A Landau fluid model for dispersive magnetohydrodynamics

T. Passot and P.L. Sulem

CNRS, Observatoire de la Côte d'Azur, Nice, France

In spatial plasmas, collisions often negligible: usual MHD questionable.

For large-scale dynamics, hydrodynamic approach nevertheless advantageous.

Interest in constructing fluid models that extend MHD equations to collisionless situations by including finite Larmor radius (FLR) corrections and Landau damping (the only fluid-particle resonance that can affect large scales).

Most useful to analyze the dynamics of the magnetosheath (buffer between the earth bow shock and the magnetopause, that plays an important role: decrease the impact of solar activity on the earth environment).

Magnetosheath displays a wide spectrum of low frequency modes (Alfvén, slow and fast magnetosonic, mirror). Cluster spacecrafts allow one to determine k-spectra and clearly identify modes (Sahraoui et al. 2004).

Size of perturbuations can be smaller than the ion gyroradius.

The plasma is relatively warm and collisionless.

Landau damping and finite Larmor radius corrections play an important role.

Evidence of coherent solitonic structures (magnetic holes and shocklets), whose origin is still debated (Tsurutani et al. 2004).

Which tool?

• Description of intermediate-scale dynamics by usual MHD is questionable.

• Numerical integration of Vlasov-Maxwell or gyrokinetic equations often beyond the capabilities of present day computers.

• Need for a reduced description that retains most of the aspects of a FLUID MODEL but INCLUDES REALISTIC APPROXIMATIONS OF THE PRESSURE TENSOR AND WAVE-PARTICLE RESONANCES.

Should remain simple enough to allow 3D numerical simulations of turbulent regime.

⋆ Gyrofluids: hydrodynamic moments obtained from gyrokinetic equations. Capture high order FLR corrections but need a specific closure and are written in a local reference frame.

⋆ Landau fluids [Hammett and co-authors (1990s)]: monofluid taking into account wave-particle resonances in a way consistent with linear kinetic theory.

Outline of the method

• Goal: Extend Landau-fluid model, to reproduce the weakly nonlinear dynamics of dispersive MHD (magnetosonic and Alfvén) waves whatever their direction of propagation, in particular of kinetic Alfvén waves (KAW) with $k\rho_L \leq 1$, by retaining FLR corrections and a generalized Ohm's law in addition to Landau damping.

• Starting point: Vlasov-Maxwell (VM) equations.

• Small parameter: ratio between the ion Larmor radius and the typical (smallest) wavelength. Field amplitudes also supposed to be small.

• Main problem: Exact hydrodynamic equations are obtained by taking moments of VM equations. The hierarchy must however be closed and the main work resides in a proper determination of the pressure tensor.

• Assumptions: Homogeneous equilibrium state with bi-Maxwellian distribution functions.

The equations

From Vlasov-Maxwell equations, derive for each species r equations for

density $\rho_r = m_r n_r \int f_r d^3v$,
velocity $u_r = \dfrac{\int v f_r d^3v}{\int f_r d^3v}$,
pressure tensor $\mathbf{p}_r = m_r n_r \int (v - u_r) \otimes (v - u_r) f_r d^3v$

heat flux tensor $\mathbf{q}_r = m_r n_r \int (v - u_r) \otimes (v - u_r) \otimes (v - u_r) f_r d^3v$,

from which a one-fluid description can possibly be obtained for
plasma density $\rho = \sum_r \rho_r$, velocity $u = \frac{1}{\rho} \sum \rho_r u_r$, total pressure $\mathbf{p} = \sum_r \mathbf{p}_r$.

But pressure equation involves heat fluxes, and so on: closure problem.

Furthermore, at hydrodynamical scales, pressure and heat flux tensors include both fast and slow components.

Basic tensors

$$\boldsymbol{\tau} = \widehat{b} \otimes \widehat{b} \qquad \mathbf{n} = \mathbf{I} - \boldsymbol{\tau}$$

where $\widehat{b} = b/B_0$

For each particle species (drop r index for simplicity)

Pressure tensor $\mathbf{p} = \mathbf{P} + \mathbf{\Pi}$ sum of a gyrotropic pressure $\mathbf{P} = p_\perp \mathbf{n} + p_\| \boldsymbol{\tau}$ (with $2p_\perp = \mathbf{p} : \mathbf{n}$ and $p_\| = \mathbf{p} : \boldsymbol{\tau}$) and of a gyroviscosity tensor $\mathbf{\Pi}$ that satisfies $\mathbf{\Pi} : \mathbf{n} = 0$ and $\mathbf{\Pi} : \boldsymbol{\tau} = 0$.

Equations for the gyrotropic pressure components,

$$\partial_t p_\perp + \nabla \cdot (u\, p_\perp) + p_\perp \nabla \cdot u - p_\perp \widehat{b} \cdot \nabla u \cdot \widehat{b} + \frac{1}{2}(\mathrm{tr}\, \nabla \cdot \mathbf{q} - \widehat{b} \cdot (\nabla \cdot \mathbf{q}) \cdot \widehat{b})$$

$$+\frac{1}{2}(\mathrm{tr}\, (\mathbf{\Pi} \cdot \nabla u)^S - (\mathbf{\Pi} \cdot \nabla u)^S : \boldsymbol{\tau} + \mathbf{\Pi} : \frac{d\boldsymbol{\tau}}{dt}) = 0$$

$$\partial_t p_\| + \nabla \cdot (u\, p_\|) + 2p_\| \widehat{b} \cdot \nabla u \ \cdot \widehat{b} + \widehat{b} \cdot (\nabla \cdot \mathbf{q}) \cdot \widehat{b} + (\mathbf{\Pi} \cdot \nabla u)^S : \boldsymbol{\tau} - \mathbf{\Pi} : \frac{d\boldsymbol{\tau}}{dt} = 0.$$

Similar decomposition of the heat flux tensor $\mathbf{q} = \mathbf{S} + \boldsymbol{\sigma}$ with the conditions $\sigma_{ijk} n_{jk} = 0$ and $\sigma_{ijk} \tau_{jk} = 0$. One has

$$S_{ijk} = \frac{1}{2}(S_i^{\perp} n_{jk} + S_j^{\perp} n_{ik} + S_k^{\perp} n_{ij} + S_l^{\perp} \tau_{li} n_{jk} + S_l^{\perp} \tau_{lj} n_{ik} + S_l^{\perp} \tau_{lk} n_{ij})$$
$$+ S_i^{\|} \tau_{jk} + S_j^{\|} \tau_{ik} + S_k^{\|} \tau_{ij} - \frac{2}{3}(S_l^{\|} \tau_{li} \tau_{jk} + S_l^{\|} \tau_{lj} \tau_{ik} + S_l^{\|} \tau_{lk} \tau_{ij})$$

which is characterized by the parallel and transverse heat flux vectors $S^{\|}$ and $S^{\perp}$ with components $S_i^{\|} = q_{ijk} \tau_{jk}$ and $2S_i^{\perp} = q_{ijk} n_{jk}$.

$\boldsymbol{\sigma}$ involves either nonlinear contributions or linear contributions of order $0(\frac{k^2}{\Omega^2} v_{th}^2)$, and thus turns out to be negligible in the equations for gyroviscosity or heat flux tensors.

Since $m_e/m_i \ll 1$: only non-gyrotropic corrections due to ions are retained.

Weakly nonlinear regime: nongyrotropic contributions $\mathbf{\Pi}$, $S_{\perp}^{\perp}$ and $S_{\perp}^{\|}$ retained at the linear level only.

Fourth order moment are written in the form

$$\rho r_{ijkl} = P_{ij}P_{lk} + P_{ik}P_{jl} + P_{il}P_{jk} + P_{ij}\Pi_{lk} + P_{ik}\Pi_{jl} + P_{il}\Pi_{jk}$$
$$+\Pi_{ij}P_{lk} + \Pi_{ik}P_{jl} + \Pi_{il}P_{jk} + \rho\widetilde{r}_{ijkl}.$$

with a gyrotropic form for the tensor $\widetilde{r}$:

$$\widetilde{r}_{ijkl} = \frac{\widetilde{r}_{\|\|}}{3}(\tau_{ij}\tau_{kl} + \tau_{ik}\tau_{jl} + \tau_{il}\tau_{jk}) + \widetilde{r}_{\|\perp}(n_{ij}\tau_{kl} + n_{ik}\tau_{jl} + n_{il}\tau_{jk}$$
$$+\tau_{ij}n_{kl} + \tau_{ik}n_{jl} + \tau_{il}n_{jk}) + \frac{\widetilde{r}_{\perp\perp}}{2}(n_{ij}n_{kl} + n_{ik}n_{jl} + n_{il}n_{jk})$$

Introducing the scalar quantities $r_{\|\|} = r_{ijlk}\tau_{ij}\tau_{kl}$, $r_{\perp\|} = \frac{1}{2}r_{ijlk}n_{ij}\tau_{kl}$ and $r_{\perp\perp} = \frac{1}{4}r_{ijlk}n_{ij}n_{kl}$ (and similar definitions for the tilde quantities),

$$\widetilde{r}_{\|\|} = r_{\|\|} - 3\frac{p_\|^2}{\rho} \qquad \widetilde{r}_{\|\perp} = r_{\|\perp} - \frac{p_\perp p_\|}{\rho} \qquad \widetilde{r}_{\perp\perp} = r_{\perp\perp} - 2\frac{p_\perp^2}{\rho}$$

$$\nabla_\perp \cdot \Pi_\perp = \frac{p_\perp^{(0)}}{2\Omega}\widehat{z} \times \Delta_\perp u - \frac{\rho^{(0)}}{2m} r_L^2 \Delta_\perp \nabla_\perp T_\perp^{(1)} - \frac{1}{2\Omega^2}\Delta_\perp \nabla_\perp \widetilde{r}_{\perp\perp} + \frac{1}{2\Omega}\widehat{z} \times \partial_t \nabla_\perp \cdot \Pi_\perp$$

$$S_\perp^\perp = \frac{2p_\perp^{(0)}}{m\Omega}\widehat{z} \times \nabla_\perp T_\perp^{(1)} - \frac{p_\perp^{(0)}}{2} r_L^2 \Delta_\perp u_\perp + \frac{2}{\Omega}\widehat{z} \times \nabla_\perp \widetilde{r}_{\perp\perp} + \frac{1}{\Omega}\widehat{z} \times \partial_t S^\perp$$

$$\Pi_z = \frac{\widehat{z}}{\Omega} \times (\nabla_\perp S_z^\perp + p_\perp^{(0)} \nabla_\perp u_z + p_\parallel^{(0)} \partial_z u_\perp - (p_\perp^{(0)} - p_\parallel^{(0)})\partial_t \widehat{b}_\perp + \partial_t \Pi_z)$$

$$-\frac{1}{\Omega^2}\partial_z\Big(\frac{p_\perp^{(0)}}{m}\nabla_\perp T_\parallel^{(1)} - 2\frac{p_\perp^{(0)} - p_\parallel^{(0)}}{m} T_\parallel^{(0)} \partial_z \widehat{b}_\perp + \nabla_\perp \widetilde{r}_{\parallel\perp}\Big)$$

$$S_\perp^\parallel = \frac{\widehat{z}}{\Omega} \times \Big(\frac{p_\perp^{(0)}}{m}\nabla_\perp T_\parallel^{(1)} - 2\frac{p_\perp^{(0)} - p_\parallel^{(0)}}{m} T_\parallel^{(0)} \partial_z \widehat{b}_\perp + \nabla_\perp \widetilde{r}_{\parallel\perp} + \partial_t S_\perp^\parallel\Big)$$

$$-\frac{2T_\parallel^{(0)}}{m\Omega^2}\partial_z(\nabla_\perp S_z^\perp + p_\perp^{(0)} \nabla_\perp u_z + p_\parallel^{(0)} \partial_z u_\perp - (p_\perp^{(0)} - p_\parallel^{(0)})\partial_t \widehat{b}_\perp).$$

$$\nabla_\perp \cdot \Pi_\perp = (\partial_x \Pi_{xx} + \partial_y \Pi_{xy}, \partial_x \Pi_{xy} + \partial_y \Pi_{yy}, 0),$$

$$\Pi_z = (\Pi_{xz}, \Pi_{yz}, \Pi_{zz} = 0).$$

Nonlinear equations for the longitudinal components of parallel and transverse heat flux vectors (retaining only lowest order nonlinearities)

$$\partial_t S_z^{\|} + \nabla \cdot (S_z^{\|} u) + 3S_z^{\|}\partial_z u_z + 3p_{\|}(\widehat{b}\cdot\nabla)(\frac{p_{\|}}{\rho}) - p_{\perp}^{(0)}\widehat{b}_{\perp}\cdot\nabla_{\perp}(\frac{p_{\|}}{\rho})$$

$$+\frac{2p_{\|}^{(0)}}{\rho^{(0)}}(p_{\|}^{(0)} - p_{\perp}^{(0)})\partial_z\widehat{b}_z + \nabla\cdot(\widetilde{r}_{\|\|}\widehat{b}) - 3\widetilde{r}_{\|\perp}\nabla\cdot\widehat{b} - (b_{\perp}\cdot\nabla_{\perp})\widetilde{r}_{\|\perp} = 0,$$

$$\partial_t S_z^{\perp} + \nabla\cdot(uS_z^{\perp}) + S_z\nabla\cdot u + p_{\|}(\widehat{b}\cdot\nabla)(\frac{p_{\perp}}{\rho}) - 2p_{\perp}^{(0)}(\widehat{b}_{\perp}\cdot\nabla_{\perp})(\frac{p_{\perp}}{\rho})$$

$$+\frac{p_{\perp}^{(0)}}{\rho^{(0)}}(\partial_x\Pi_{xz} + \partial_y\Pi_{yz}) + \nabla\cdot(\widetilde{r}_{\|\perp}\widehat{b})$$

$$+(\frac{p_{\perp}(p_{\|} - p_{\perp})}{\rho} - \widetilde{r}_{\perp\perp} + \widetilde{r}_{\|\perp})(\nabla\cdot\widehat{b}) - (\widehat{b}_{\perp}\cdot\nabla_{\perp})\widetilde{r}_{\perp\perp} = 0.$$

Fourth order moment closure

Turn to kinetic theory. Compute various hydrodynamic quantities using linearly perturbed distribution function, at second order in ω/Ω. When comparing $\widetilde{r}_{\parallel\parallel}$ with $S_z^{\parallel}$ or $T_\parallel^{(1)}$, one gets ($\zeta = \frac{k_\parallel}{\Omega}\sqrt{\frac{m}{2T_\parallel^{(0)}}}$).

$$\widetilde{r}_{\parallel\parallel} = \sqrt{\frac{2T_\parallel^{(0)}}{m}} \frac{2\zeta^2(1+2\zeta^2 R(\zeta)) + 3(R(\zeta)-1) - 12\zeta^2 R(\zeta)}{2\zeta(1-3R(\zeta)+2\zeta^2 R(\zeta))} S_z^{\parallel}.$$

$$\widetilde{r}_{\parallel\parallel} = \frac{p_\parallel^{(0)} T_\parallel^{(0)}}{m} \frac{2\zeta^2(1+2\zeta^2 R(\zeta)) + 3(R(\zeta)-1) - 12\zeta^2 R(\zeta)}{1-R(\zeta)+2\zeta^2 R(\zeta)} \frac{T_\parallel^{(1)}}{T_\parallel^{(0)}}.$$

Proceeding as in Snyder et al. (1997) , we write

$$\widetilde{r}_{\parallel\parallel} = \beta_\parallel p_\parallel^{(0)} \frac{T_\parallel^{(0)}}{m} [\frac{T_\parallel^{(1)}}{T_\parallel^{(0)}} - D_\parallel \sqrt{\frac{2T_\parallel^{(0)}}{m}} i \frac{k_z}{|k_z|} S_z^{\parallel}],$$

where $\beta_\parallel = \frac{32-9\pi}{3\pi-8}$ and $D_\parallel = \frac{2\sqrt{\pi}}{3\pi-8}$ are determined by matching with the exact kinetic expressions in the isothermal $|\zeta| \ll 1$ and adiabatic limits $|\zeta| \gg 1$.

$\widetilde{r}_{\parallel\perp}$ can be expressed in terms of $S_z^{\perp}$ and the parallel current j_z.

$$\widetilde{r}_{\parallel\perp} = \sqrt{\frac{2T_{\parallel}^{(0)}}{m}} \frac{1 - R(\zeta) + 2\zeta^2 R(\zeta)}{2\zeta R(\zeta)} [S_z^{\perp} + (\Gamma_0(b) - \Gamma_1(b)) \frac{p_{\perp}^{(0)} p_{\parallel}^{(0)}}{\rho^{(0)} v_A^2} (\frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} - 1) \frac{j_z}{en^{(0)}}],$$

or using a three-pole Padé approximant for R,

$$(\frac{d}{dt} - \frac{2}{\sqrt{\pi}} \sqrt{\frac{2T_{\parallel}^{(0)}}{m}} \mathcal{H}_z \partial_z) \widetilde{r}_{\parallel\perp} + \frac{2T_{\parallel}^{(0)}}{m} \partial_z [S_z^{\perp} + \frac{p_{\perp}^{(0)}}{v_A^2} (\frac{T_{\perp}^{(0)} - T_{\parallel}^{(0)}}{m_p}) \frac{j_z}{en^{(0)}}] = 0$$

where $b = \frac{k_{\perp}}{\Omega} \sqrt{\frac{m}{2T_{\perp}^{(0)}}}$, $\Gamma_n(b) = e^{-b} I_n(b) \approx (\frac{b}{2})^n$ as $b \rightarrow 0$.

$\widetilde{r}_{\perp\perp}$ negligible in the large scale limit.

Validation

- For parallel Alfvén waves,

Leading order (proton) gyroviscous tensor is sufficient

$$\pi^{(1)}_{xx} = -\pi^{(1)}_{yy} = -\frac{p_\perp}{2\Omega}(\partial_y u_x + \partial_x u_y), \qquad \pi^{(1)}_{zz} = 0, \qquad \pi^{(1)}_{xy} = -\frac{p_\perp}{2\Omega}(\partial_y u_y - \partial_x u_x)$$

$$\pi^{(1)}_{yz} = \frac{1}{\Omega}[2p_\parallel \partial_z u_x + p_\perp(\partial_x u_z - \partial_z u_x)], \qquad \pi^{(1)}_{xz} = -\frac{1}{\Omega}[2p_\parallel \partial_z u_y + p_\perp(\partial_y u_z - \partial_z u_y)]$$

Only the longitudinal components $S^\perp_z$ and $S^\parallel_z$ of the transverse and parallel heat transfer vectors are relevant.

The long-wave reductive perturbative expansion performed on the resulting Landau-fluid model reproduces the KDNLS equation derived from Vlasov-Maxwell, up to the replacement of the plasma response functions by the corresponding two- or four-pole approximants.

Consequence: modulational type instabilities (including filamentation) of Alfvén waves and their weakly nonlinear developments are correctly reproduced.

- Alfvén waves at finite angle of propagation:

FLR corrections of order $1/\Omega_p^2$ are to be retained.
The governing equation is linear and reads, assuming $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$, (adiabatic protons and isothermal electrons) and $\beta \ll 1$ (ξ: stretched coordinate along the propagation)

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2}[\frac{\cos^3\alpha}{\sin^2\alpha} + \sqrt{\beta}\sqrt{\frac{\pi}{2}}\sqrt{\frac{m_e}{m_p}}\cos^3\alpha(\tan^2\alpha + \frac{1}{\tan^2\alpha})\mathcal{H}]\partial_{\xi\xi\xi}\frac{b_y}{B_0} = 0,$$

- Kinetic Alfvén waves ($\cos^2\alpha \ll \beta$):

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2}\cos\alpha[-\beta(1 + \frac{3}{4}\frac{T_p^{(0)}}{T_e^{(0)}}) + \sqrt{\beta}\sqrt{\frac{\pi}{2}}\sqrt{\frac{m_e}{m_p}}\mathcal{H}]\partial_{\xi\xi\xi}\frac{b_y}{B_0} = 0.$$

agreement with Akhiezer et al. '75, Hasegawa and Chen '76.

- Magnetosonic waves:

Landau damping rate is (assuming $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$)

$$\gamma = -\sqrt{\beta}\sqrt{\frac{\pi}{8}}\sqrt{\frac{m_e}{m_p}}\frac{\sin^2\alpha}{\cos\alpha}\frac{(\omega^2-\beta\cos^2\alpha)^2+\beta^2\cos^4\alpha}{(2\omega^2-\beta-1)(\omega^2-\beta\cos^2\alpha)}kv_A,$$

an expression identical to that found by a direct derivation from the Vlasov-Maxwell equations.

The long-wave equation is KdV+damping term.

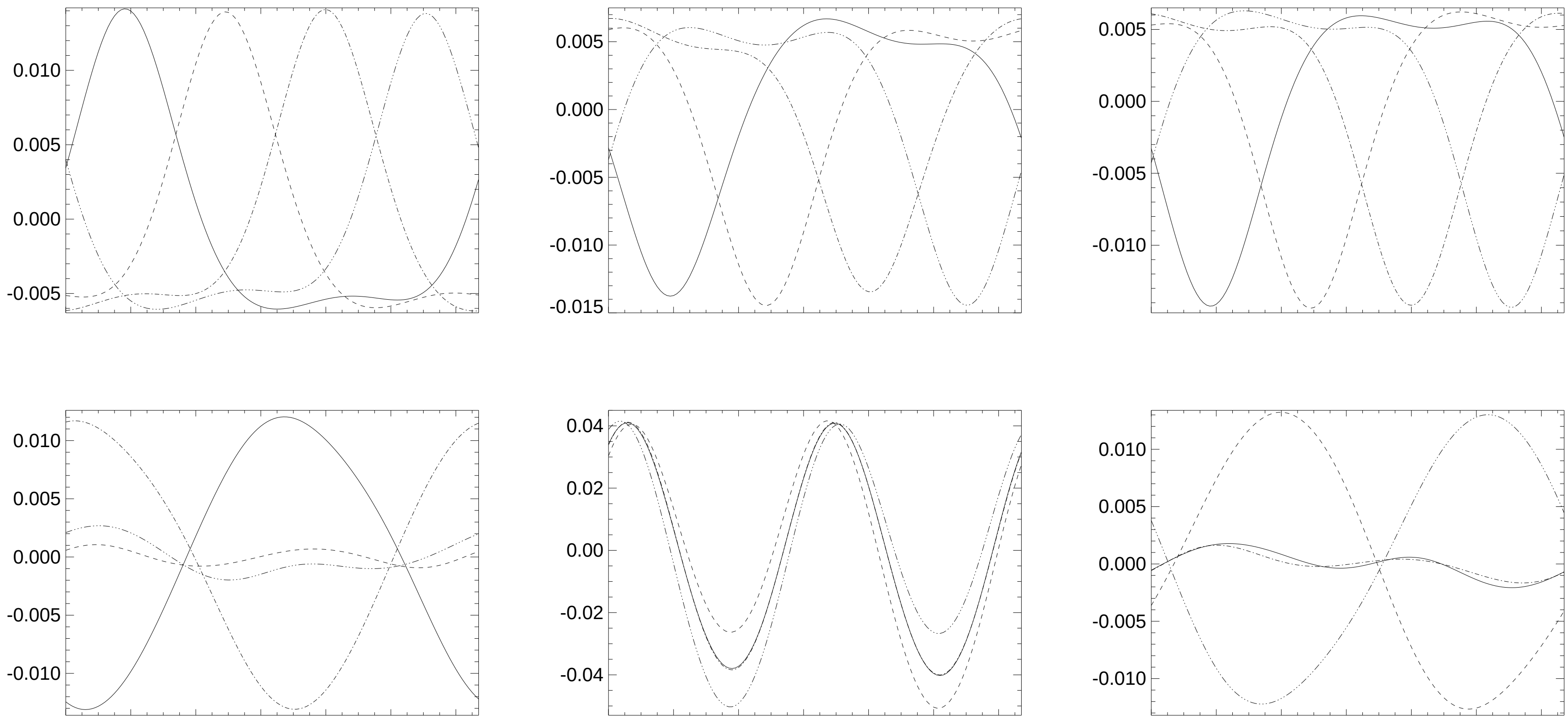

Figure 1: Profiles of the $\widetilde{u}_z$ component (left), of the density perturbation (middle) and of the magnetic field perturbation (right) at successive times (denoted by solid, dashed, dotted-dashed and triple-dotted-dashed lines) separated by 1.5 units starting at $t = 104.3$ (top) and at $t = 1502.3$ (bottom), in the case $a_0 = 10^{-2}$, $R_p = 10^{-1}$ and $\beta = 10^{-3}$.

Formation of SOLITONIC STRUCTURES with a hump for the parallel velocity and density depressions correlated with magnetic holes.

Eventually, the profile evolves to a quasi-stationary wave.

Dissipation of the magnetosonic wave very weak and Alfvén waves subdominant.

Larger amplitude regime

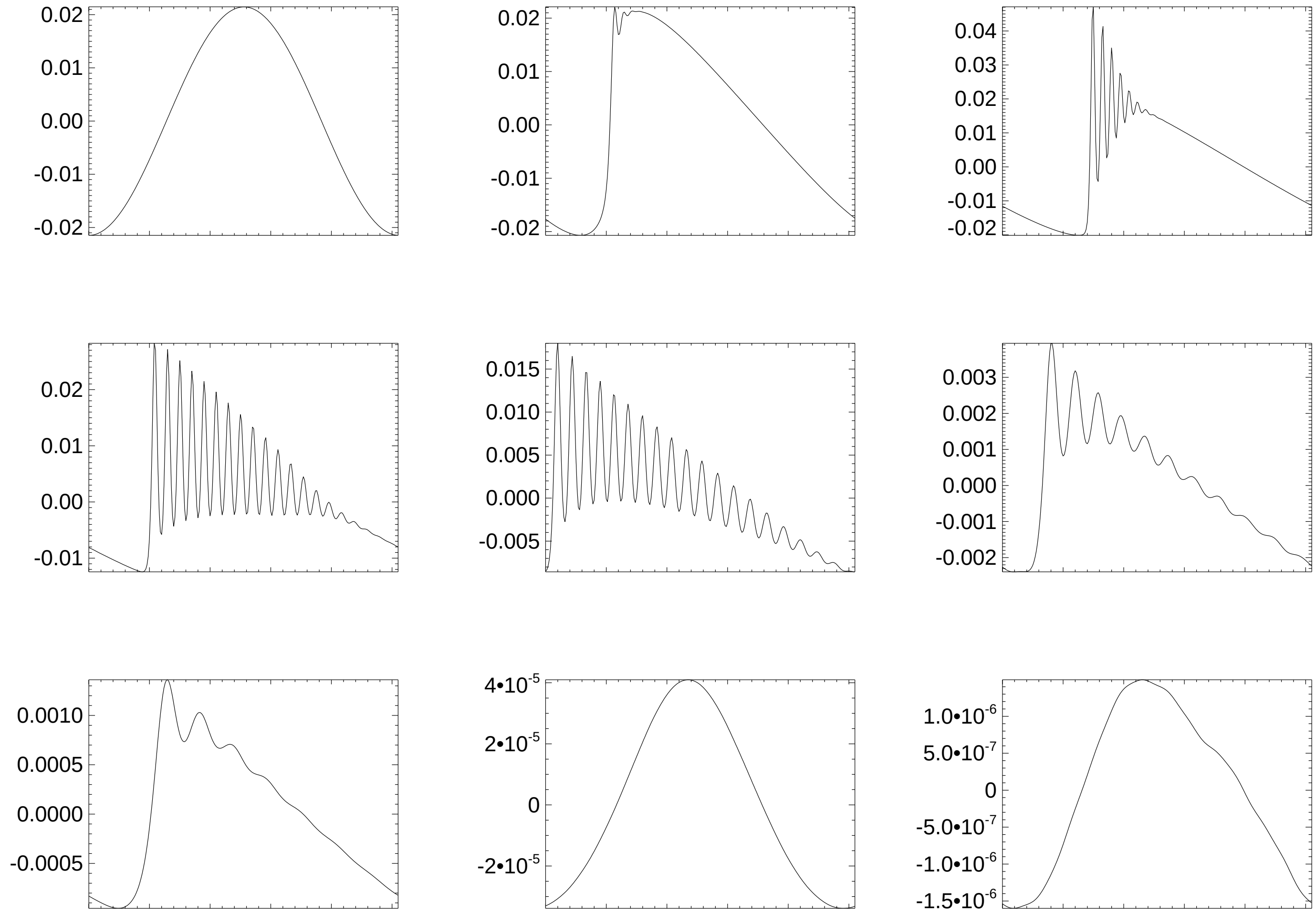

Figure 2: Snapshots at times $t = 1, 32, 45, 101, 150, 501, 1002, 4000$ and 17000 of the longitudinal velocity component $\widetilde{u}_z$ for a magnetosonic wave of initial amplitude $a_0 = 2.2\ 10^{-2}$, with $\beta = 10^{-2}$ and $R_p = 7\ 10^{-3}$.

Formation and dissipation of the high frequency modes in a dispersive shocks. Magnetosonic contribution is completely dissipated. Increase of $T_{\|e}$ by 20%.

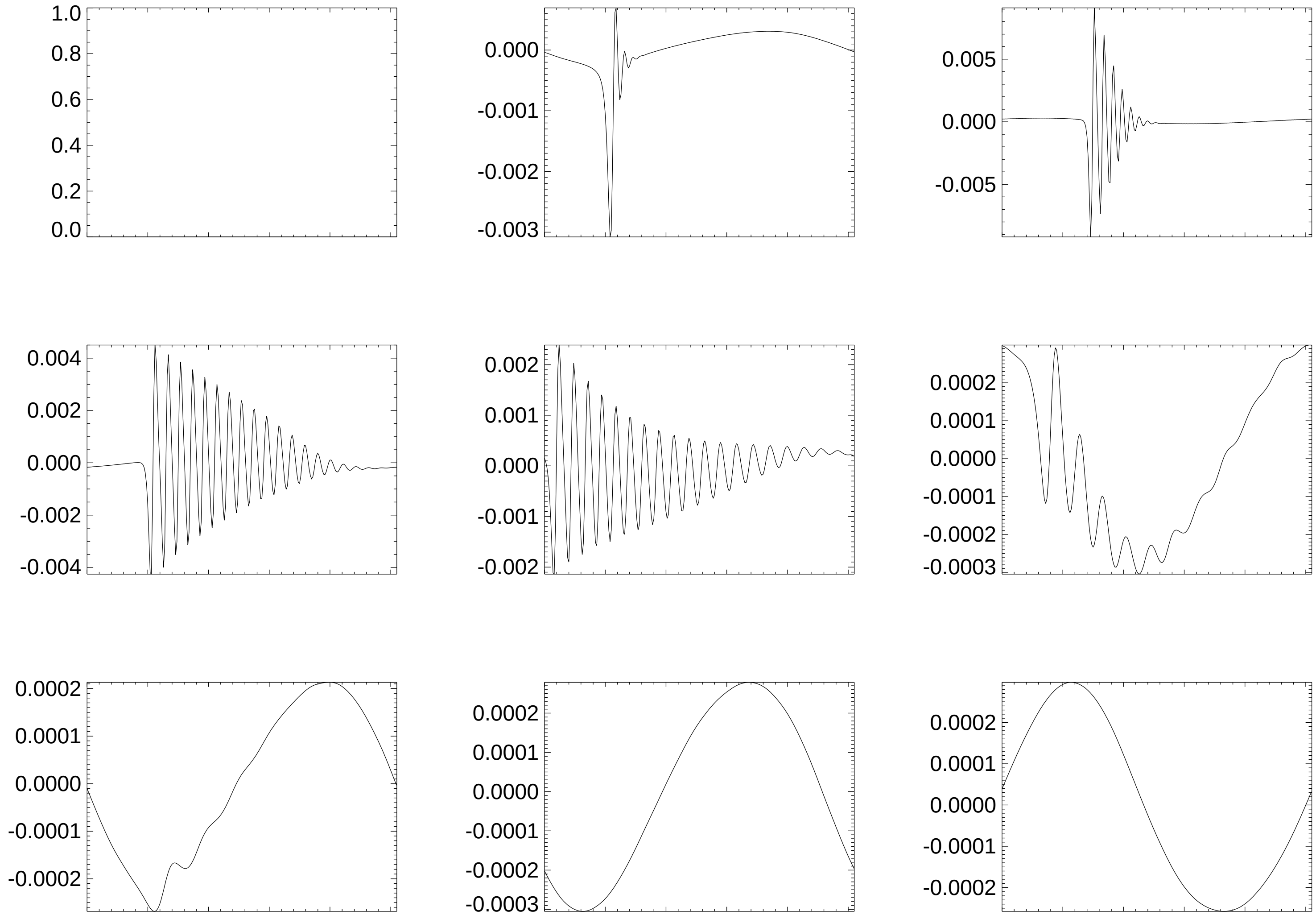

Figure 3: Same as before for the transverse velocity component $\widetilde{u}_y$.

An Alfvén wave is generated that evolves to a large-scale profile. Small-scale oscillations being damped at a rate that scales like k^3/R_p^2. Resulting state: an Alfvén wave essentially insensitive to Landau damping.

- Transverse magnetosonic waves: (isotropic equilibrium temperatures)

Dispersion relation:

$$\omega^2 = k^2 v_A^2 (1+\beta) \left(1 - \frac{1}{4} k_\perp^2 r_L^2 \frac{1+\beta_e+5/2\beta_i}{1+\beta_e+\beta_i} \right)$$

with $\beta_r = \dfrac{8\pi p_r^{(0)}}{B_0^2}$, $\beta = \beta_i + \beta_e$

In agreement with kinetic theory (Mikhailovskii and Smolyakov 1985)

Perspectives

• Benchmark the model by comparison with gyrokinetic and Vlasov-Maxwell simulations .

• Explore the nonlinear stage of parametric instabilities.

• Modelisation of coherent structures (magnetic holes and shocklets) observed in the solar wind and magnetosheath.

• Simulation of dispersive Alfvén wave turbulence

• Explore the possible description of nonlinear Landau damping.

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