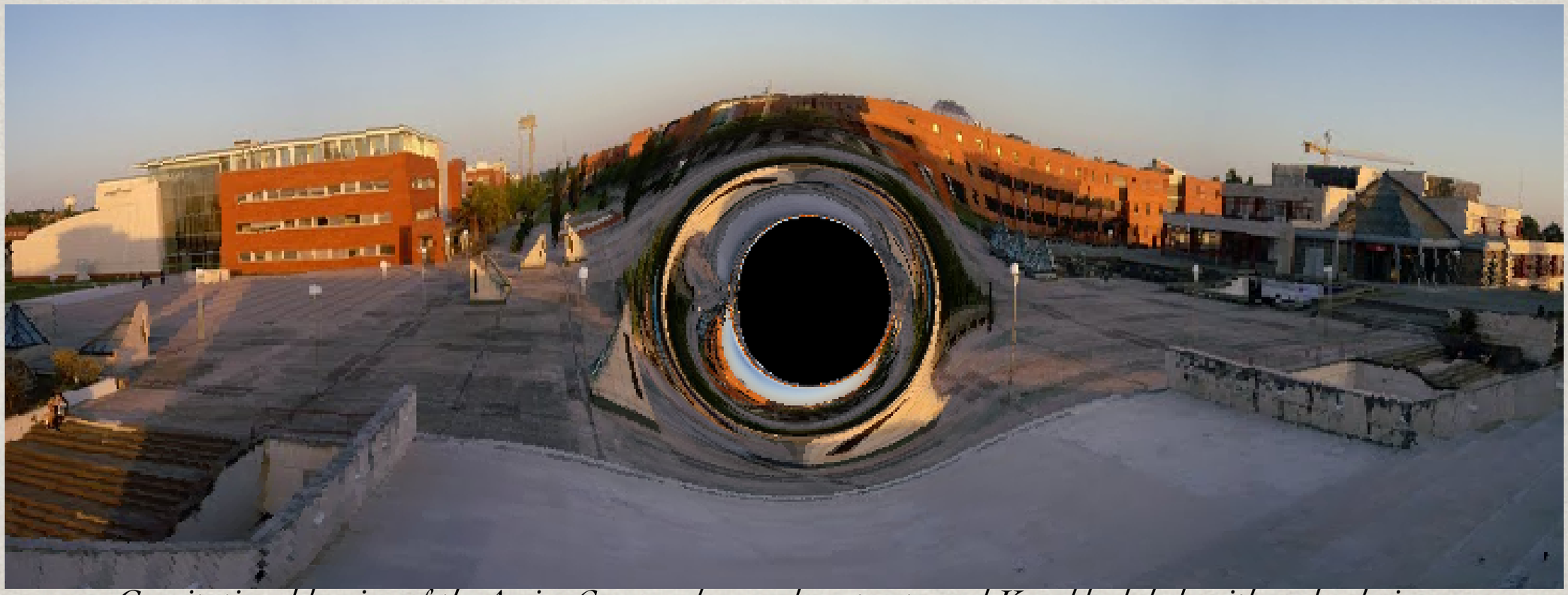
Kerr black holes with scalar hair:

theory and phenomenology

C. Herdeiro
Departamento de Física da Universidade de Aveiro, Portugal



Gravitational lensing of the Aveiro Campus by an almost extremal Kerr black hole with scalar hair

Gravitation and scalar fields
Meudon, Paris, October 6th 2015

based on
PRL112(2014)221101
IJMPD9(2015)1542014; CQG32(2015)14
arXiv:1509.00021

with E. Radu,
P. Cunha, H. Rúnarsson

Plan:

1) Introduction and message

2) Kerr black holes with scalar hair
   i) Boson Stars
   ii) Scalar clouds around Kerr black holes
   iii) Phenomenology: shadows

3) Outlook

1) Introduction and message

D=4, asymptotically flat, regular (on and outside the event horizon)

black hole (BH) solutions of Einstein's gravity

Vacuum:

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}R$$

Kerr Kerr 1963

Uniqueness Israel 1967; Carter 1970; Hawking 1972

No (independent-multipolar) hair

Electro-vacuum:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

Kerr-Newman Newman et al. 1965

Uniqueness Israel 1968; Robinson 1975, 1977

No (independent-multipolar) hair

Many no-scalar-hair theorems:

(only scalars, D=4, asymptotically flat)

C.H., Radu, 2015

Theory Lagrangian density $\mathcal{L}$	No-hair theorem	Known scalar hairy BHs with regular geometry on and outside $\mathcal{H}$ (primary or secondary hair; regularity)
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase[22]	
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein[11]	
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– –Sudarsky[61]	Herdeiro–Radu[136,137] (primary, regular); generalizations:[159]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– –Zannias[32] Zannias[33]	Bocharova–Bronnikov–Melnikov– –Bekenstein (BBMB)[16–18] (secondary, diverges at $\mathcal{H}$); generalizations:[87]
V-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler[46,47,50] Bekenstein[26] Sudarsky[51]	Many, with non-positive definite potentials:[71–75,78–80] (typically secondary, regular)
P-scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– –Jha[62]	
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa\lvert\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b\rvert^2$		Droz–Heusler–Straumann[126] (primary but topological; regular); generalizations:[129,131]
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$ Charmousis' and	Hawking[27] Saa[34,35] Sotiriou– –Faraoni[31]	
Sotiriou's talks Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– –Nicolis[45]	Sotiriou-Zhou[43] (secondary; regular) Babichev–Charmousis[88,90] (secondary[88] or primary,[90] diverges at $\mathcal{H}^+$ or $\mathcal{H}^-$);

Assumptions for an influential Bekenstein theorem:

Many no-scalar-hair theorems:

(only scalars, D=4, asymptotically flat)

Theory Lagrangian density $\mathcal{L}$	No-hair theorem	Known scalar hairy BHs with regular geometry on and outside $\mathcal{H}$ (primary or secondary hair; regularity)
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase[22]	
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein[11]	
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– –Sudarsky[61]	Herdeiro–Radu[136,137] (primary, regular); generalizations:[159]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– –Zannias[32] Zannias[33]	Bocharova–Bronnikov–Melnikov– –Bekenstein (BBMB)[16–18] (secondary, diverges at $\mathcal{H}$); generalizations:[87]
V-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler[46,47,50] Bekenstein[26] Sudarsky[51]	Many, with non-positive definite potentials:[71–75,78–80] (typically secondary, regular)
P-scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– –Jha[62]	
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa\lvert\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b\rvert^2$		Droz–Heusler–Straumann[126] (primary but topological; regular); generalizations:[129,131]
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$	Hawking[27] Saa[34,35] Sotiriou– –Faraoni[31]	
Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– –Nicolis[45]	Sotiriou-Zhou[43] (secondary; regular) Babichev–Charmousis[88,90] (secondary[88] or primary,[90] diverges at $\mathcal{H}^+$ or $\mathcal{H}^-$);

Assumptions for an influential Bekenstein theorem:

Assumption 1:
minimally coupled scalar

Assumption 2:
energy condition on V

Assumption 3:
scalar field inherits isometries

Violating assumption 3 in

Massive-complex-scalar-vacuum:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi\right)$$

$$-V(|\Phi|) = -\lambda\Phi^4$$

Runarsson's talk

There are BH solutions which are:

- asymptotically flat
- regular on and outside the horizon
- continuously connecting to the Kerr solution
- with an independent scalar charge (primary hair)

Kerr Black Holes with scalar hair

C.H. and Radu, PRL 2014

2) Kerr black holes with scalar hair

i) Boson stars

Boson stars:

Kaup (1968); Ruffini and Bonazzola (1969)
Review: Liebling and Palenzuela (2012)

Einstein-Klein-Gordon theory:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \Phi^*_{,a}\Phi^{,a} - \mu^2\Phi^*\Phi\right]$$

Rotating boson stars:
Yoshida and Eriguchi (1997)
Schunck and Mielke (1998)

$$ds^2 = -e^{2F_0(r,\theta)}dt^2 + e^{2F_1(r,\theta)}\left(dr^2 + r^2d\theta^2\right) + e^{2F_2(r,\theta)}r^2\sin^2\theta\left(d\varphi - W(r,\theta)dt\right)^2$$

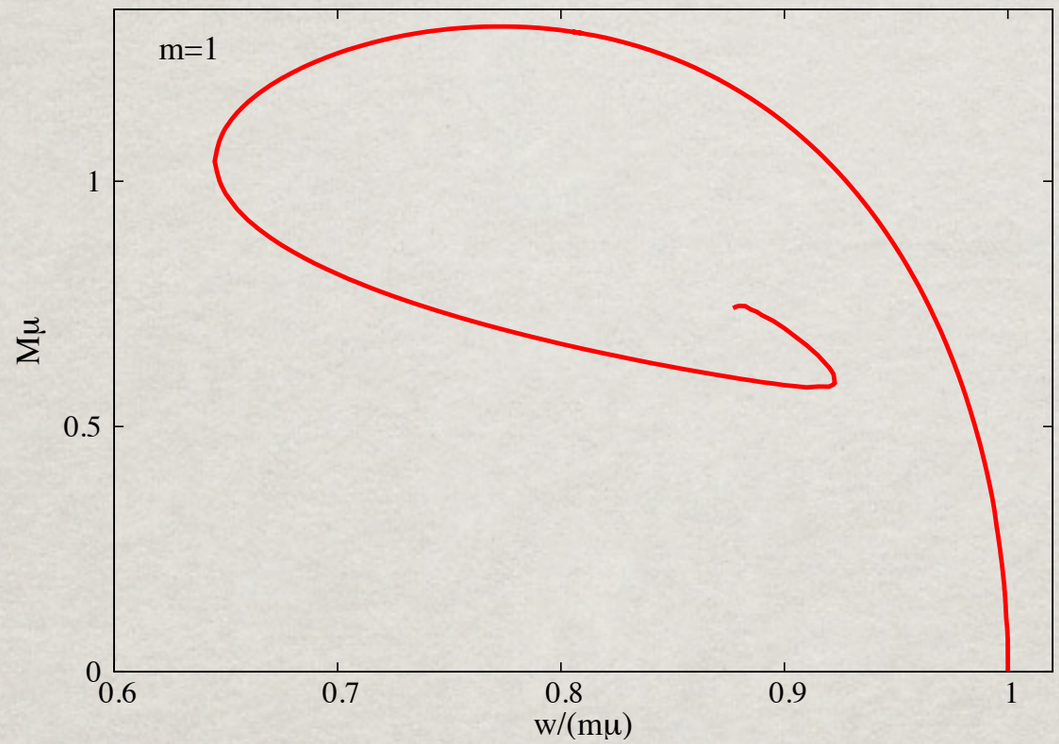
$$\Phi = \phi(r,\theta)e^{i(m\varphi - wt)}$$

Three input parameters: (w,m,n)

Solutions preserved by a single helicoidal Killing vector field:

$$\frac{\partial}{\partial t} + \frac{w}{m}\frac{\partial}{\partial\varphi}$$

Boson stars phase space (nodeless):



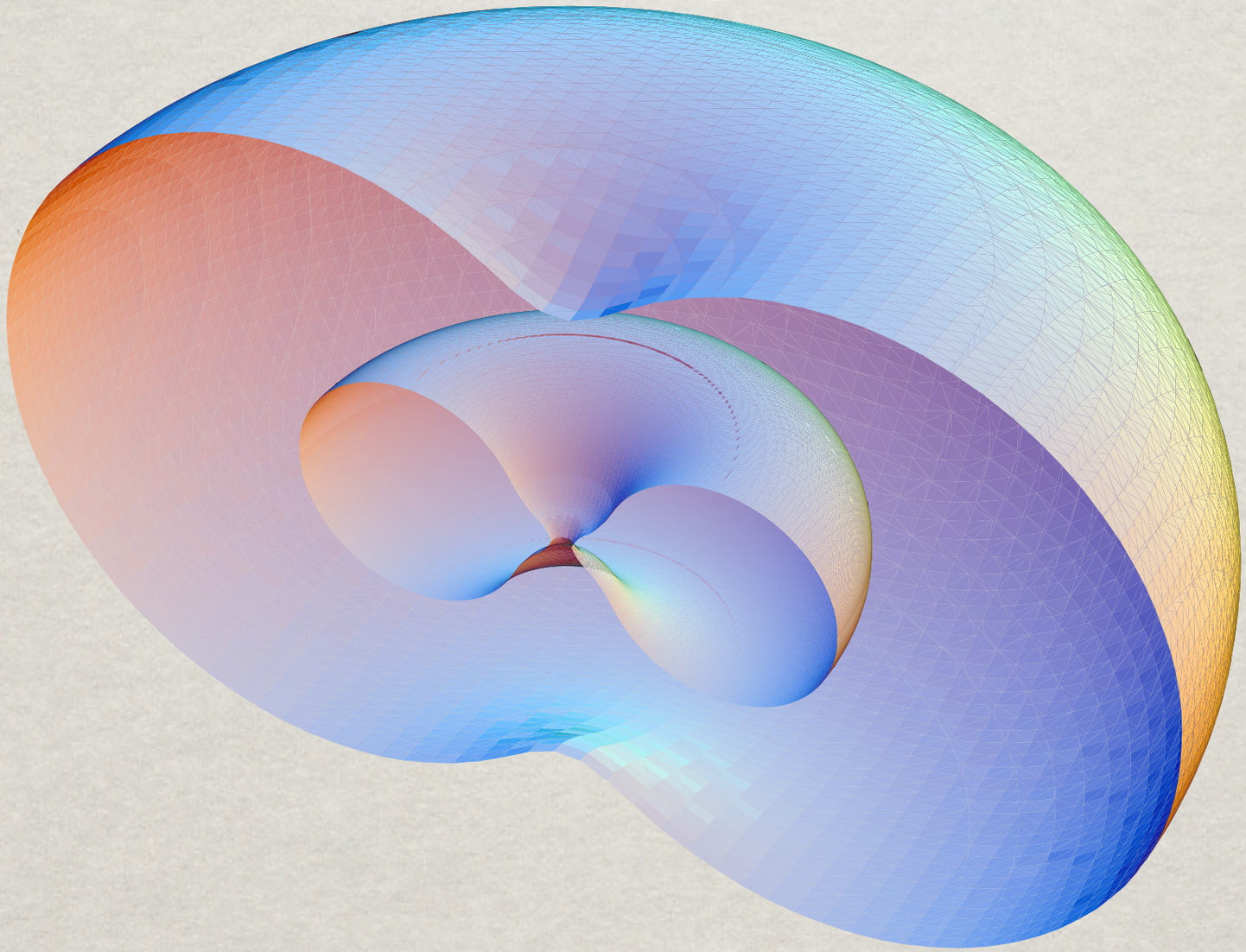
Conserved Noether charge: $$Q = \int_\Sigma dr d\theta d\varphi j^t \sqrt{-g}$$

For rotating boson stars: $$J = mQ$$

Schunck and Mielke (1998)

Convenient parameter: $$q \equiv \frac{mQ}{J}$$

Surfaces of constant scalar energy density



2) Kerr black holes with scalar hair

ii) Scalar clouds around Kerr black holes

Linear analysis: Klein-Gordon equation in Kerr

$$\Box\Phi = \mu^2\Phi \qquad\qquad \Phi = e^{-iwt}e^{im\varphi}S_{\ell m}(\theta)R_{\ell m}(r)$$

Radial Teukolsky equation: Teukolsky (1972); Brill et al. (1972)

$$\frac{d}{dr}\left(\Delta\frac{dR_{\ell m}}{dr}\right) = \left(a^2w^2 - 2maw + \mu^2r^2 + A_{\ell m} - \frac{K^2}{\Delta}\right)R_{\ell m}$$

$$\Delta \equiv r^2 - 2Mr + a^2$$

$$K \equiv (r^2 + a^2)w - am$$

Generically one obtains *quasi*-bound states:

$$\omega = \omega_R + i\omega_I$$

critical frequency

$$w_c = m\Omega_H$$

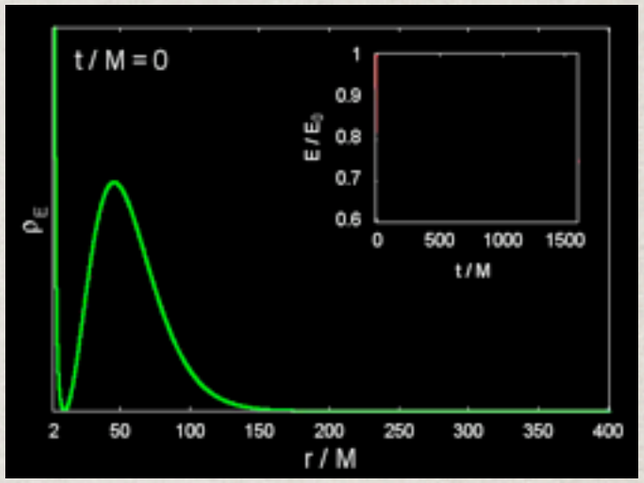
$$\omega = \omega_R + i\omega_I$$

critical frequency

$$w_c = m\Omega_H$$

$$w_I < 0 \quad \text{if} \quad w_R > w_c$$

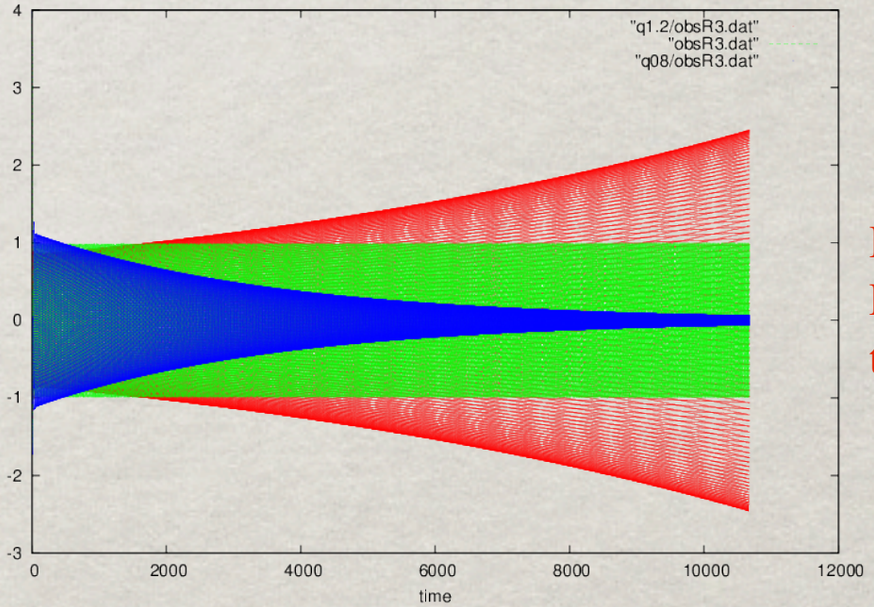
decay



Degollado et. al.,
PRL 109 (2012) 081102

$$w_I = 0 \quad \text{if} \quad w = w_c$$

true bound states: *stationary clouds*

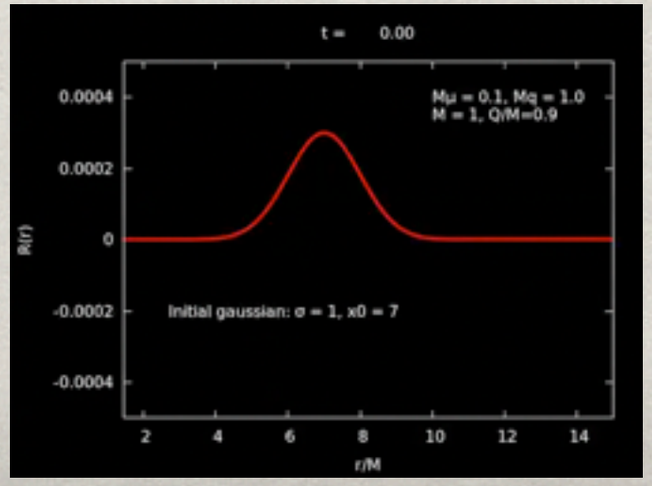


Degollado, CH, Runarsson, to appear

$$w_I > 0 \quad \text{if} \quad w_R < w_c$$

grow

Press and Teukolsky (1972)

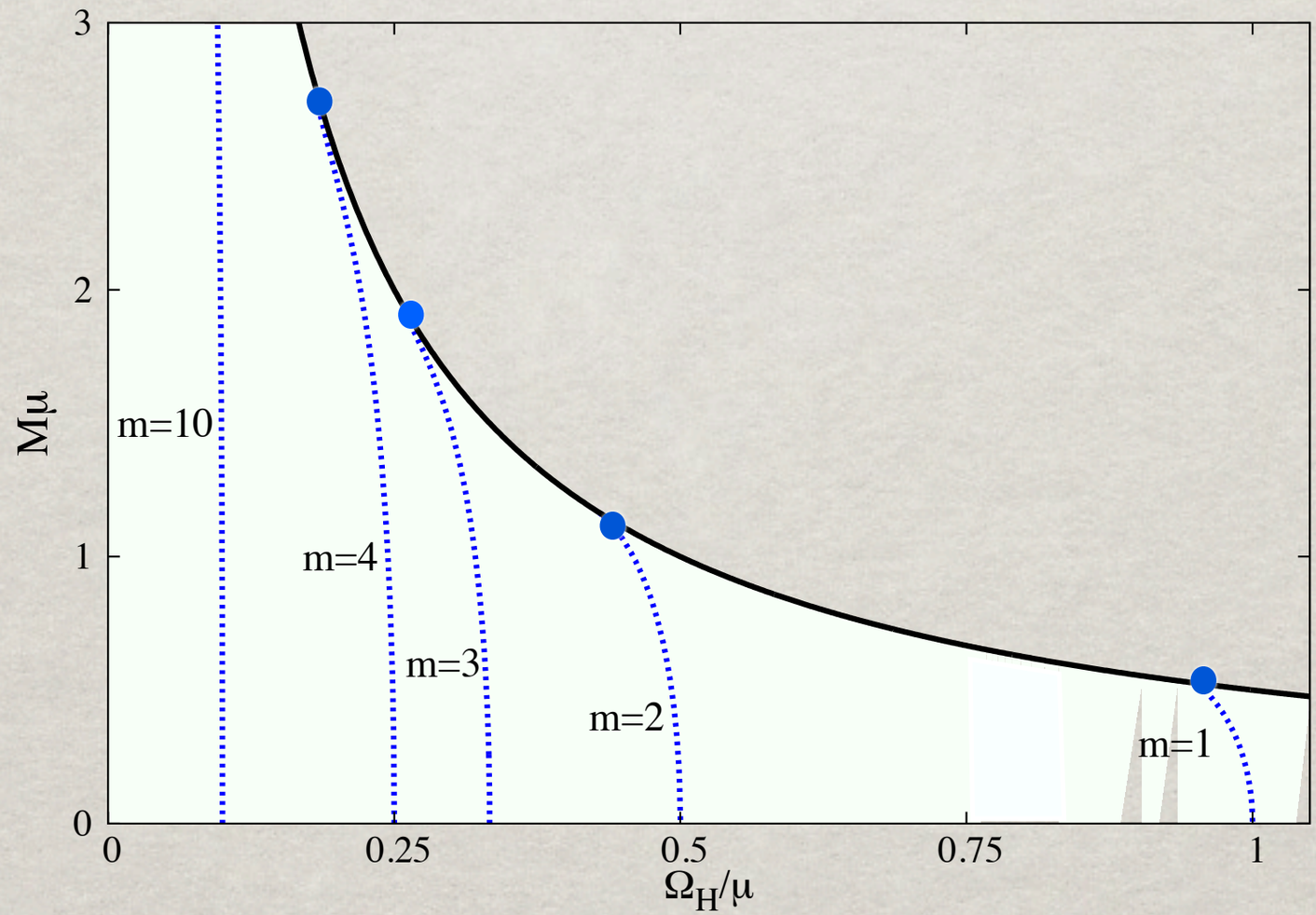


Degollado and C.H. 2014

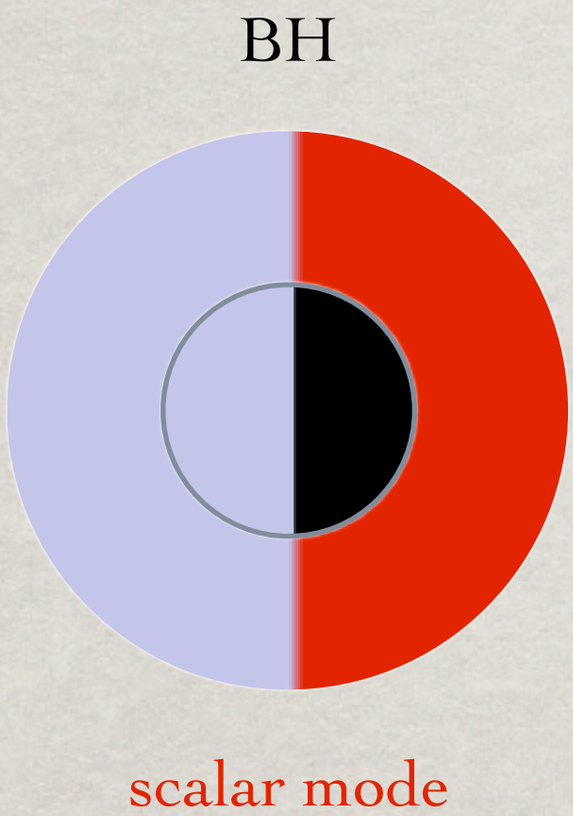
Klein-Gordon (linear) stationary clouds around Kerr:

Damour, Deruelle and Ruffini (1976); Zouros and Eardley (1979); Detweiler (1980); Hod 2012; (...); Yakov Shilapentokh-Rothman (2014)

Clouds for Kerr: discrete set labelled by (n,l,m) subject to one quantization condition which yields BH mass,spin. Hod (2012)

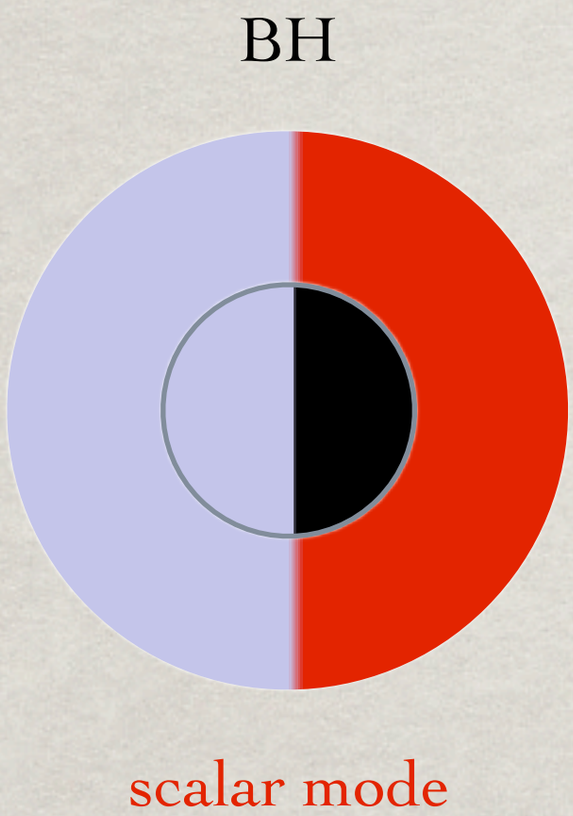


Stability: stationary clouds



$\frac{w}{m} = \Omega_H$ is a rotation synchronization condition

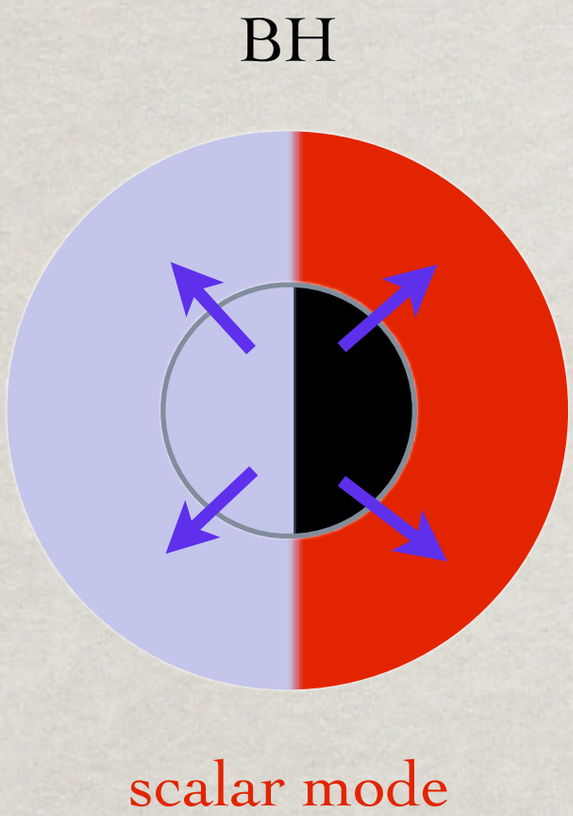
Stability: stationary clouds



$$\frac{w}{m} < \Omega_H$$

Superradiant regime
black hole decreases angular velocity

Stability: stationary clouds

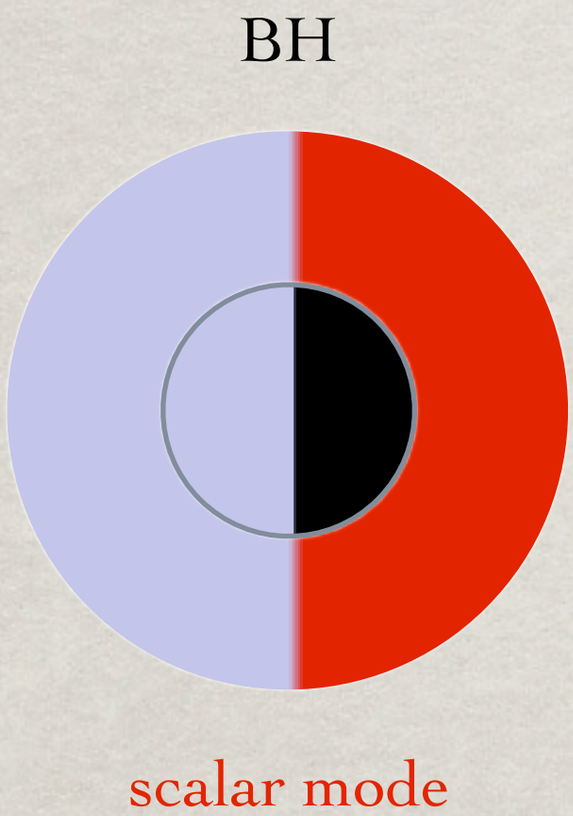


Transfer of rotational energy from BH to scalar cloud

$$\frac{w}{m} < \Omega_H$$

Superradiant regime
black hole decreases angular velocity

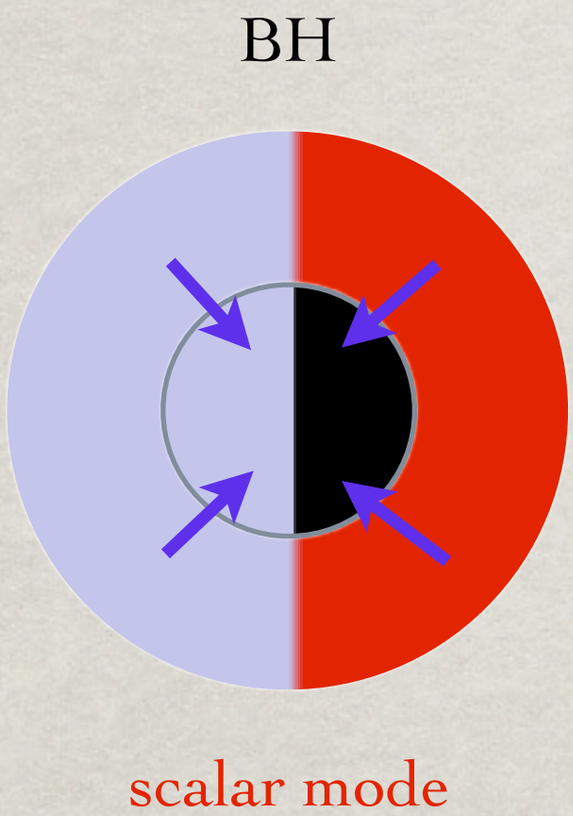
Stability: stationary clouds



$$\frac{w}{m} > \Omega_H$$

decaying regime
black hole increases angular velocity

Stability: stationary clouds

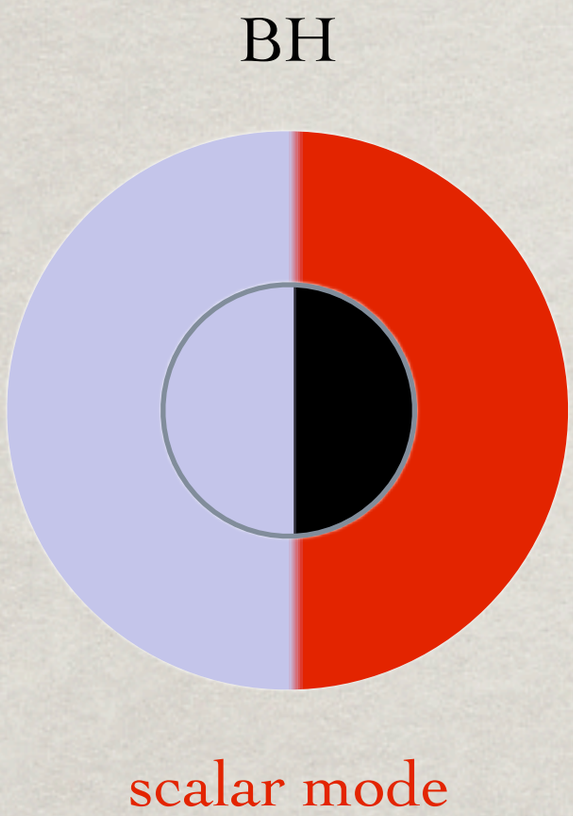


Transfer of rotational energy from scalar cloud to BH

$$\frac{w}{m} < \Omega_H$$

Superradiant regime
black hole decreases angular velocity

Stability: stationary clouds



Suggests:
clouds as dynamical attractors
Synchronization locking (cf. tidal locking for earth-moon)

Benone, Crispino, C.H. and Radu, PRD, 2014

Backreacting clouds
yield
Kerr black holes with scalar hair

Einstein Klein-Gordon: non-linear setup

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} N dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \, (d\varphi - W(r,\theta) dt)^2 \qquad N = 1 - \frac{r_H}{r}$$

$$\Phi = \phi(r,\theta) e^{i(m\varphi - wt)}$$

Single KVF BH c.f.
Dias, Horowitz and Santos (2011)

Asymptotically:

$$g_{tt} = -1 + \frac{2M}{r} + \ldots, \quad g_{\varphi t} = -\frac{2J}{r} \sin^2\theta + \ldots$$

$$\phi = f(\theta) \frac{e^{-\sqrt{\mu^2 - w^2} r}}{r} + \ldots$$

take: $w < \mu$

Near the horizon:

$$x \equiv \sqrt{r^2 - r_H^2}$$

$$F_i = F_i^{(0)}(\theta) + x^2 F_i^{(2)}(\theta) + \mathcal{O}(x^4)$$

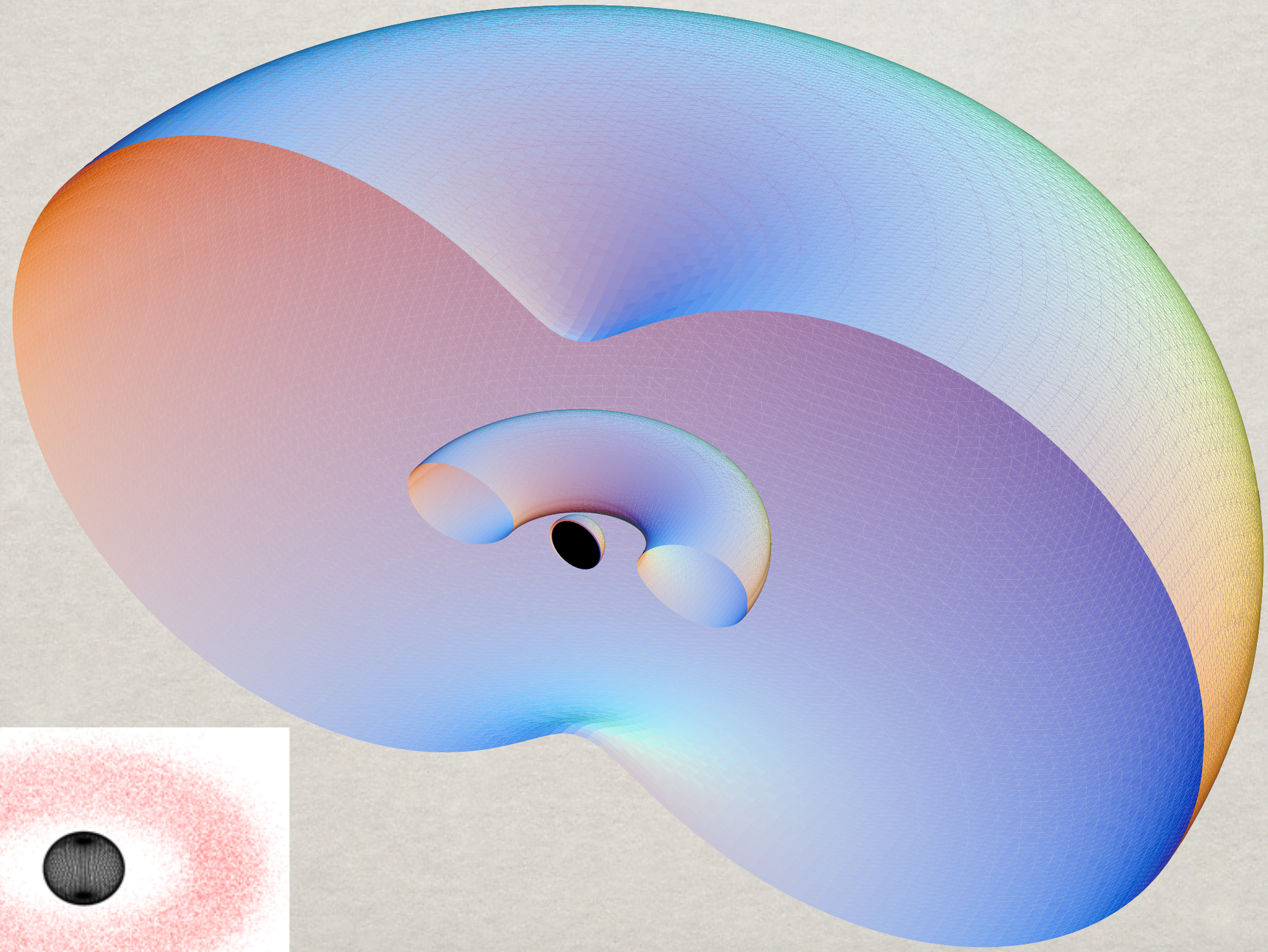
$$W = \Omega_H + \mathcal{O}(x^2)$$

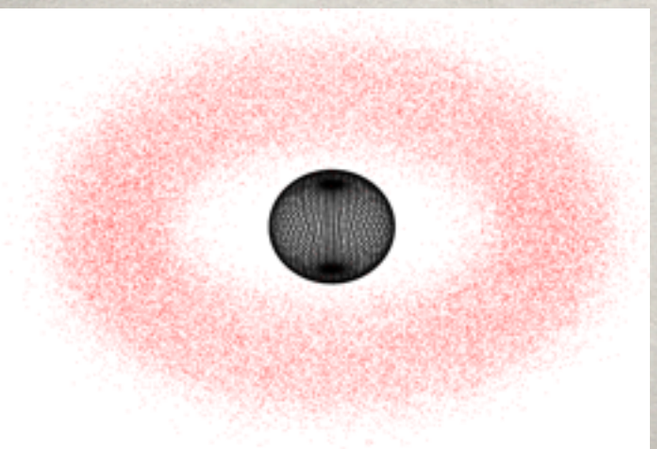
$$\phi = \phi_0(\theta) + \mathcal{O}(x^2)$$

take: $\Omega_H = \frac{w}{m}$

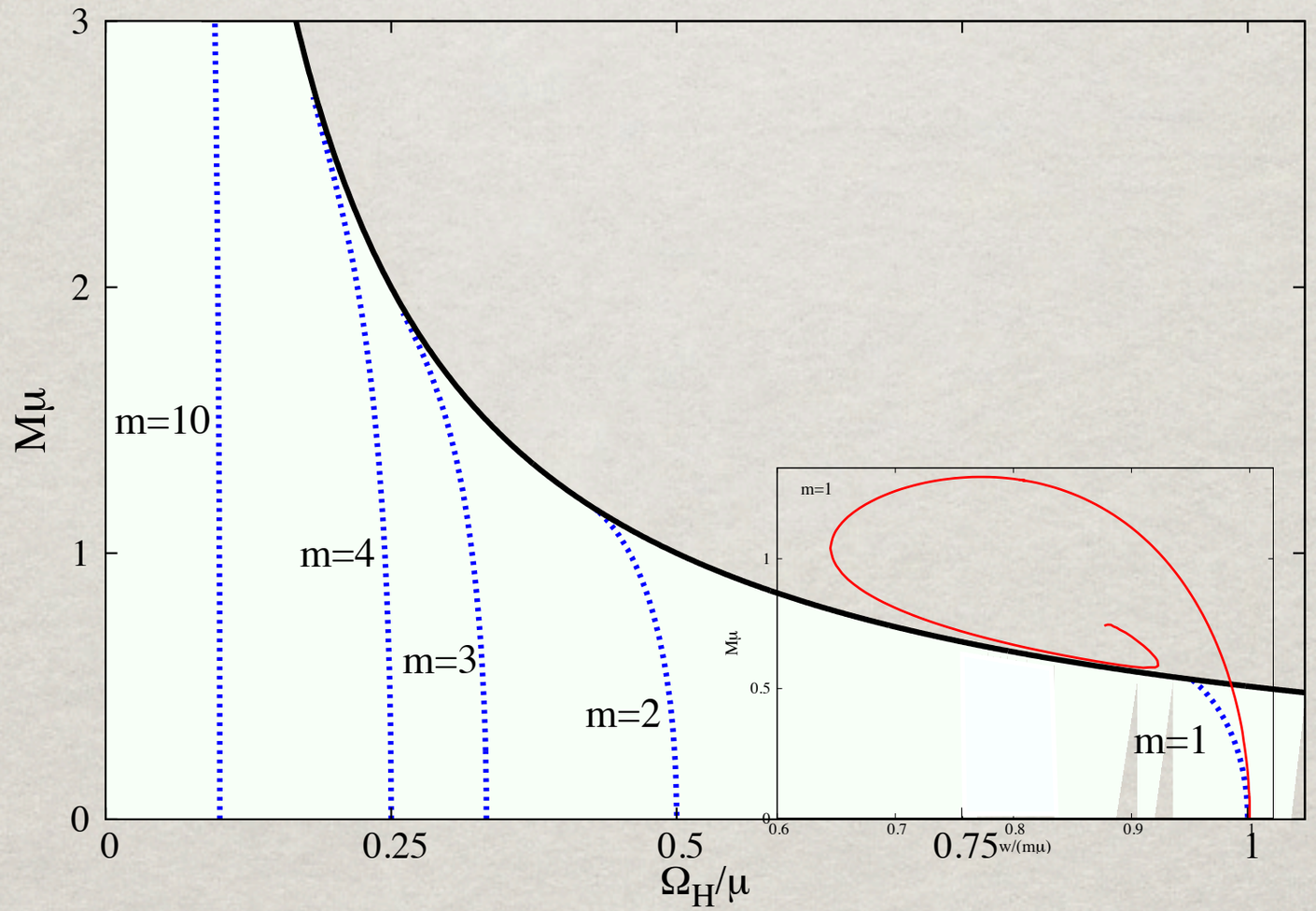
Four input parameters: m, w, r_H, n

Hairy black holes

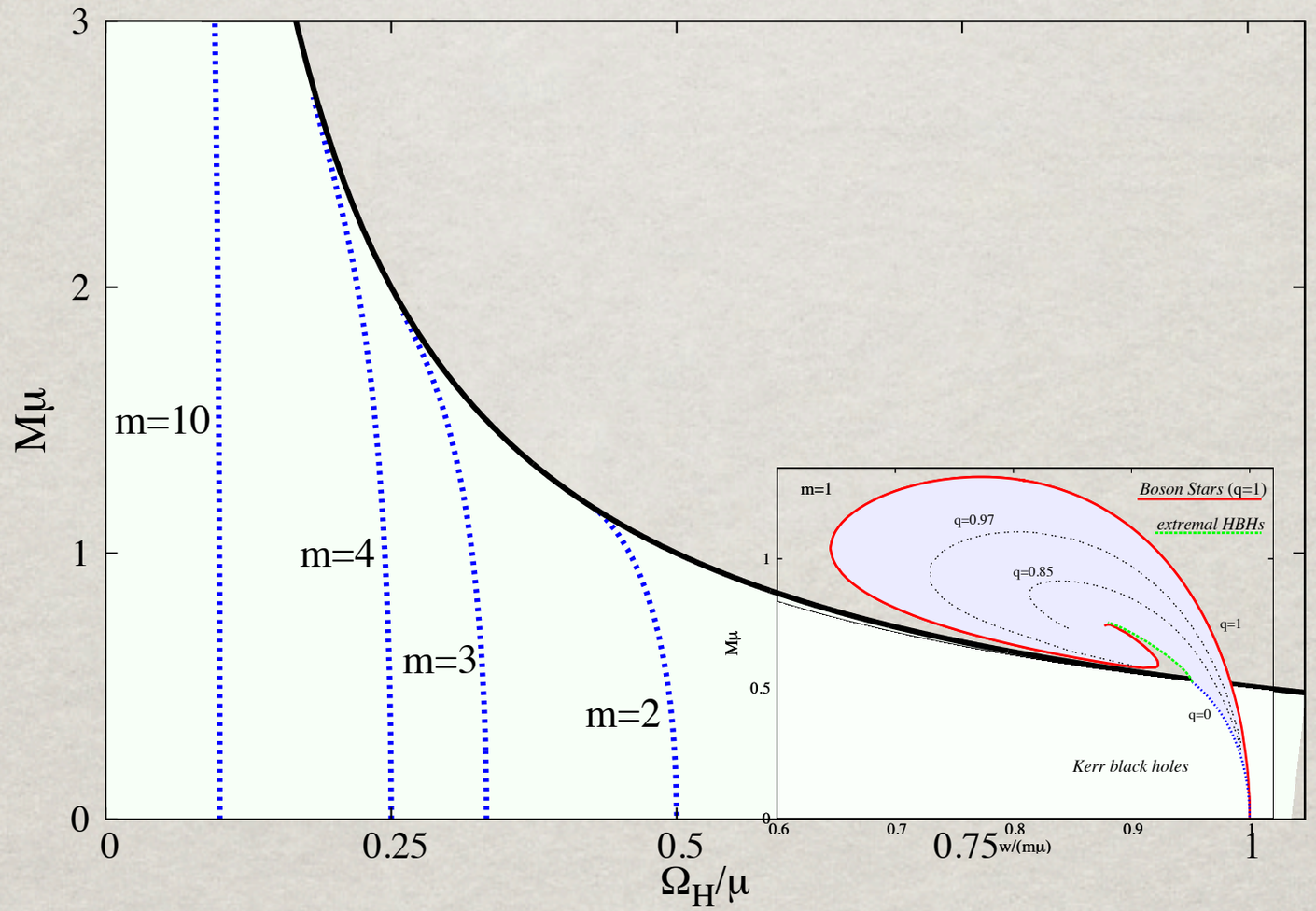




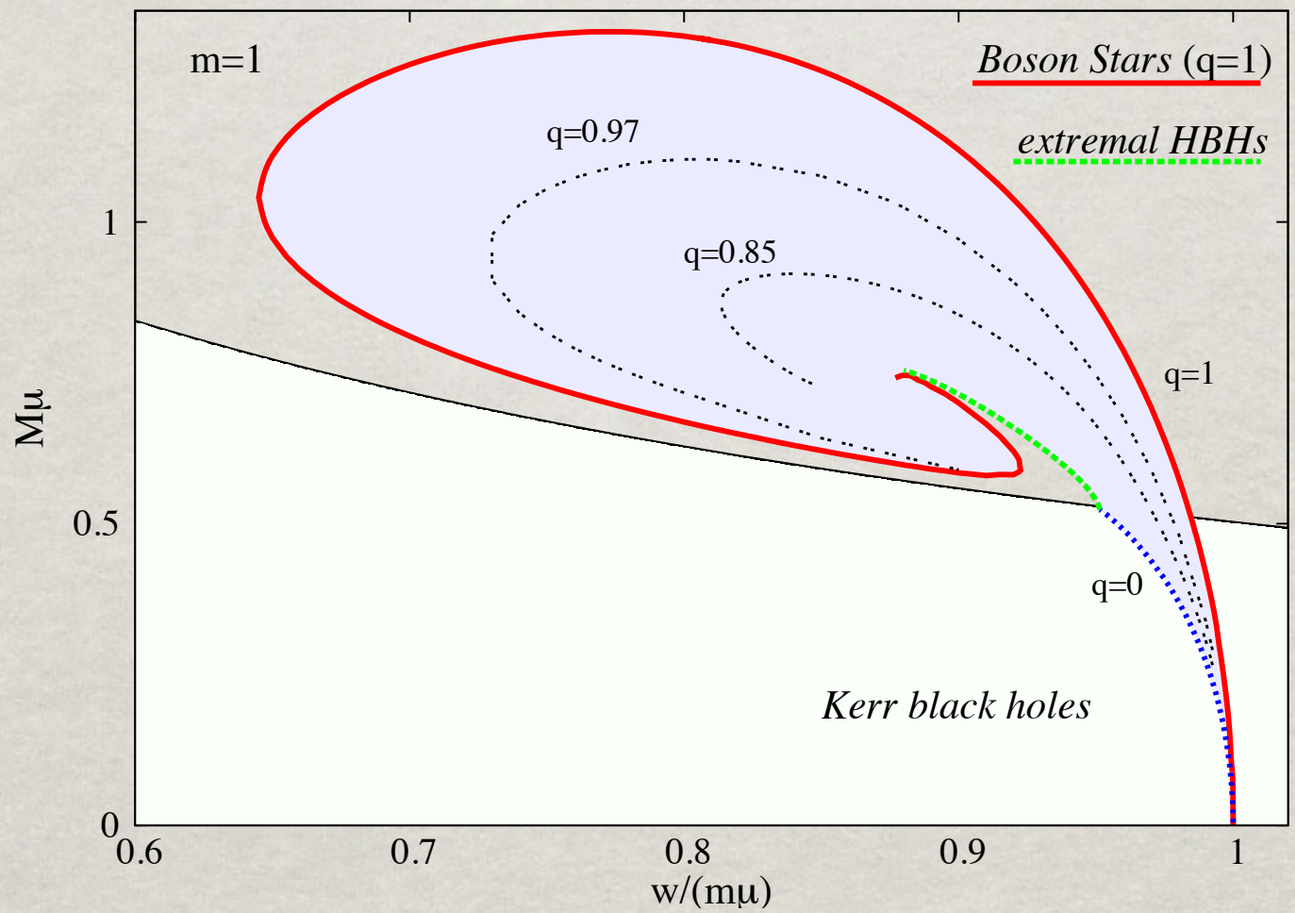
Hairy black holes phase space



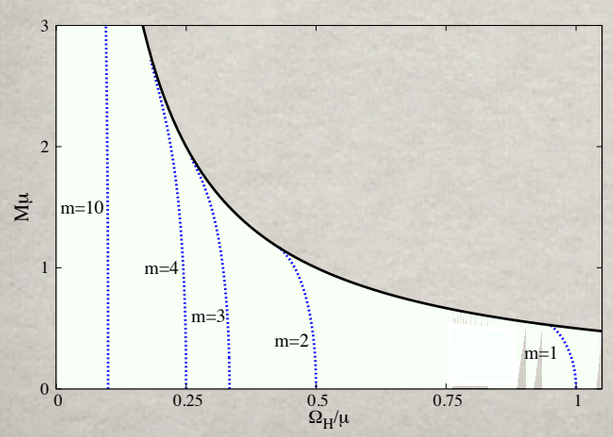
Hairy black holes phase space



Hairy black holes phase space



$$q \equiv \frac{mQ}{J}$$



Five parameters family of solutions:
3 continuous parameters (M,J,q)
2 discrete parameters (m,n)

2) Kerr black holes with scalar hair
iii) Phenomenology

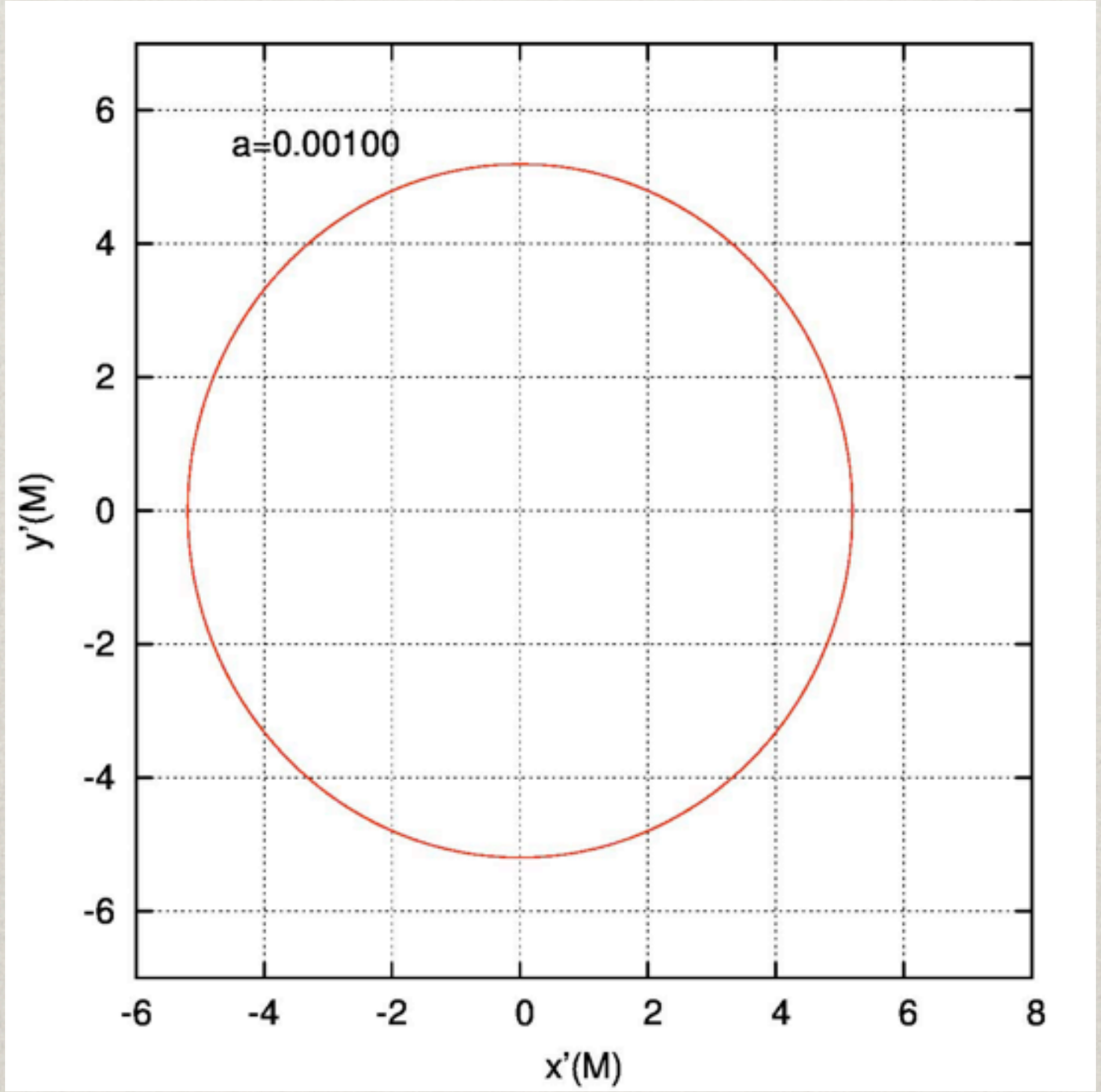
There is non uniqueness

(different solutions for same ADM M,J);
but degeneracy raised with q

Can we distinguish by a local measurement degenerate configurations?

Shadow of a Kerr black hole:

(equatorial plane observation)



Cunha, M.Sc. Thesis

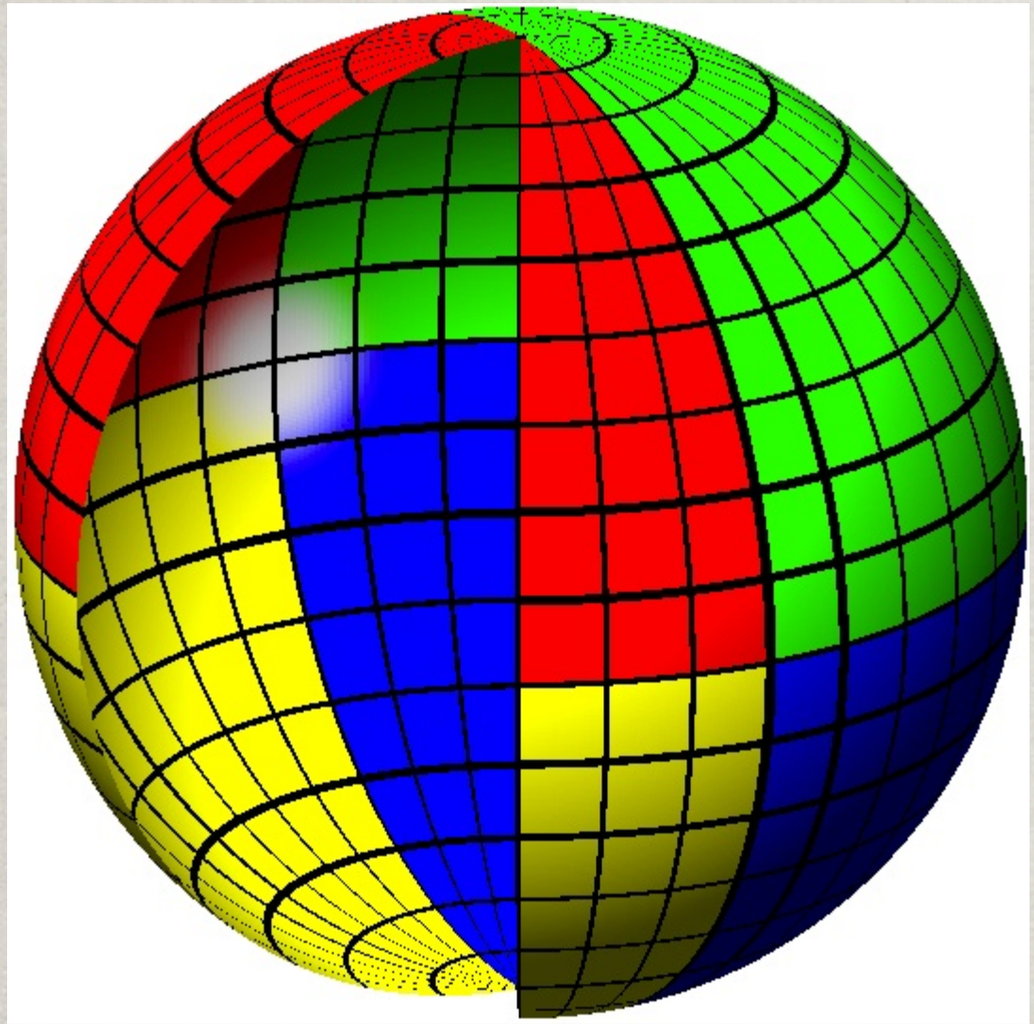
Technique: backwards ray-tracing

camera

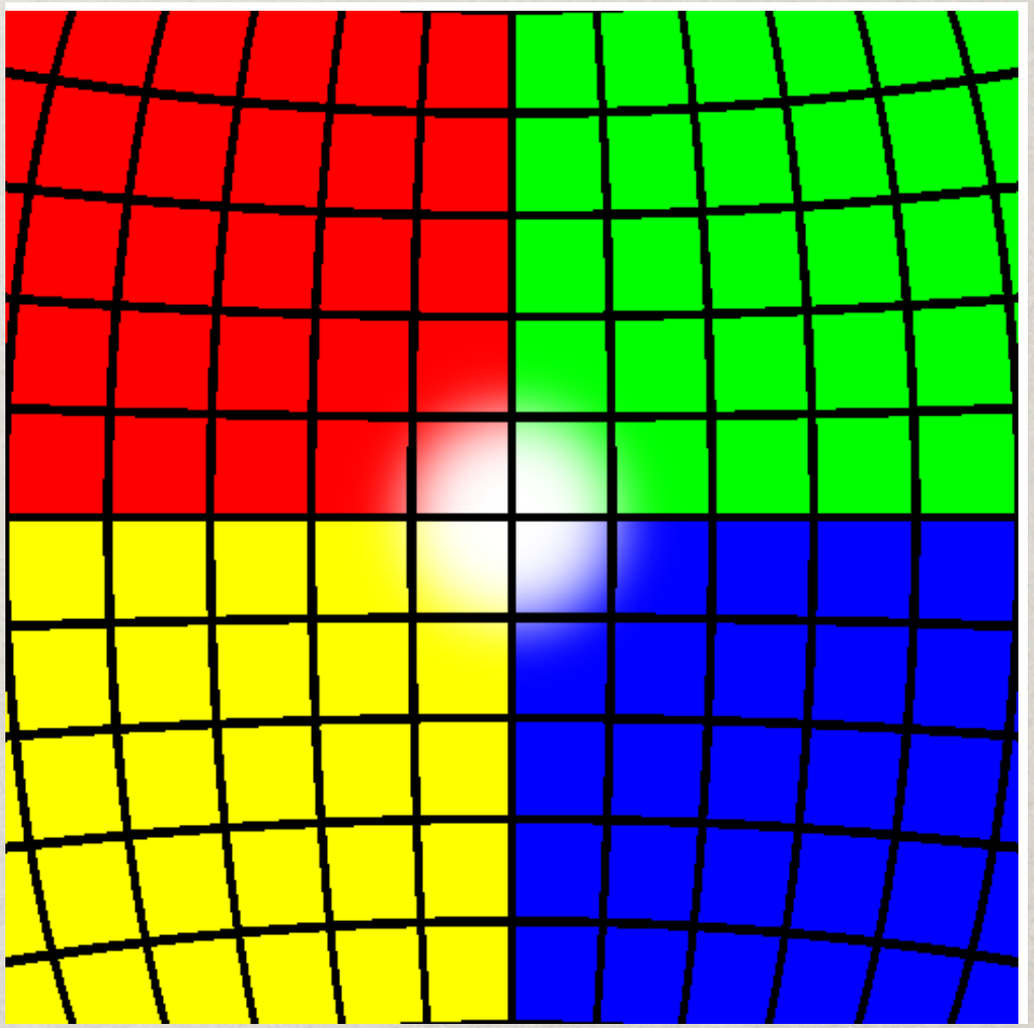
•

Cunha, M.Sc. Thesis

We have performed ray tracing to compute lensing and shadows.



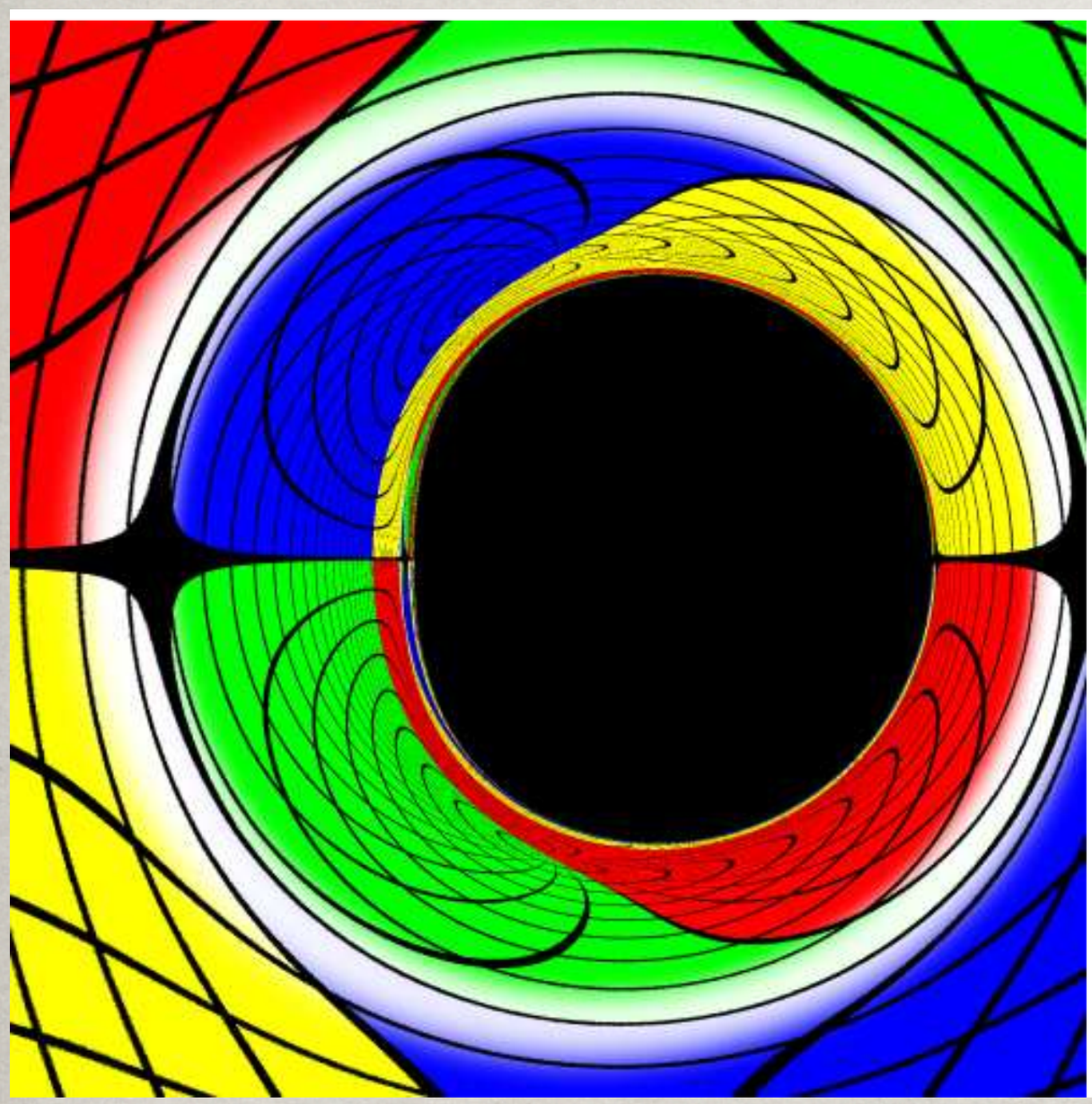
The full celestial sphere

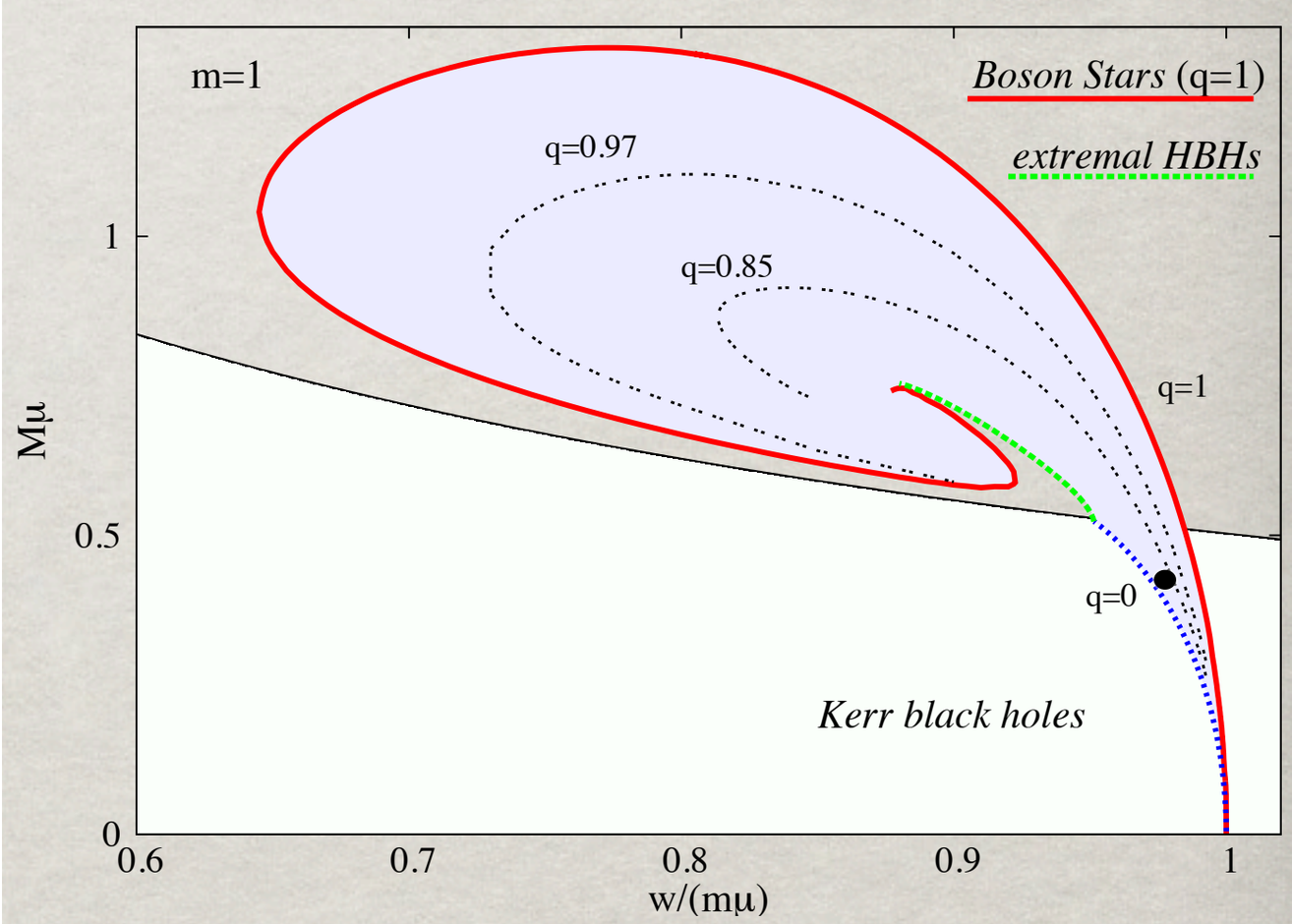


The “camera” opening angle

Following A. Bohn et al. arXiv:1410.7775

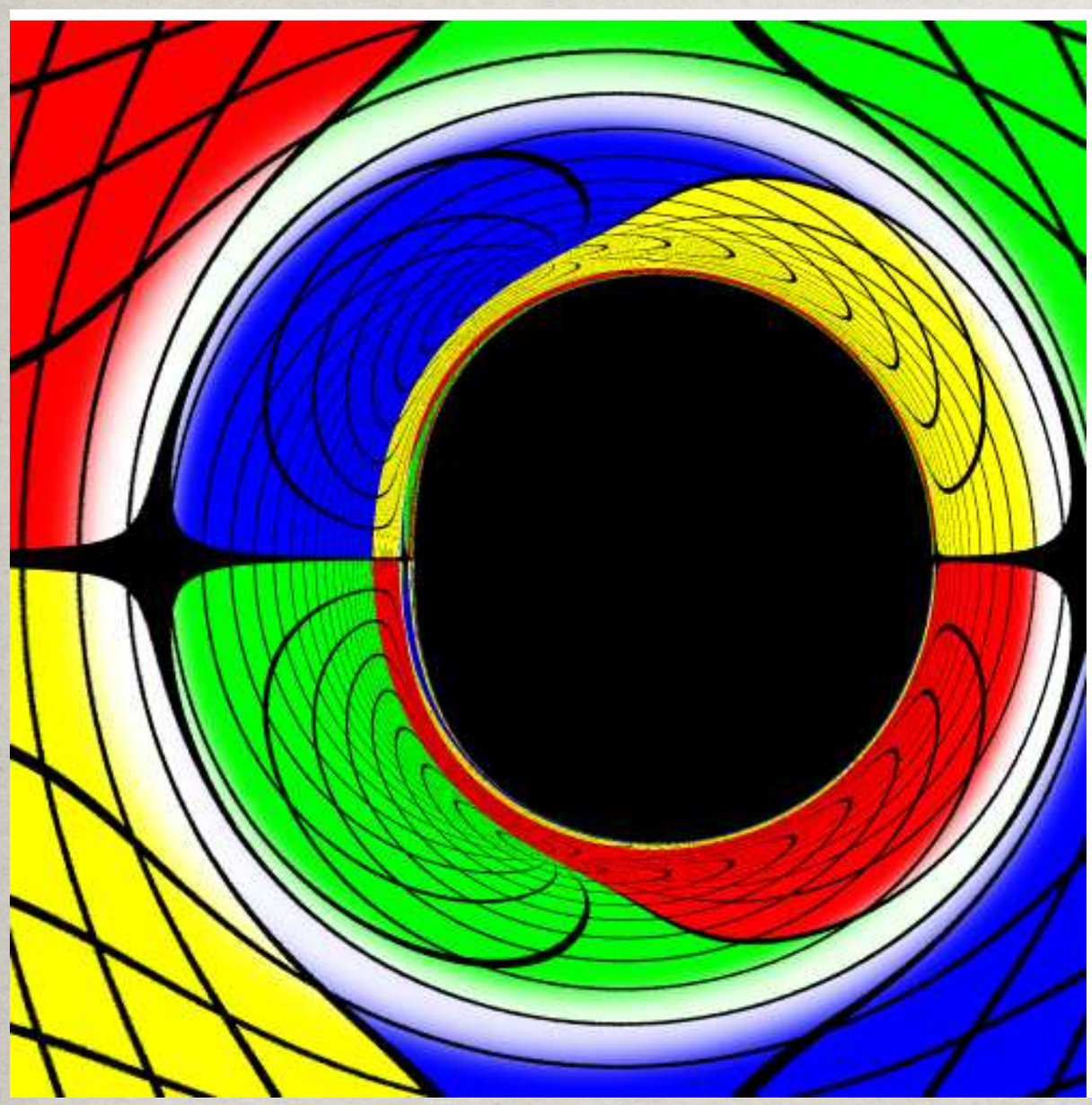
A Kerr-like hairy black hole



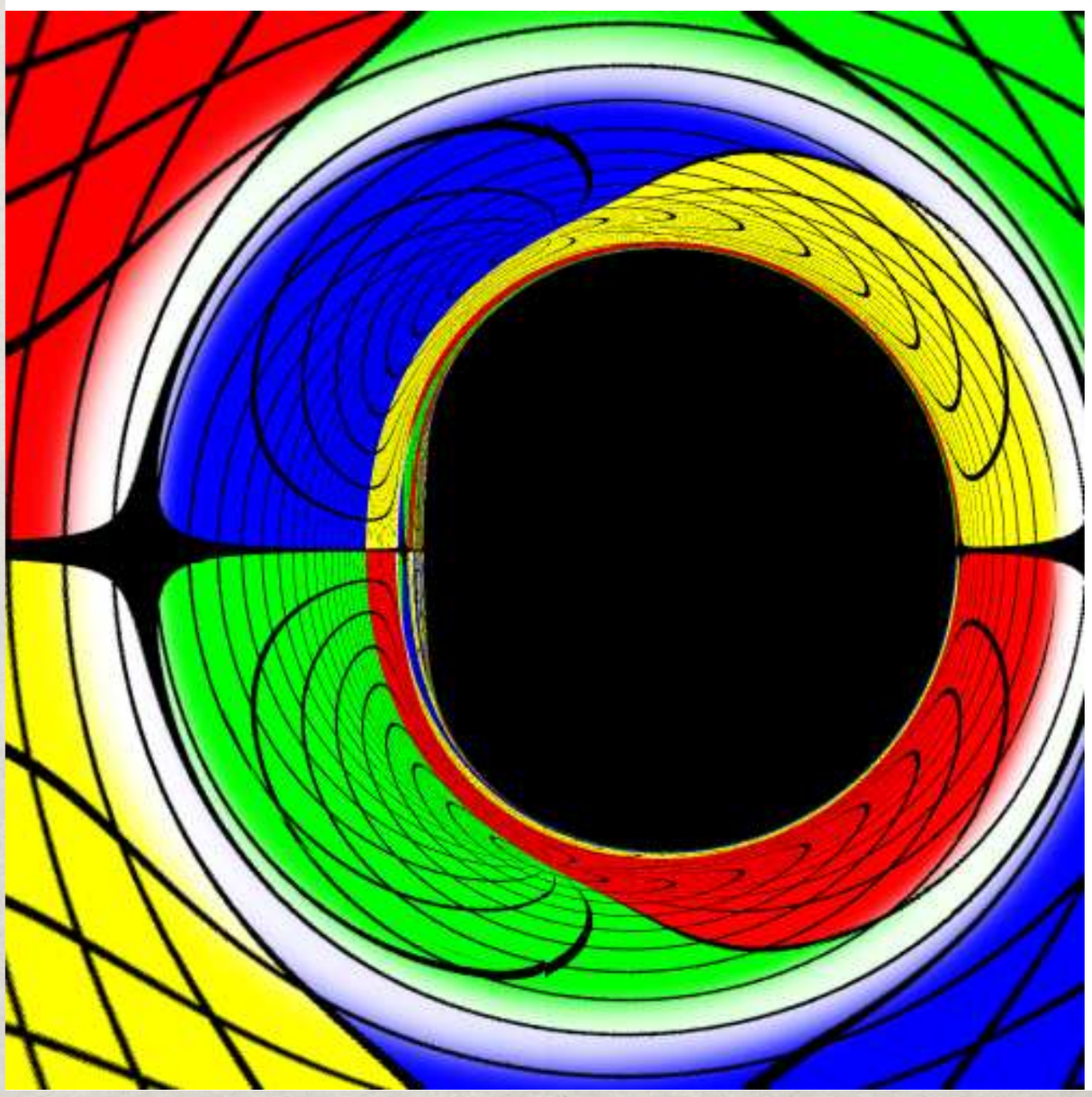


5% of mass;
13% of angular momentum
is stored in the scalar field

A Kerr-like Kerr BH with scalar hair

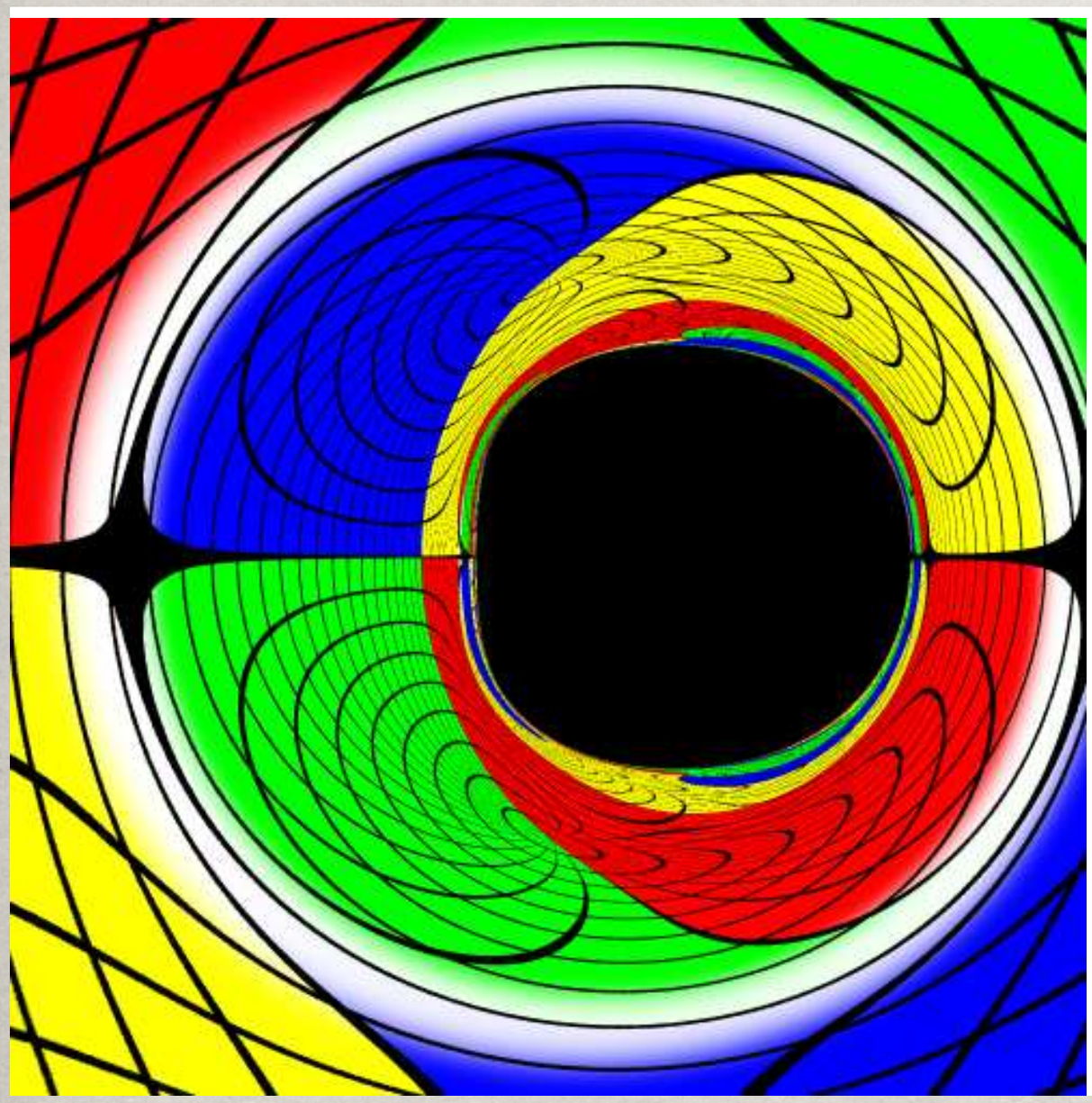


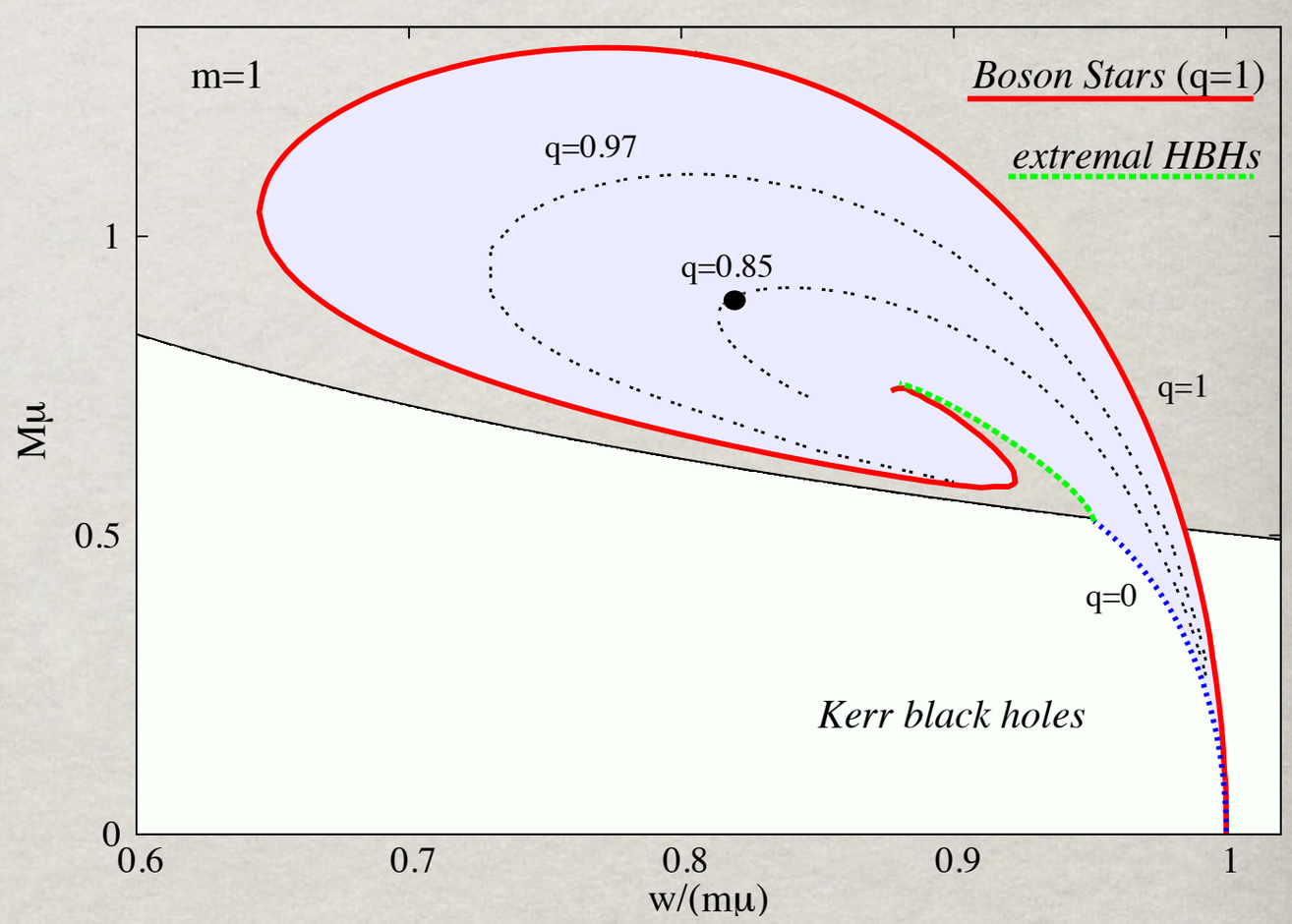
Kerr BH with scalar hair
M=0.393; J=0.15 (horizon)
M=0.022; J=0.022 (scalar field)



Vacuum Kerr BH
M=0.415; J=0.172

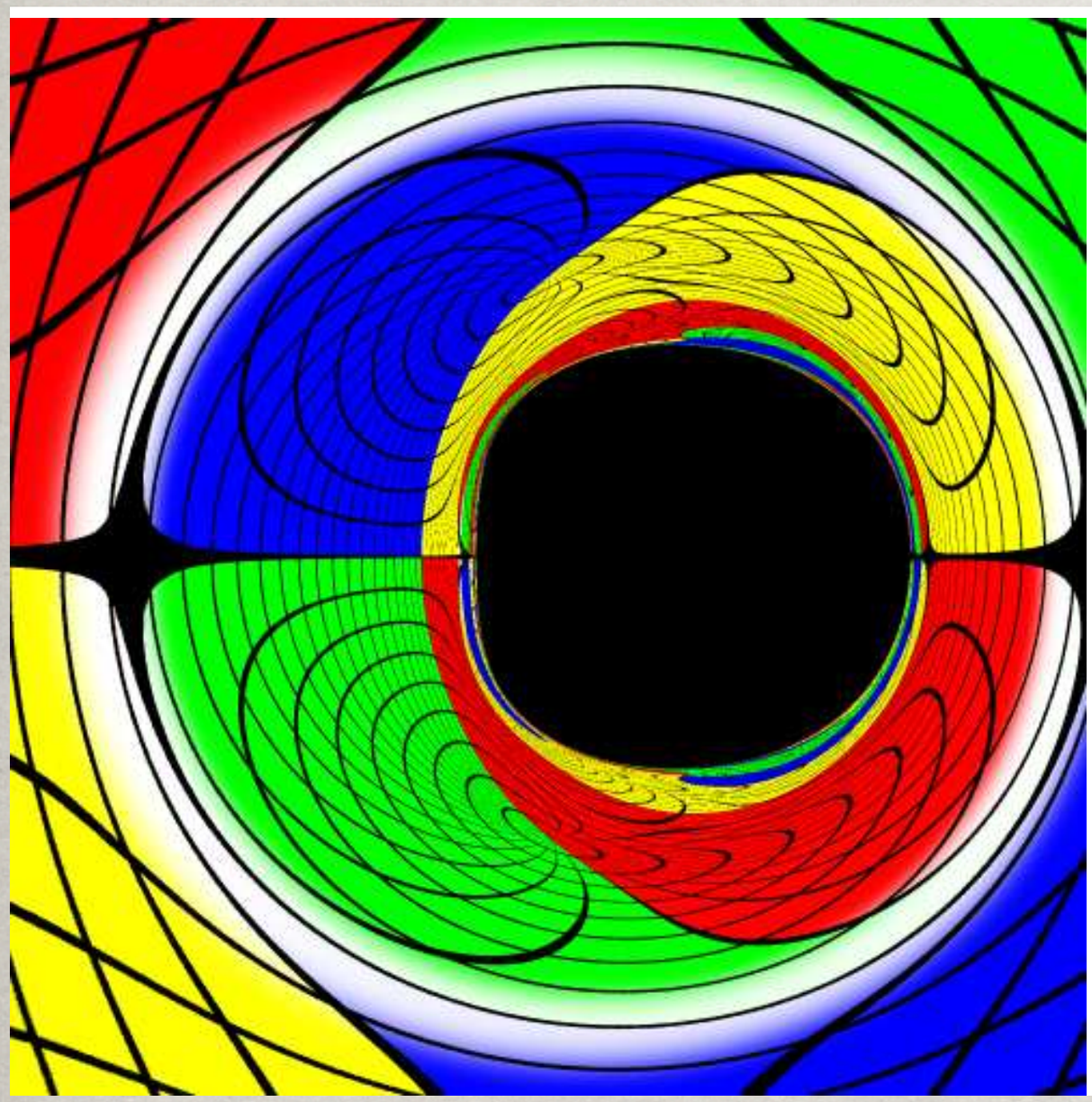
A non-Kerr-like hairy black hole



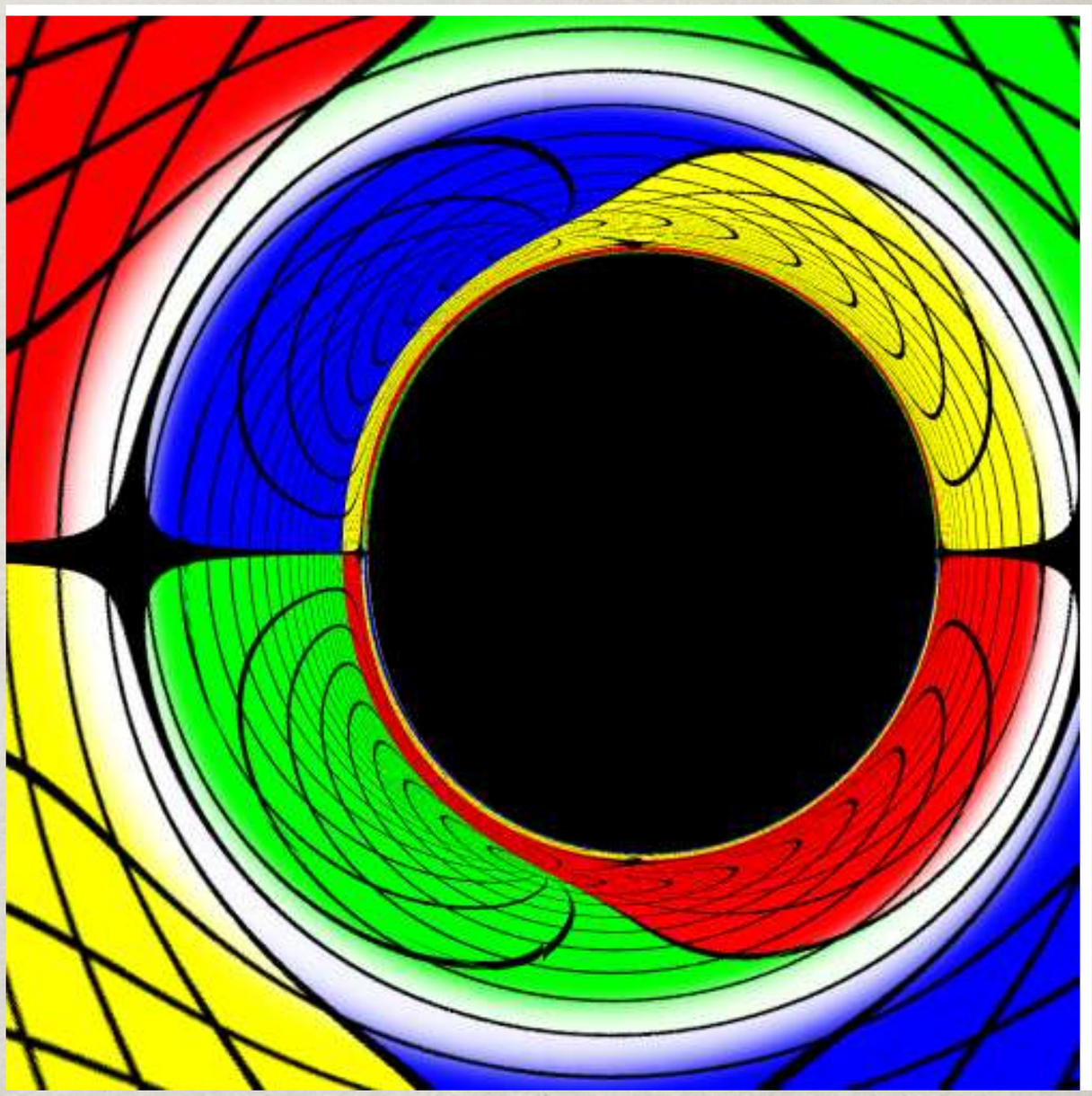


75% of mass;
85% of angular momentum
is stored in the scalar field

A non-Kerr-like hairy black hole

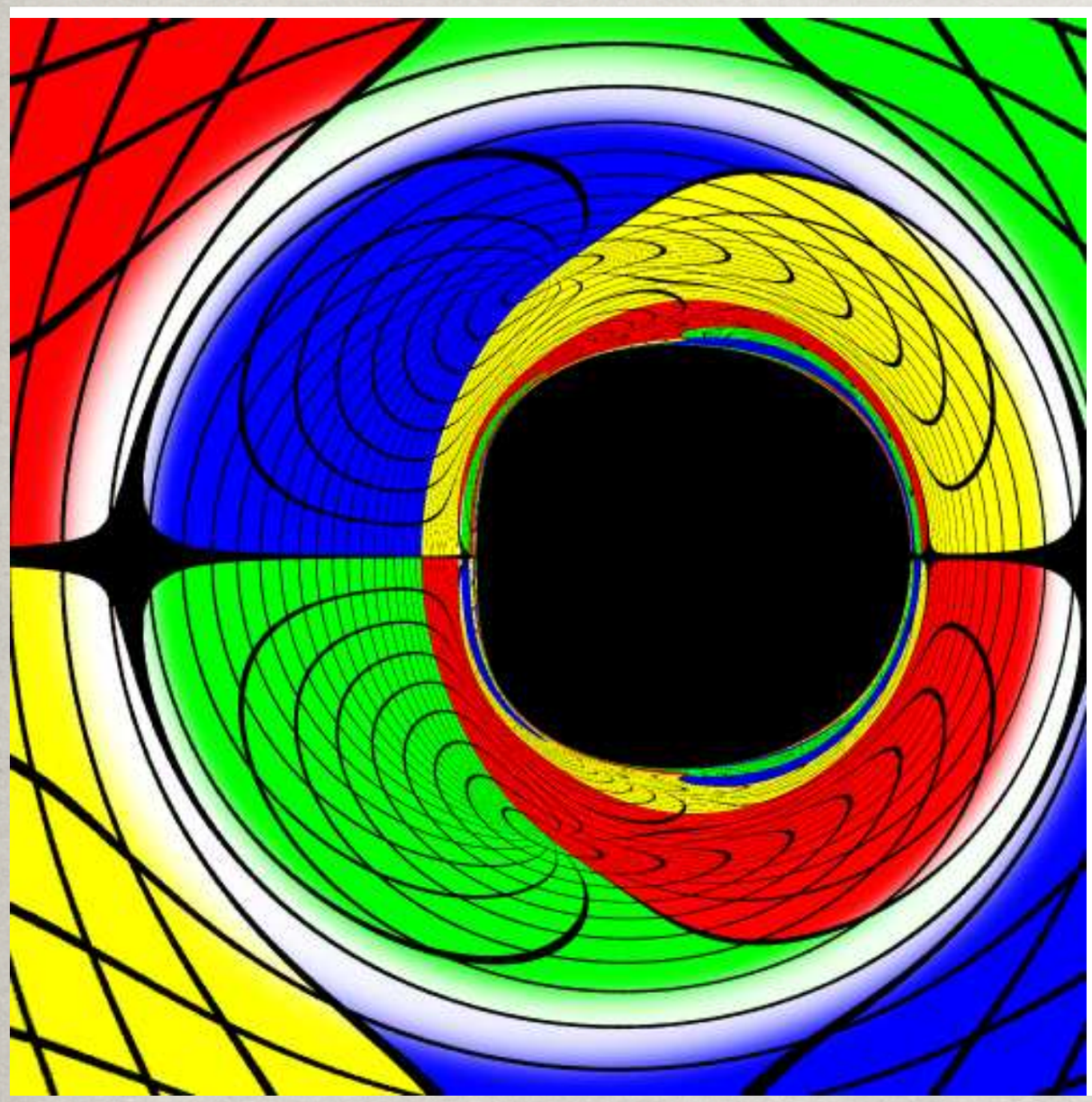


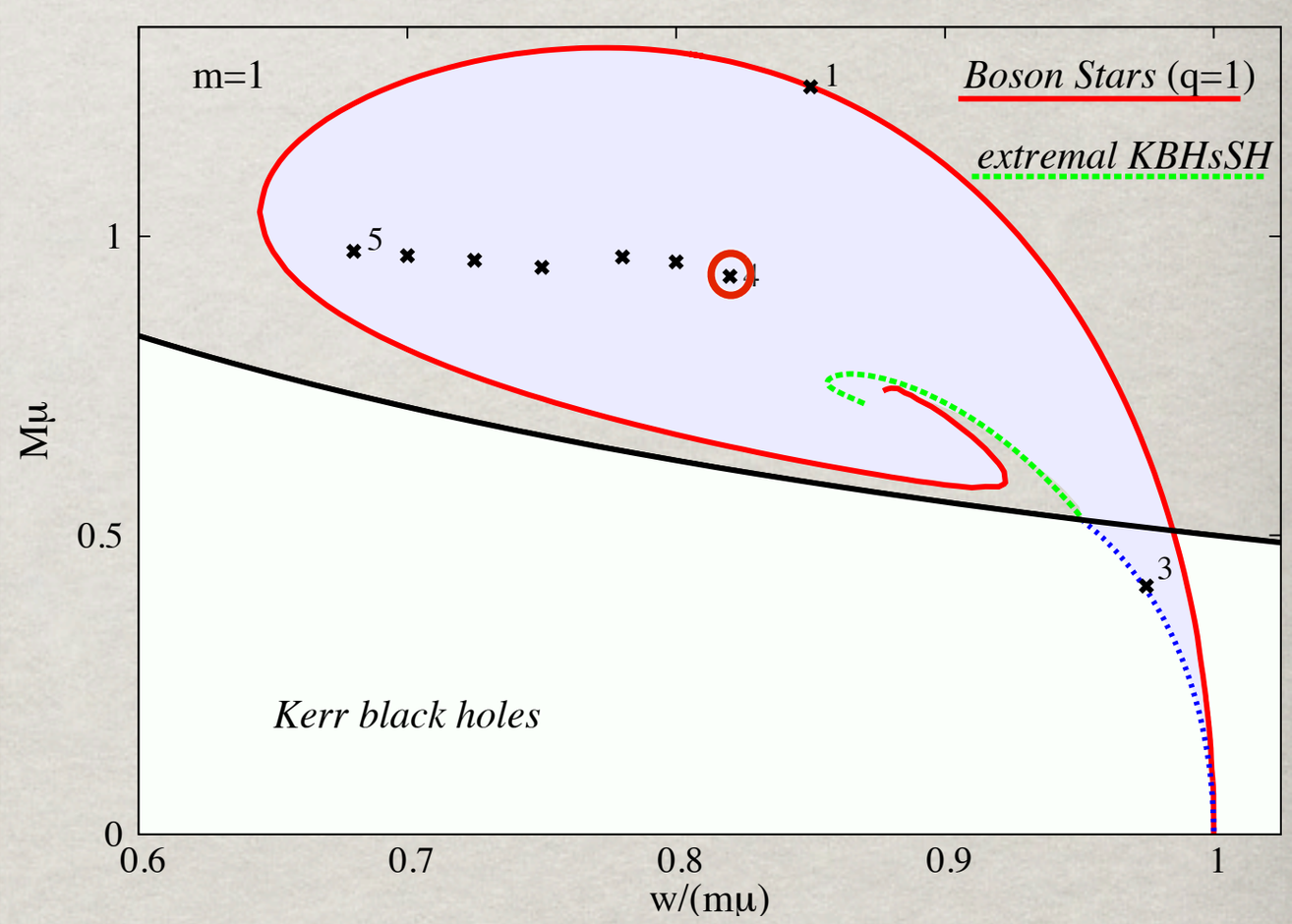
Kerr BH with scalar hair
M=0.234; J=0.114 (horizon)
M=0.699; J=0.625 (scalar field)

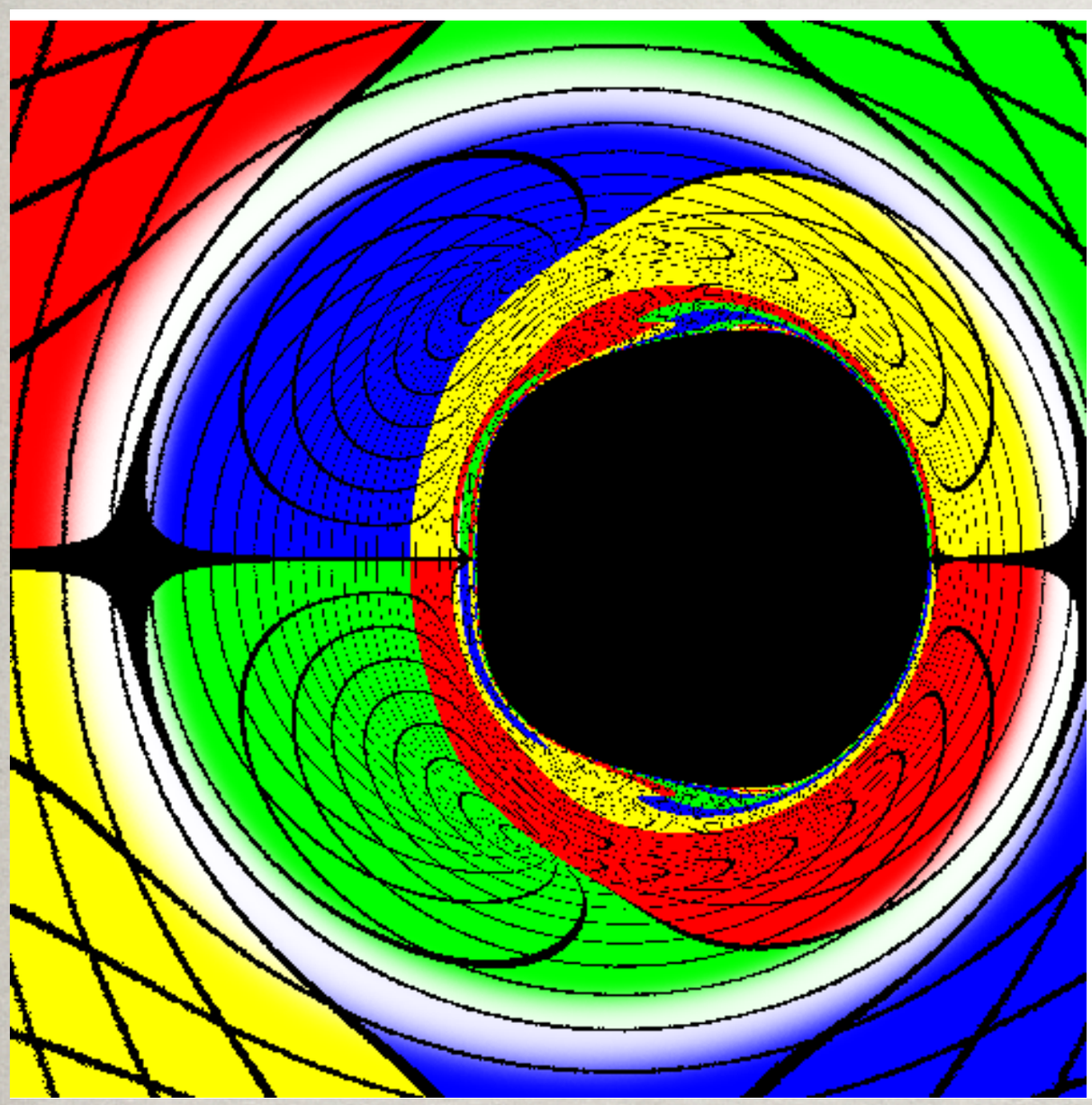


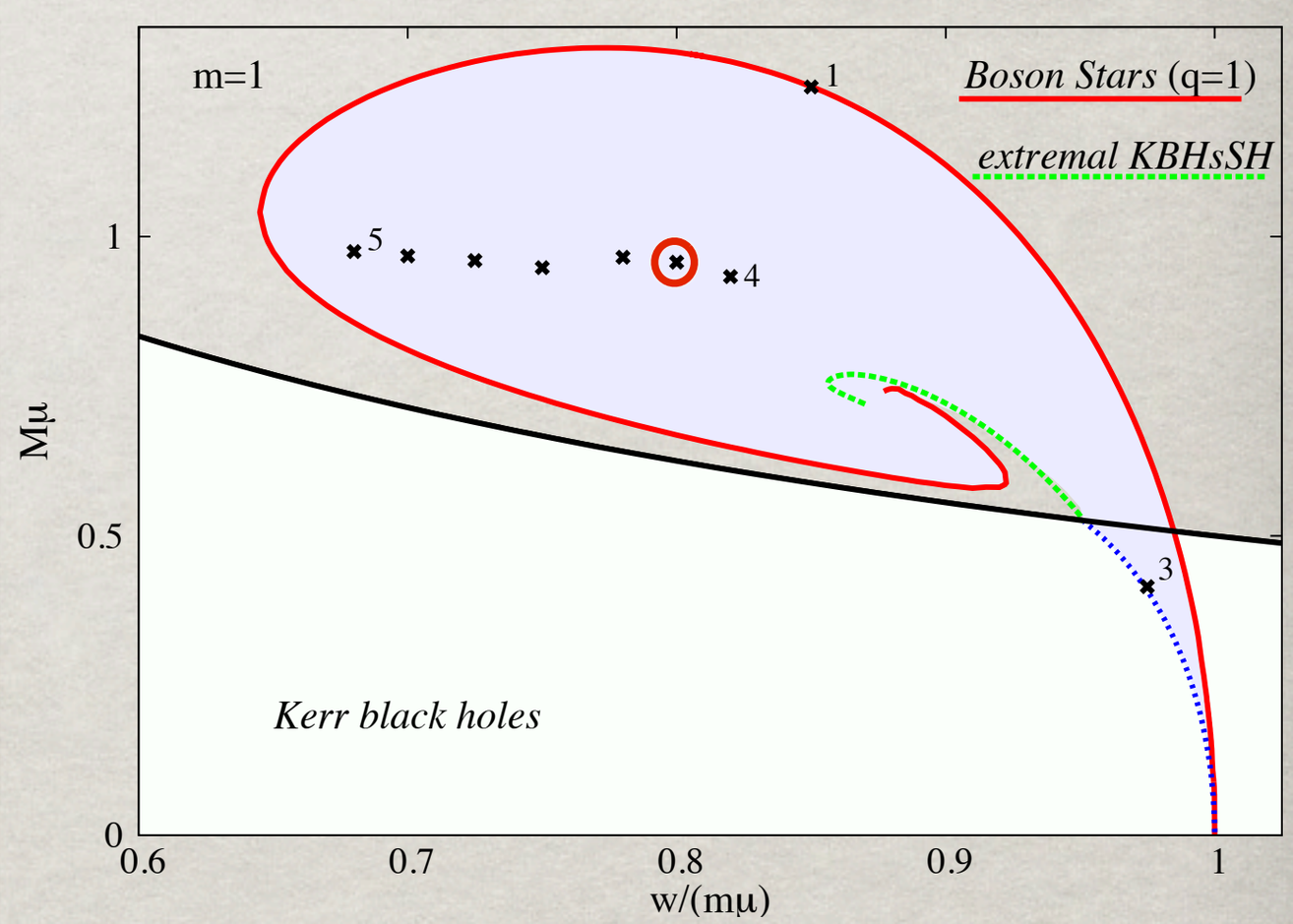
Vacuum Kerr BH
M=0.933; J=0.739

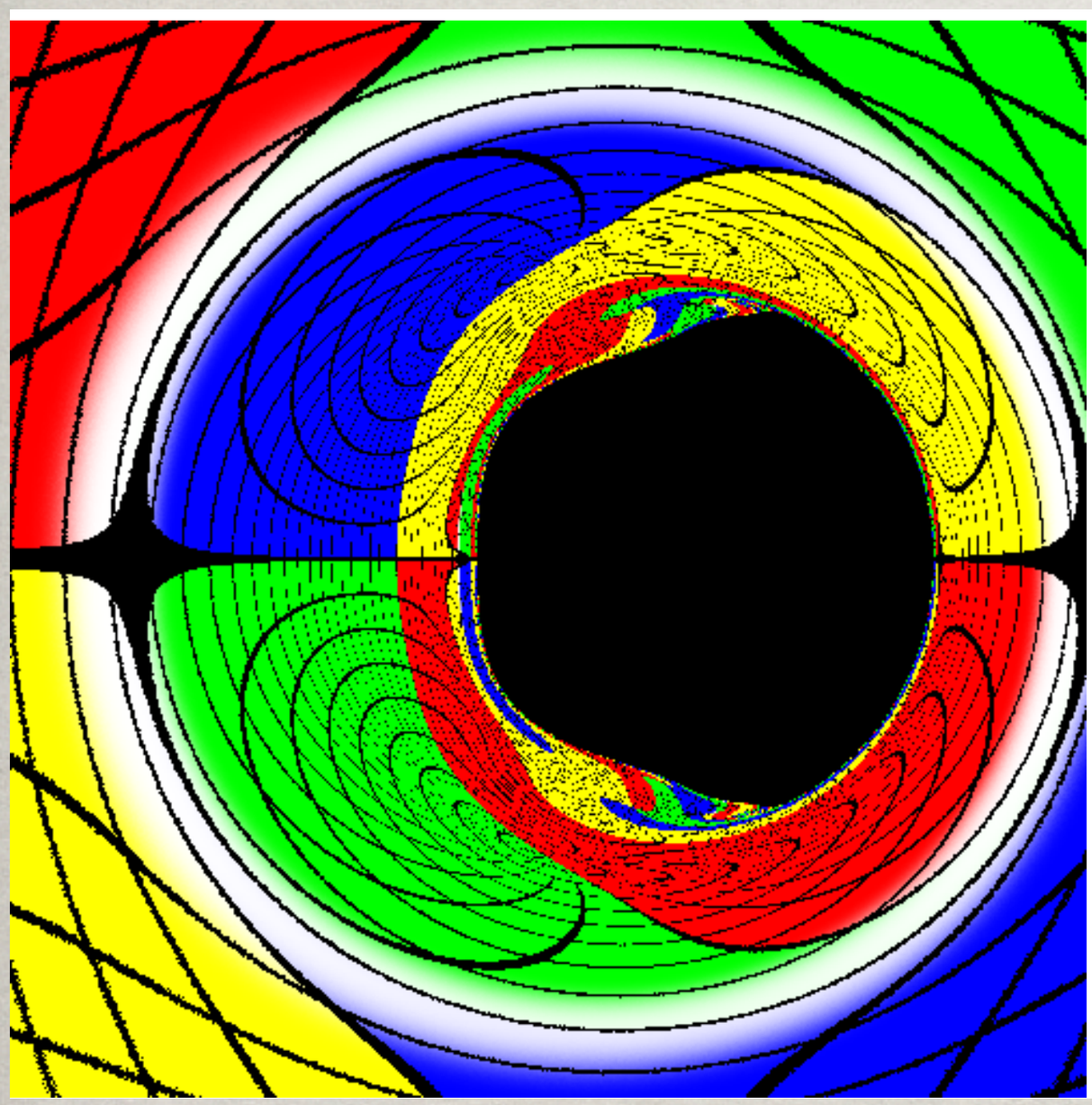
More non-Kerr-like hairy black holes

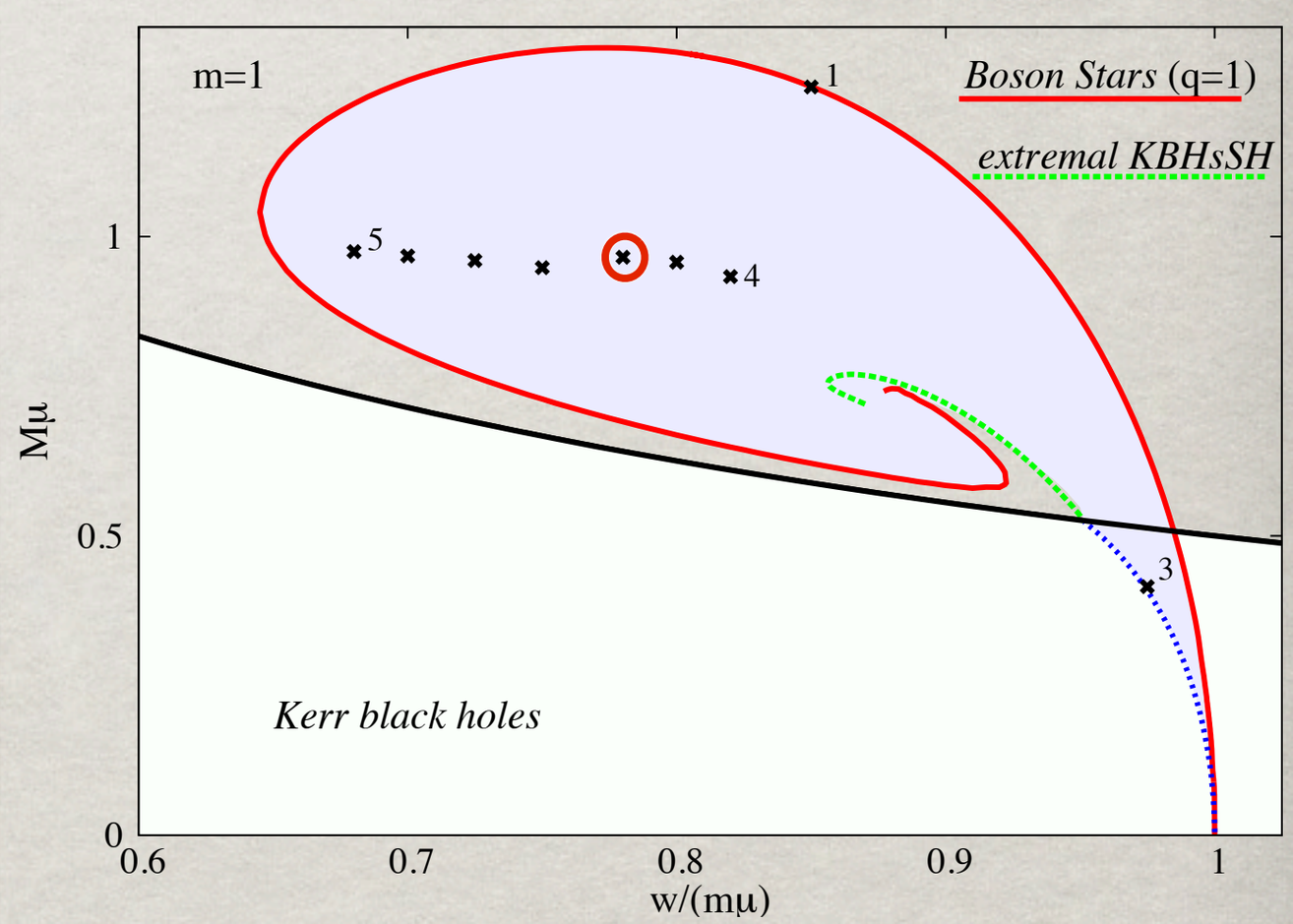


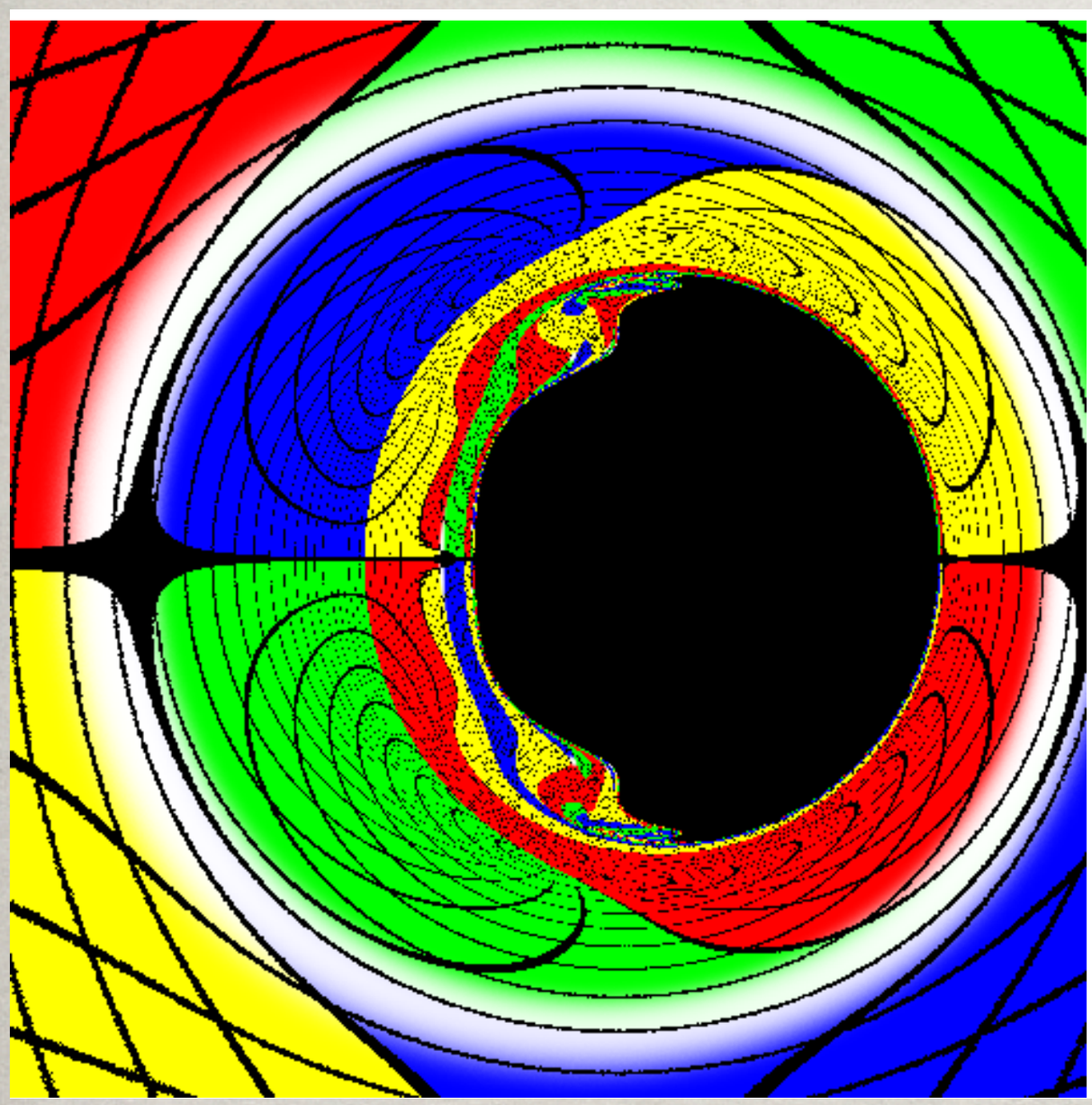


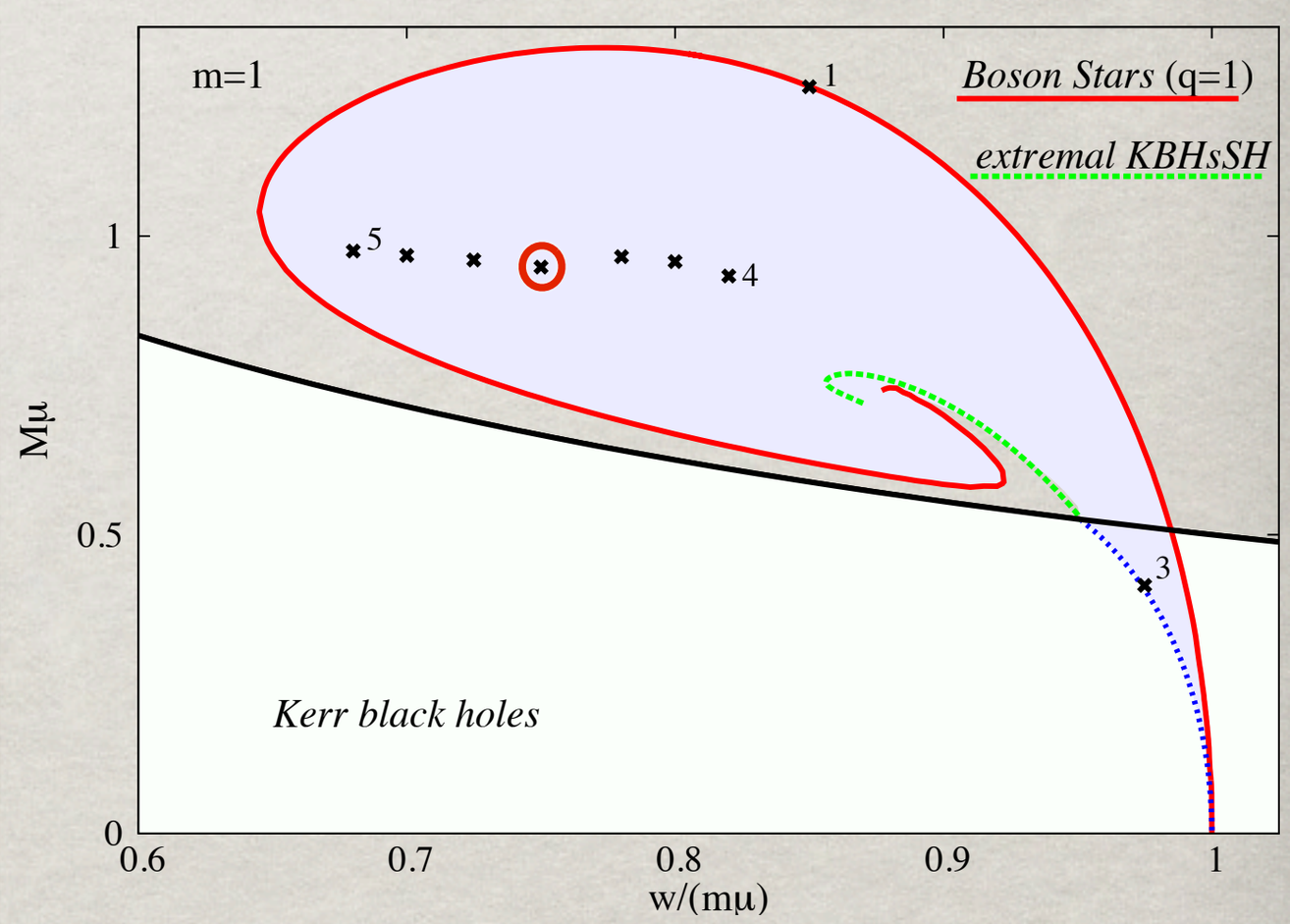


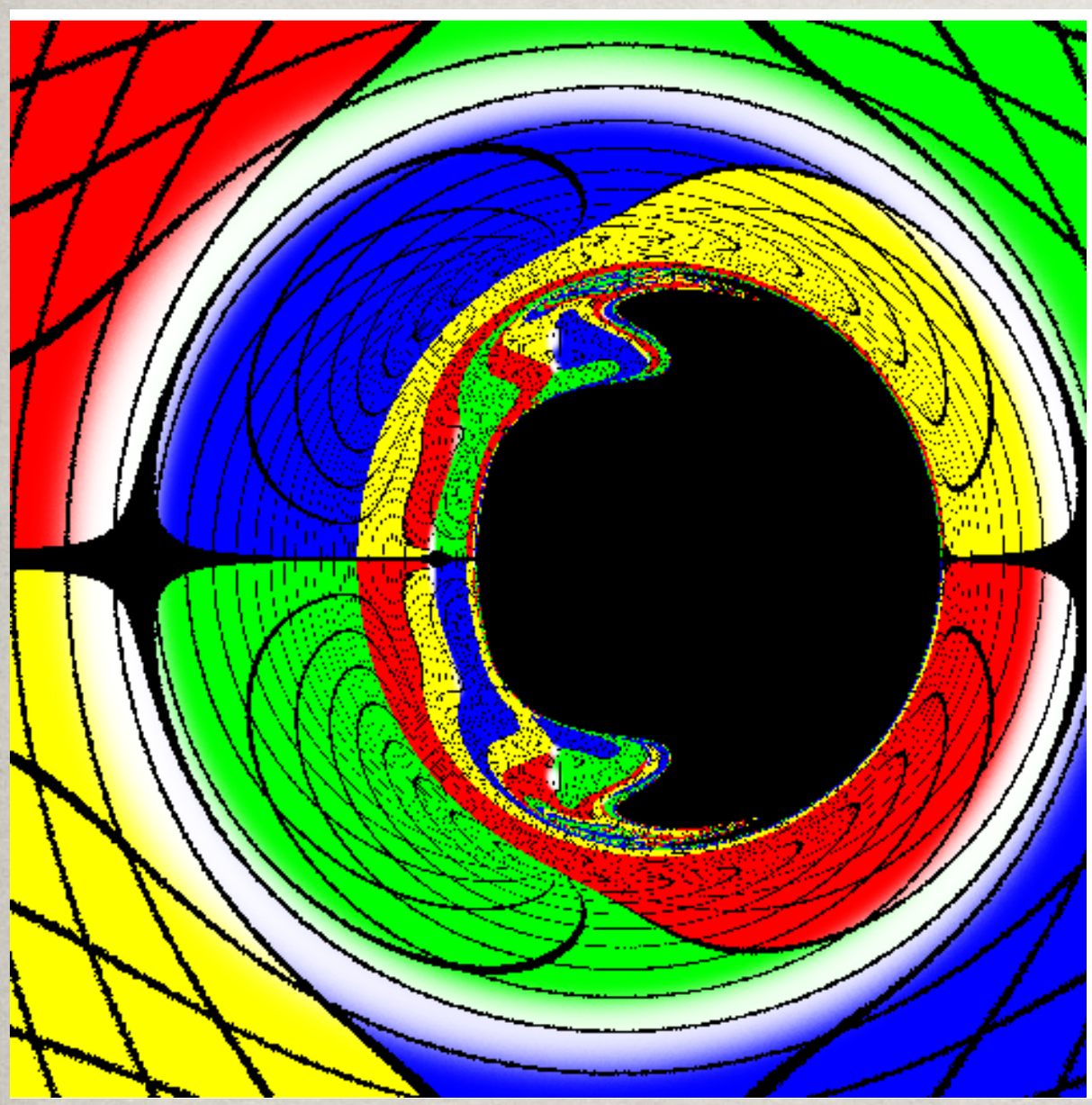

m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
2
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1

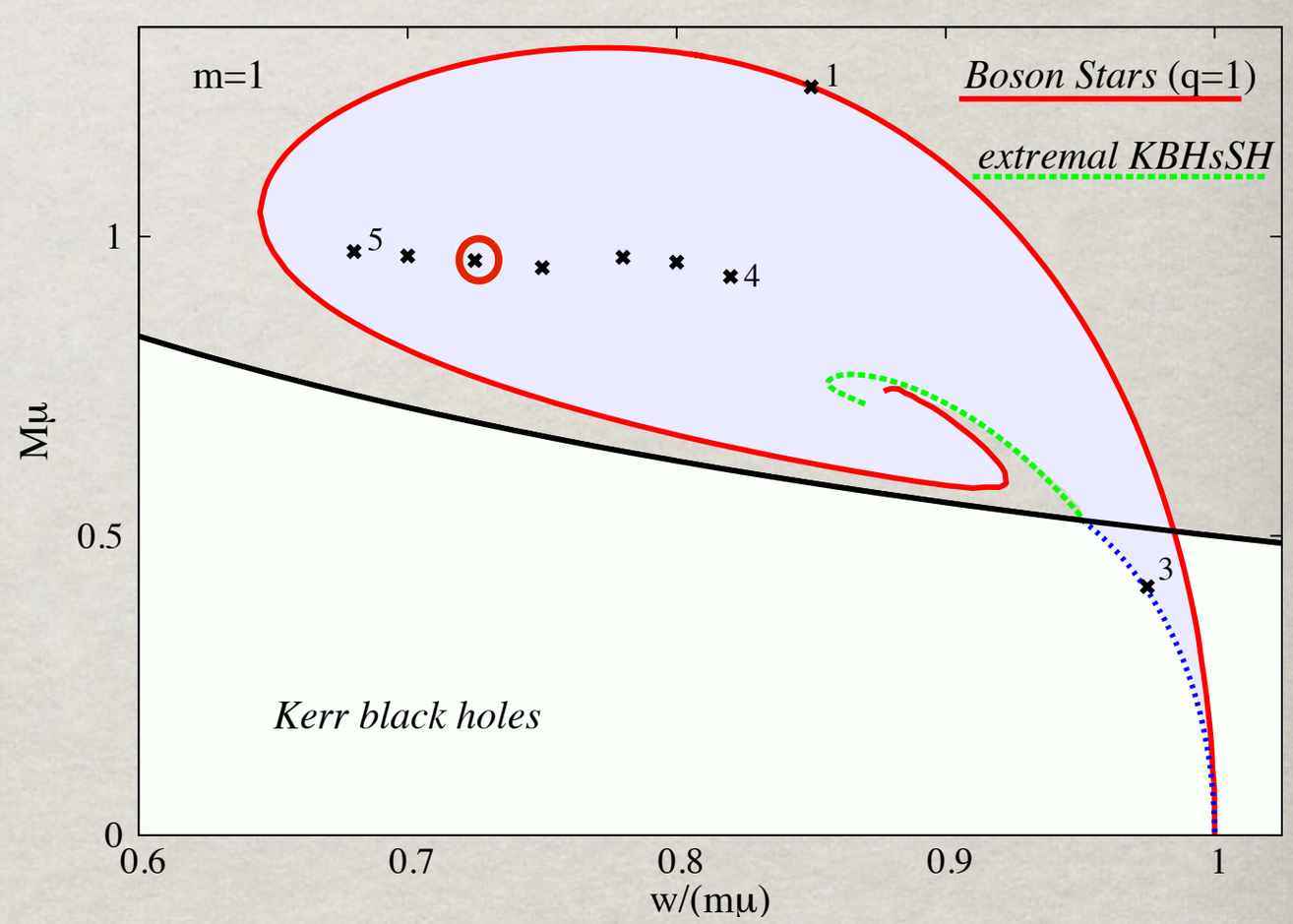


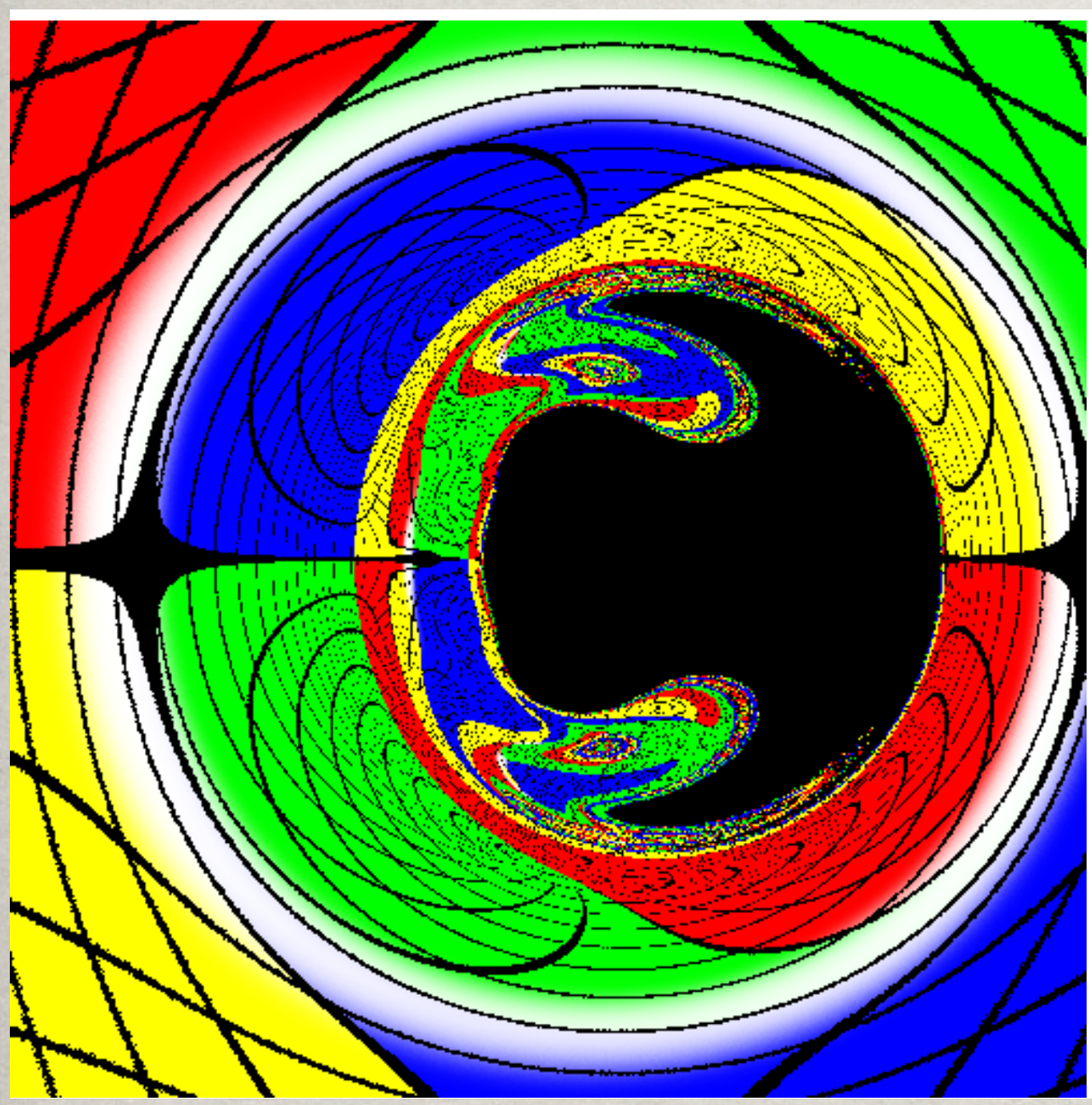

m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
2
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1

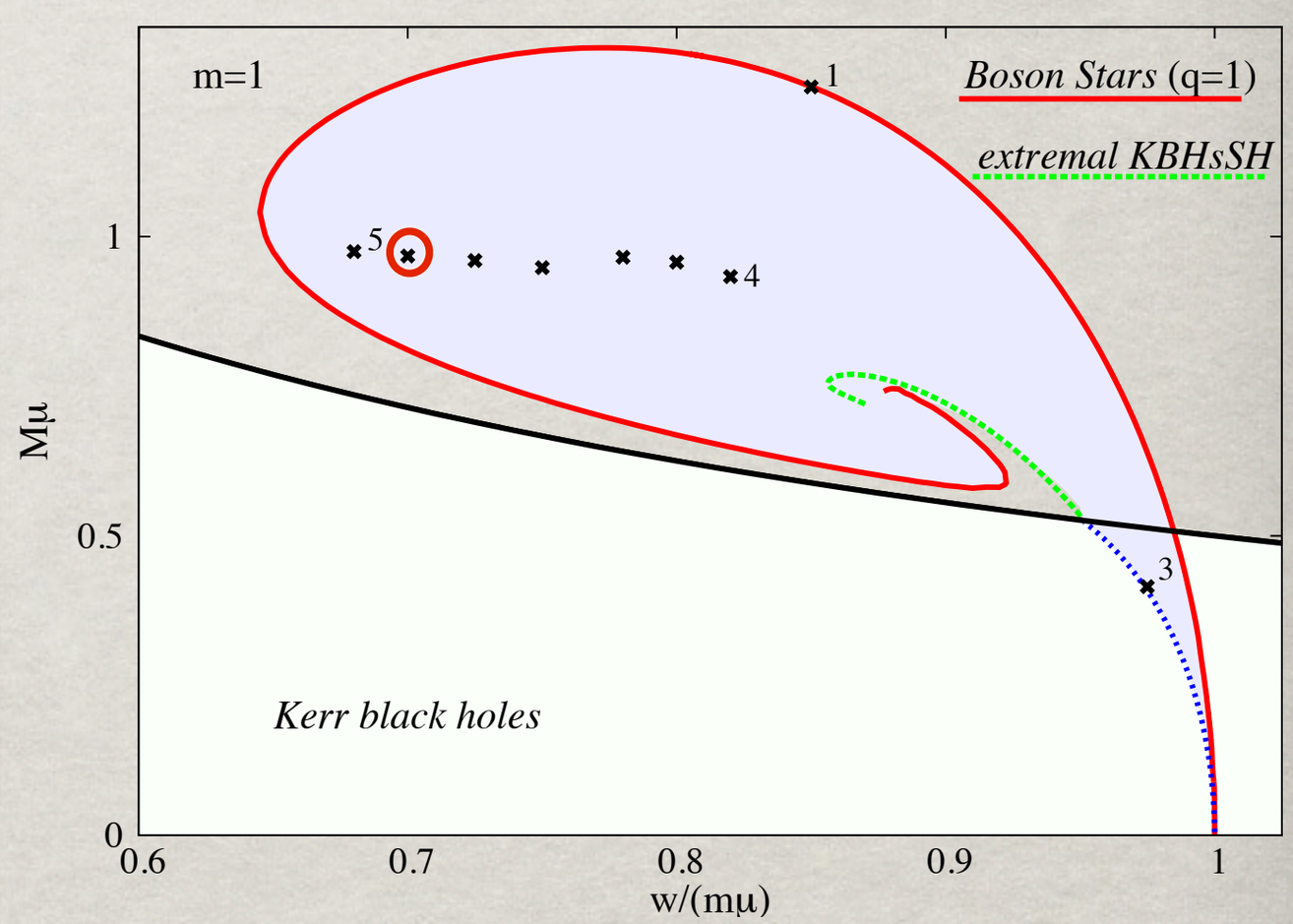


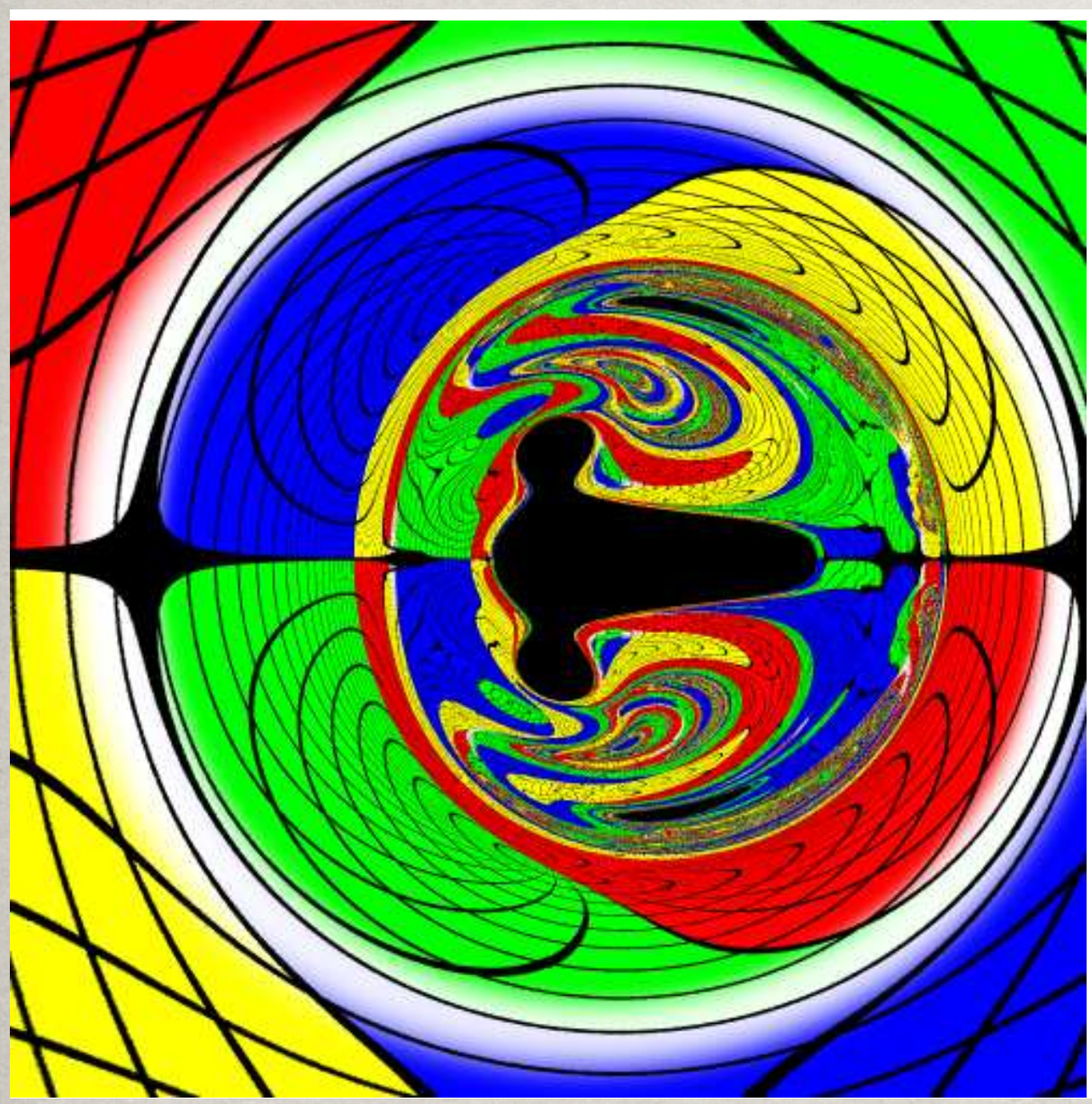

m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
2
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1

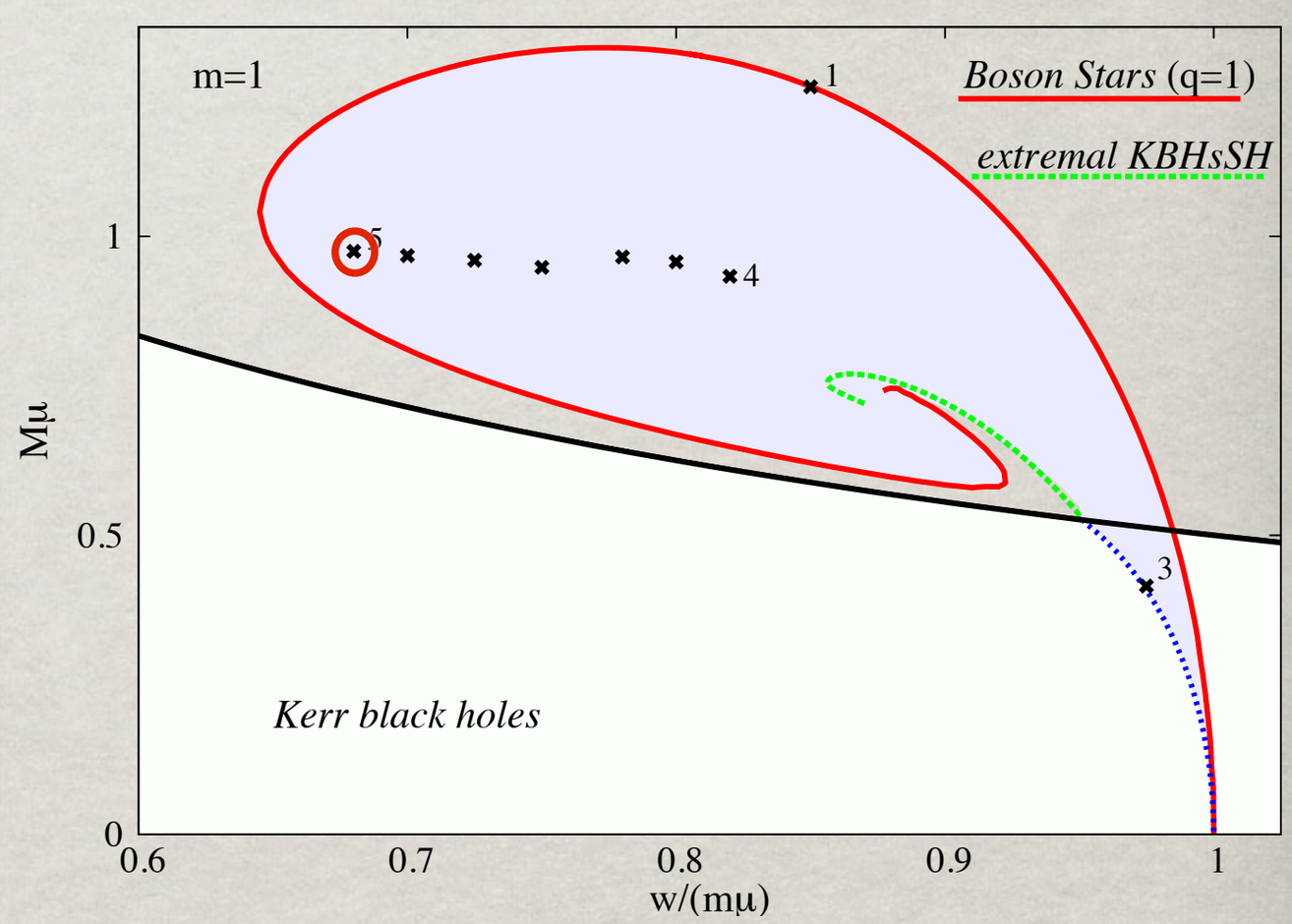



m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
2
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1

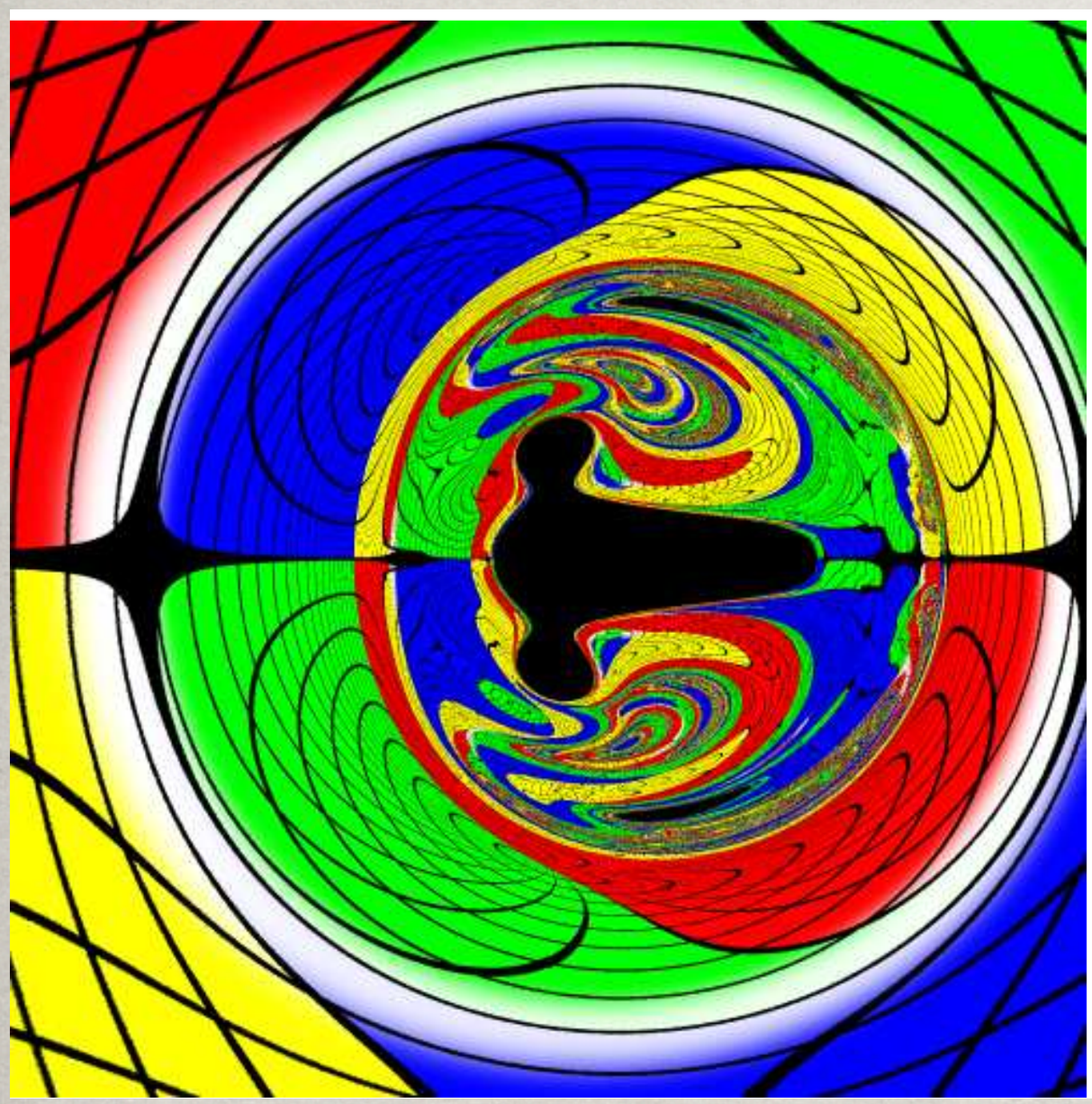


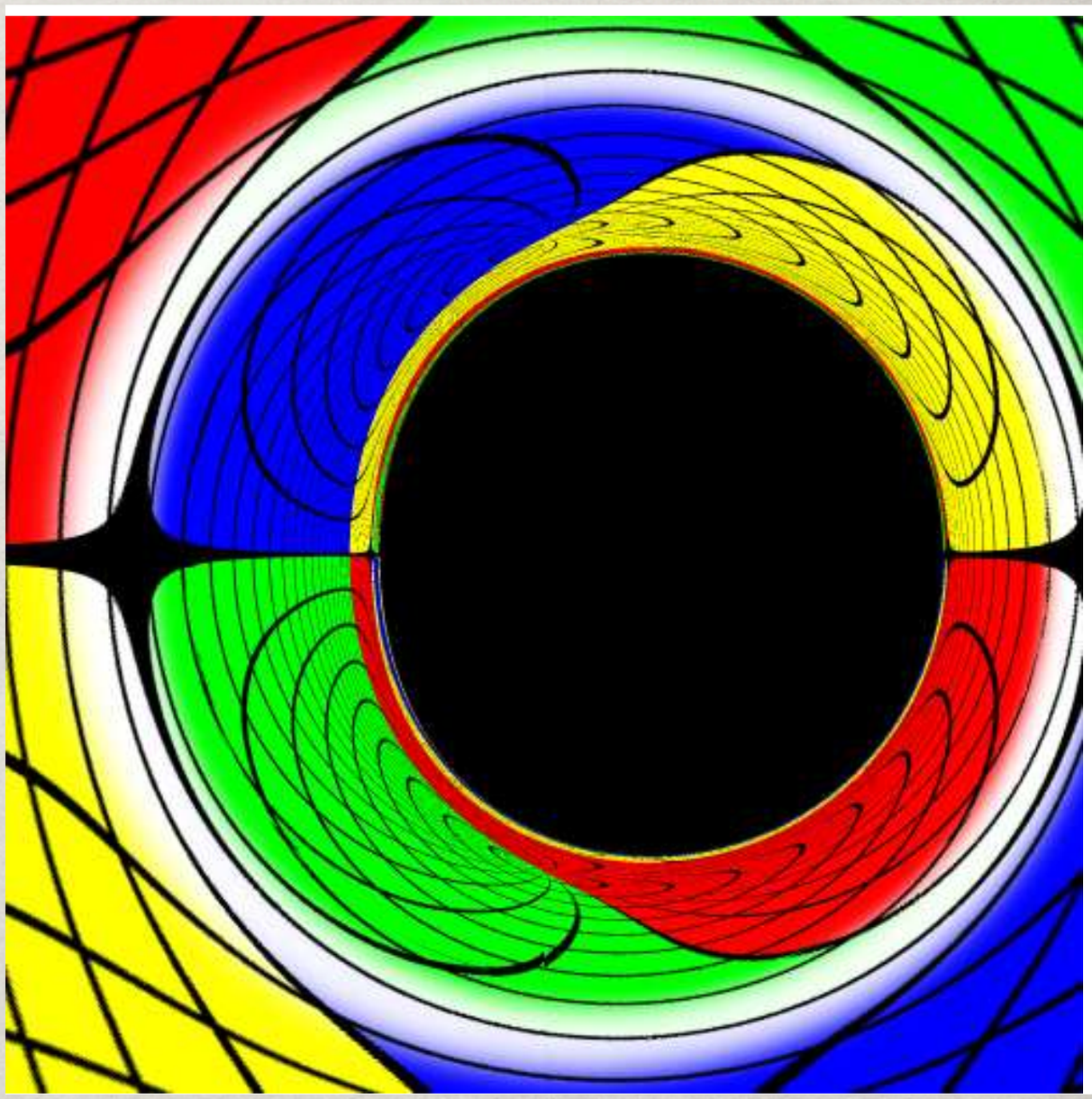

m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
2
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1




m=1
Boson Stars (q=1)
extremal KBHsSH
Kerr black holes
1
2
3
4
5
Mμ
w/(mμ)
0
0.5
1
0.6
0.7
0.8
0.9
1

A very non-Kerr-like hairy black hole





Qualitatively new feature:
multiple shadows of a single black hole

3) Outlook

Hairy black holes interpolate between Kerr and boson stars.

branching of Kerr black holes towards a new family of solutions due to superradiant instability.

Mechanism:
A (hairless) BH which is afflicted by the superradiant instability of a given field for which the energy-momentum tensor is time-independent, allows a hairy generalization with that field.

Yet another example:

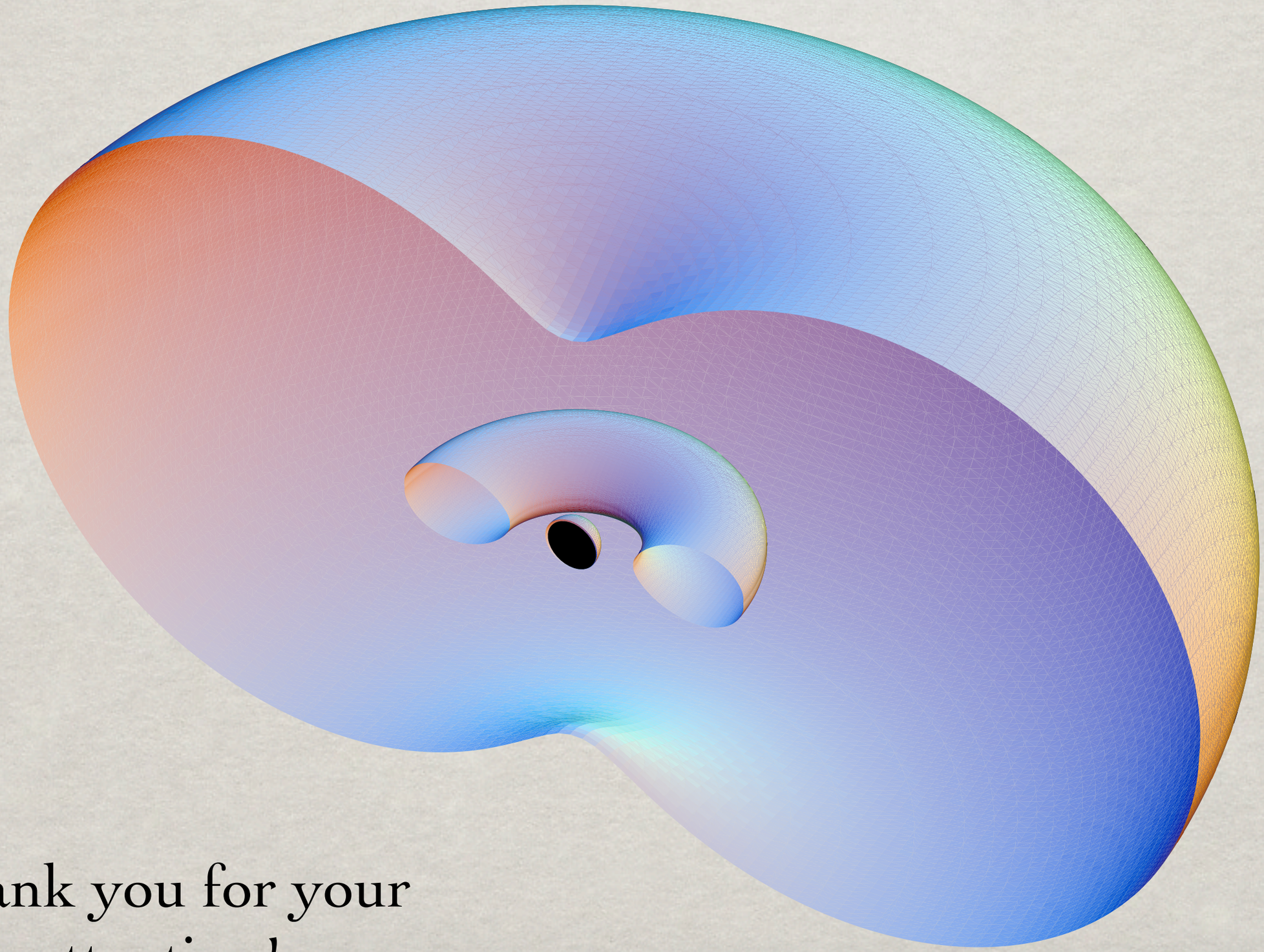
Proca stars

Brito, Cardoso, C.H., Radu, arXiv:1508.05395

and

Kerr black holes with Proca hair

To appear



Thank you for your attention!