non-local observables out of equillibrium.

- Boson stars in Lifshitz background

Gravity/gauge duality: condensed matter S.A Hartnoll'11

Linear dilaton

Linear dilaton field and 2-form fields

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left(R[g] + \Lambda - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{4}e^{\lambda\phi}F^2_{(2)}\right).$$

The Einstein equations are

$$R_{mn} = -\frac{\Lambda}{3}g_{mn} + \frac{1}{2}(\partial_m\phi)(\partial_n\phi) + \frac{1}{4}e^{\lambda\phi}\left(2F_{mp}F^p_n\right) - \frac{1}{12}e^{\lambda\phi}F^2 g_{mn}.$$

The scalar field equation

$$\Box\phi = \frac{1}{4}\lambda e^{\lambda\phi}F^2, \quad \text{with} \quad \Box\phi = \frac{1}{\sqrt{|g|}}\partial_m(g^{mn}\sqrt{|g|}\partial_n\phi).$$

The gauge field obeys the following equation

$$D_m\left(e^{\lambda\phi}F^{mn}\right) = 0.$$

Black hole (brane) solutions

$$ds^2 = e^{2\nu r}\left(-f(r)dt^2 + dx^2\right) + e^{2r}\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{f(r)},$$

where $f(r) = 1 - me^{-(2\nu+2)r}$. Aref'eva,AG, Gourgoulhon'15

$$F_{(2)} = \frac{1}{2}q dy_1 \wedge dy_2, \quad \phi = \phi(r), \quad e^{\lambda\phi} = \mu e^{4r}.$$

$\nu = 4$, $\lambda = \pm\frac{2}{\sqrt{3}}$, $\Lambda = 90$, $\mu q^2 = 240$.

SUGRA IIA on $M = X_{(1)5} \times X_{(2)5}$,: $F_{(2)}, F_{(4)}, H_{(3)}$.

with

$$F_{(2)} = \frac{1}{2}q dy_1 \wedge dy_2, \quad F_{(4)} \sim \text{const}, \quad H_3 = 0,$$

Azeyanagi et al. '09

5d GAUGED $U(1)^3$ SUGRA $\Rightarrow$ SUGRA IIB

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From Lifshitz to AdS asymptotycs

- Let $\phi = const$ and $F_2 = 0$

Black hole solutions with AdS asymptotics

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Corresponds to the UV limit.

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Out of equillibrium

Construction of Lifshitz-Vaidya spacetimes

The Lifshitz-like metric $z = \frac{1}{r^{\nu}}$

$$ds^2 = z^{-2}\left(-f(z)dt^2 + dx^2\right) + z^{-2/\nu}(dy_1^2 + dy_2^2) + \frac{dz^2}{z^2 f(z)},$$
$$f = 1 - mz^{2/\nu+2}.$$

The Eddington-Finkelstein coordinates:

$$dv = dt + \frac{dz}{f(z)}.$$

A matter shell infalling in Lifshitz background

$$ds^2 = -z^{-2}f(z)dv^2 - 2z^{-2}dvdz + z^{-2}dx^2 + z^{-2/\nu}(dy_1^2 + dy_2^2),$$
$$f = 1 - m(v)z^{2/\nu+2}, v < 0 - \text{inside the shell}, v > 0 - \text{outside},$$
$$F_2 = \tfrac{1}{2}q dy_1 \wedge dy_2, \quad \lambda\phi = 4r + r_0.$$

Thermalization

Def.

Thermalization time at scale l is the time at which the tip of the geodesic with endpoints $(-l/2)$ and $(l/2)$ grazes the middle of the shell.

The Lagrangian of the pointlike probe

$$\mathcal{L} = \sqrt{-\frac{f(z,v)}{z^2}\frac{dv}{d\tau}\frac{dv}{d\tau} - \frac{2}{z^2}\frac{dv}{d\tau}\frac{dz}{d\tau} + \frac{1}{z^2}\frac{dx}{d\tau}\frac{dx}{d\tau} + \frac{1}{z^{2/\nu}}\left(\sum \frac{dy_i}{d\tau}\frac{dy_i}{d\tau}\right)}$$

$\tau = x$ or $\tau = y_i$, $i = 1, 2$.

Thermalization time

Let $\tau = x$, the Lagrangian $\mathcal{L} = \sqrt{\mathcal{R}}/z$

The integrals of motion

$$\mathcal{J} = -\frac{1}{z\sqrt{\mathcal{R}}} = -\frac{1}{z^2\mathcal{L}} \quad \mathcal{I}_1 = \frac{f(z)v_x^{'} + z_x^{'}}{z\sqrt{\mathcal{R}}},$$

$$\mathcal{I}_2 = \frac{z^{-2/\nu}y_{1,x}^{'}}{\sqrt{\mathcal{R}}}, \quad \mathcal{I}_3 = \frac{z^{-2/\nu}y_{2,x}^{'}}{\sqrt{\mathcal{R}}},$$

where $\mathcal{R} = 1 - f(z)(v_x^{'})^2 - 2v_x^{'}z_x^{'} + z^{2-2/\nu}((y_{1,x}^{'})^2 + (y_{2,x}^{'})^2)$.

$$z_x^{'} = \pm\sqrt{f(z)\left(\frac{1}{z^2\mathcal{J}^2} - z^{2/\nu}\left(\frac{\mathcal{I}_2^2}{\mathcal{J}^2} + \frac{\mathcal{I}_3^2}{\mathcal{J}^2}\right) - 1\right) + \frac{\mathcal{I}_1^2}{\mathcal{J}^2}},$$

$$x = \pm\int\frac{dz}{\sqrt{f(z)\left(\frac{1}{z^2\mathcal{J}^2} - z^{2/\nu}\left(\frac{\mathcal{I}_2^2}{\mathcal{J}^2} + \frac{\mathcal{I}_3^2}{\mathcal{J}^2}\right) - 1\right) + \frac{\mathcal{I}_1^2}{\mathcal{J}^2}}}.$$

Thermalization time

The turning point can be found from

$$f(z_*)\left(\frac{1}{z_*^2} - z_*^{2/\nu}\left(\mathcal{I}_2^2+\mathcal{I}_3^2\right) - \mathcal{J}^2\right) + \mathcal{I}_1^2 = 0.$$

$$x_{\mathcal{I}_1=\mathcal{I}_2=\mathcal{I}_3=0} = \pm\int_{\epsilon}^{z_*}\frac{dz}{\sqrt{f(z)\left(\frac{1}{z^2\frac{1}{z_*^2}}-1\right)}}.$$

The thermalization time t_{therm} at scale l

$$t_{therm} = \int_{\varepsilon}^{z_*}\frac{dz}{f(z)},\quad \ell = 2\int_{\epsilon}^{z_*}\frac{dz}{\sqrt{f(z)\left(\frac{1}{z^2\frac{1}{z_*^2}}-1\right)}}.$$

Thermalization in the x-direction

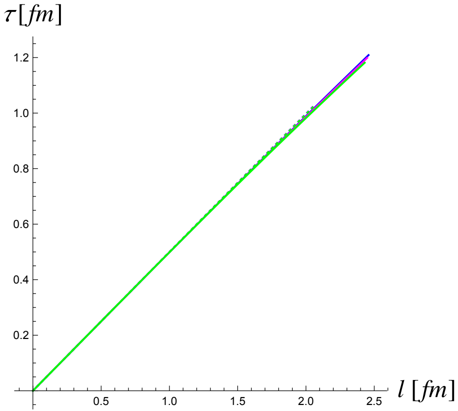

Figure : Dependencies of τ on ℓ for the 5-dimensional Lifshitz metric for $\nu = 2$, $\nu = 3$, $\nu = 4$ with $m = 0.1$ and $m = 0, 5$.

$\tau = y_i$

$$\mathcal{L} = \frac{\sqrt{\mathcal{R}}}{z}, \quad \mathcal{R} = 2z^{2-2/\nu} - f(z)(\dot{v}_y)^2 - 2\dot{v}_y\dot{z}_y + (\dot{x})^2.$$

The integrals of motion read

$$\mathcal{J} = -\frac{2z^{1-2/\nu}}{\sqrt{\mathcal{R}}} = -\frac{2}{z^{2/\nu}\mathcal{L}}, \quad \mathcal{I}_1 = \frac{f(z)\dot{v}_y + \dot{z}_y}{z\sqrt{\mathcal{R}}}, \quad \mathcal{I}_2 = -\frac{\dot{x}_y}{z\sqrt{\mathcal{R}}} \tag{5.1}$$

$$\ell = 2\int_{\epsilon}^{z_*} \frac{dz}{\sqrt{2z^{2-2/\nu}f(z)\left(\frac{z_*^{2/\nu}}{z^{2/\nu}} - 1\right)}}, \quad t_{therm} = \int_{\epsilon}^{z_*} \frac{dz}{f(r)}.$$

Thermalization in the y_i-direction

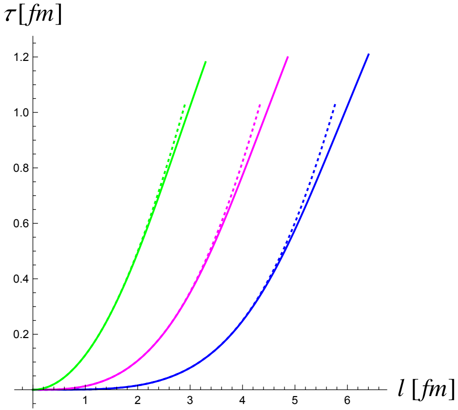

Figure : Dependencies of the thermalization times τ on ℓ for the Lifshitz metric for $\nu = 2$, $\nu = 3$ and $\nu = 4$ (left to right). The solid and dotted curves correspond to $m = 0.5$ and $m = 0.1$, respectively

Summary and Outlook

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2. Black brane and Vaidya solutions in Lifshitz-like backgrounds
3. Computation of thermalization time
4. Wilson loops and Entanglement entropy in black brane background

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1. Non-local observables in Lifshitz-Vaidya background
2. Lifshitz boson stars and interpretation condensed matter ?
3. The underlying theory ???
4. Interpolating solutions $Lif_5 \Rightarrow AdS_5$, $AdS_2/CFT_1, \quad 1d \quad CFT = SQM$.

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Thank you for your attention!