Scale-Relativistic Estimate of the SU(3), SU(2) and U(1) Couplings

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Abstract. The value of the SU(3) coupling constant is estimated in the framework of the scale-relativistic minimal standard model. It is evolved, thanks to its renormalization group equation, from the GUT scale (which is identified with the Planck mass in scale relativity) where its value is conjectured to be $1/4\pi^2$, to the Z boson scale, where its value is predicted to be 0.115. We also discuss the emergence of new definite structures, in particular of a fundamental mass-scale at 123.23 GeV.

1. Introduction

In a recent paper [1], we have applied the scale-relativistic method to the problem of the estimate of the low energy fine structure constant. The aim of the present letter is to consider the case of the SU(3), SU(2) and U(1) couplings and to point out the emergence of a new mass scale at 123.228(8) GeV in this theory.

Let us first recall that scale relativity [2] is based on the postulate that the equations of physics must be written in a scale-covariant way and that the Planck length-scale is invariant under scale transformations (i.e., dilatation and contractions). We send the reader to refs [2] for a full justification of these postulates. We shall simply recall here the main new relations obtained in this framework, which generalize the standard theory relations.

Consider a relevant field whose variation in terms of scale is given in the standard renormalization group by a power law $\varphi = \varphi_o\,(\lambda_o/r)^{\delta}$, where the anomalous dimension δ is usually assumed to be constant. In scale relativity, this law takes the scale-covariant form:

$$\varphi = \varphi_o\,(\lambda_o/r)^{\delta(r)}\,, \tag{1}$$

with the anomalous dimension δ now varying with scale (for resolution $r \leq \lambda_o$) as

$$\delta(r) = \frac{\delta_o}{\sqrt{1 - ln^2(\lambda_o/r)\,/\,ln^2(\lambda_o/\mathbb{A})}}\,, \tag{2}$$

where λ_o is the Compton length of the system considered, and where $\mathbb{A}$ is the Planck length:

$$\mathbb{A} = (\hbar G/c^3)^{1/2} = 1.61605(10) \times 10^{-35}\ \text{m}, \tag{3}$$

now interpreted as a limiting, lower length-scale, impassable, invariant under dilatations and contractions. The energy-scale and length-scale are no longer directly inverse, but related by the scale-relativistic generalized Compton formula

$$ln\,\frac{m}{m_o} = \frac{ln\ (\lambda_o/\lambda)}{\sqrt{1 - \dfrac{ln^2(\lambda_o/\lambda)}{ln^2(\lambda_o/\mathbb{A})}}}\,, \tag{4}$$

i.e., $m/m_o=(\lambda_o/\lambda)^{\delta(\lambda)}$. A similar generalization holds for the Heisenberg relations [2].

Concerning marginal fields, their variation with *length-scale* is unaffected by scale-relativistic corrections [2], but this is no longer the case in terms of mass-scale. The passage to mass-scale is now performed by using Eq.(4). For example, the (formal) QED inverse coupling reads, from its renormalization group equation [3-5], to lowest order [1]

$$\bar{\alpha}(m) = \bar{\alpha}(m_Z) - \frac{10+N_H}{6\pi}\,\frac{ln(m/m_Z)}{\sqrt{1 + ln^2(m/m_Z)/\mathbb{C}_Z^{\,2}}}\,, \tag{5}$$

in terms of a running mass-energy scale m. Here N_H is the number of Higgs doublets and $\mathbb{C}_Z = ln(\lambda_Z/\mathbb{A}) = 39.75585(24)$, from $m_Z = 91.182(23)$ GeV [8,15] and Eq. (4). Note that scale relativity introduces a Lorentzian structure for the generalized renormalization group, but with values of the "constants" $\mathbb{C}$'s which are constant only for a given Compton (more generally de Broglie) scale, i.e., once given the mass and state of motion of the system considered.

2. First results of scale relativity: a reminder

Let us briefly recall the results which have already been obtained in this new framework [1,2].

(i) The charges and the self-energies now have finite non-zero values at infinite energy scale.

(ii) A new fundamental scale emerges, which is given by the length-scale corresponding to the Planck energy. This new scale is given to lowest order by $ln(\lambda_Z/\lambda) = \mathbb{C}_Z/\sqrt{2}$, and is thus $\approx 10^{-12}$ times smaller than W/Z length-scale. In other words, this is but the GUT scale (10^{14} GeV in the standard theory).

(iii) As a consequence, the *four* fundamental couplings, U(1), SU(2), SU(3) *and* gravitational converge in the new framework towards about the same scale, *which now corresponds to the Planck energy.*

(iv) The GUT energy now being of the order of the Planck one ($\approx 10^{19}$GeV), the predicted lifetime of the proton ($\propto m_{GUT}^4/m_p^5 >> 10^{38}$ yrs) becomes compatible with experimental results ($> 5.5 \times 10^{32}$ yrs, [6]).

(v) The formal QED inverse coupling $\bar{\alpha}_0 = \frac{3}{8}\bar{\alpha}_2 + \frac{5}{8}\bar{\alpha}_1 = \frac{3}{8}\bar{\alpha}$ has been shown to converge towards the value $4 \times (3.1411 \pm 0.0019)^2 \approx 4\pi^2$ at infinite energy. Conversely, the conjecture that the corresponding "bare" charge is $1/2\pi$ allowed us to estimate the low energy fine structure constant to better than 1‰ [1], provided the number of Higgs doublets, which contributes to 2.11 N_H in the final value of $\bar{\alpha}$, is fixed to $N_H = 1$.

3. Scale-relativistic Estimate of the SU(3) Coupling

The fact that, in scale relativity, the four couplings are now convergent towards about the same energy provides us with a new opportunity of analysis of "Grand Unification". In particular, we may now consider the intersection of the U(1), SU(2) and SU(3) couplings with the gravitational one.

Following Einstein's construction of general relativity, the gravitational coupling is directly given by mass-energy-momentum, so that it becomes running with scale in the quantum relativistic domain as $\alpha_g = Gm^2/\hbar c = (m/m_{\mathbb{P}})^2$, where $m_{\mathbb{P}} = (\hbar c/G)^{1/2} = 1.22105(8) \times 10^{19}$ GeV. The SU(3) running inverse coupling is given by its renormalization group equation [4,5]. To next to leading order, its solution for 3 families of leptons can be written as

$$\bar{\alpha}_3(r) = \bar{\alpha}_3(\lambda_Z) + \frac{7}{2\pi} ln\frac{\lambda_Z}{r} + \frac{11}{4\pi(40+N_H)} ln\{1 - \frac{40+N_H}{20\pi} \alpha_1(\lambda_Z) ln\frac{\lambda_Z}{r}\}$$

$$- \frac{27}{4\pi(20-N_H)} ln\{1 + \frac{20-N_H}{12\pi} \alpha_2(\lambda_Z)\, ln\frac{\lambda_Z}{r}\} + \frac{13}{14\pi} ln\{1 + \frac{7}{2\pi} \alpha_3(\lambda_Z) ln\frac{\lambda_Z}{r}\} \quad . \tag{6}$$

The experimental value of $\bar{\alpha}_3$ at the Z scale has been recently improved [7]:

$$\alpha_3(m_Z) = 0.112 \pm 0.003 \tag{7}$$

i.e. $\bar{\alpha}_3(\lambda_Z) = 8.93 \pm 0.23$. Basing ourselves on our result of ref. [1], we shall consider only the case of 1 Higgs doublet in what follows. To lowest order, the equation of intersection of the SU(3) and gravitational couplings writes in terms of mass scale:

$$\bar{\alpha}_3(m_Z) + \frac{7}{2\pi} \frac{ln(m/m_Z)}{\sqrt{1 + ln^2(m/m_Z)/\mathbb{C}_Z^2}} = \left(\frac{m_{\mathbb{P}}}{m}\right)^2, \tag{8}$$

From the above value of $\alpha_3(m_Z)$, we find that they cross at an energy $m = 1.94 \times 10^{18}$ GeV, i.e., $m_{\mathbb{P}}/2\pi$. This is confirmed by the value of the common inverse coupling at crossing, $\bar{\alpha}_{3g} = 39.36 \pm 0.23$, in good agreement with $4\pi^2 = 39.478$ (see Fig. 1).

Conversely we shall now make the conjecture that the inverse coupling is indeed $4\pi^2$ at the crossing energy. From this conjecture we can thus predict the value of $\bar{\alpha}_3$ at the Z energy. We find to lowest order $\bar{\alpha}_3 = 9.045$. The second order variation depends on the values of $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$. As in ref. [1], we do not need high precision for these quantities to be input in Eq. (6). So we assume as first approximation that all 4 couplings converge towards the same energy $m_{GUT} = m_{\mathbb{P}}/2\pi$, and use the lowest order solutions to their renormalization group equations (neglecting also the Higgs doublet) to obtain the couplings at Z scale:

$$\bar{\alpha}_1(\lambda_Z) = \bar{\alpha}_1(\lambda_{GUT}) + \frac{2}{\pi} ln(\lambda_Z/\lambda_{GUT}) \tag{9a}$$

$$\bar{\alpha}_2(\lambda_Z) = \bar{\alpha}_2(\lambda_{GUT}) - \frac{5}{3\pi} ln(\lambda_Z/\lambda_{GUT}) \tag{9b}$$

From $\mathbb{V}_{GUT} = ln(\lambda_Z/\lambda_{GUT}) = 27.32$, we find $\bar{\alpha}_1(\lambda_Z) \approx 57$, and $\bar{\alpha}_2(\lambda_Z) \approx 25$ (the current values are 59.22(14) and 30.10(23) [5]). Inserting these values in Eq. (6) and solving for $\bar{\alpha}_3$ finally yields: $\bar{\alpha}_3(m_Z) = 8.653 \pm 0.018$, where the quoted uncertainty is a rough estimate of the third order contribution; in comparison, the effect of the error on $\bar{\alpha}_1$ and $\bar{\alpha}_2$ (which intervene in the second order terms only) can be shown to be negligible. This result corresponds to a value of the coupling constant:

$$\alpha_3(m_Z) = 0.1155 \pm 0.0002\ , \tag{10}$$

where the uncertainty is more than ten times smaller than the present experimental one, so that such a prediction is falsifiable by future precision experiments.

4. Emergence of a New Fundamental Scale

One of the main mysteries left open in the standard model is that of the origin of the two fundamental symmetry breaking scales, namely the electroweak and GUT scales. We have already seen that scale relativity suggests a solution for the nature of the GUT scale, which becomes the Planck mass-scale in its framework. What about the electroweak scale?

We have no completely satisfying solution to this problem yet, but we note that definite structures seem to arise in the plane {log of scale / couplings} (see Fig.1). Several results (see hereabove and ref. [1]) lead us to the conjecture that the value $4\pi^2$ is critical for the inverse couplings. Now it happens that the WZ mass-scale is precisely $exp(4\pi^2)$ smaller than the Planck scale. Indeed, from the recently measured values at LEP, $m_Z = 91.182(7)$ GeV and $m_W = 79.9(2)$ GeV [8], we find:

$$\frac{1}{3}\, m_W + \frac{2}{3}\, m_Z = 87.42(11) \text{ GeV} \tag{11}$$

while

$$m_{WZ} \equiv m_{\mathbb{P}}\, e^{-4\pi^2} = 87.393(7) \text{ GeV} \quad . \tag{12}$$

Future improvements in the measurement of the W mass will allow one to test whether this is only pure numerology, or whether this is a genuine, though unexplained, relation between the Planck scale and the W/Z scale. The agreement is already remarkable, since $ln\{3m_{\mathbb{P}}/(2m_Z+m_W)\} = 39.4781$, differing relatively by only 10^{-5} from $4\pi^2 = 39.4784$. Assuming perfect equality between Eqs. (11) and (12), i.e.,

$$\frac{m_W + 2\, m_Z}{3\, m_{\mathbb{P}}} = e^{-4\pi^2}\ , \tag{13}$$

could yield a prediction for the W mass, $m_W = 79.815 \pm 0.015$ GeV. However such a small uncertainty may be misleading, since if a physical justication is once found for Eq. (13), it is expected to be corrected by higher order terms to this precision.

Now the scale-relativistic splitting between length-scales and mass-scales implies that another scale linked to the critical value $4\pi^2$ is defined by the equation

$$\mathbb{C}_V = ln(\lambda_V/\mathbb{A}) = 4\pi^2 \ , \qquad (14)$$

i.e., $ln(\lambda_e/\lambda_V) = \mathbb{C}_e - 4\pi^2$, where $\mathbb{C}_e = ln(\lambda_e/\mathbb{A}) = 51.52797(7)$. From Eq. (4), this corresponds to a mass-scale:

$$ln(\frac{m_V}{m_e}) = \frac{\mathbb{C}_e}{2\sqrt{2\pi}} \ \frac{\mathbb{C}_e - 4\pi^2}{\sqrt{\mathbb{C}_e - 2\pi^2}} \Rightarrow m_V = 123.228(8) \ \text{GeV} \qquad (15)$$

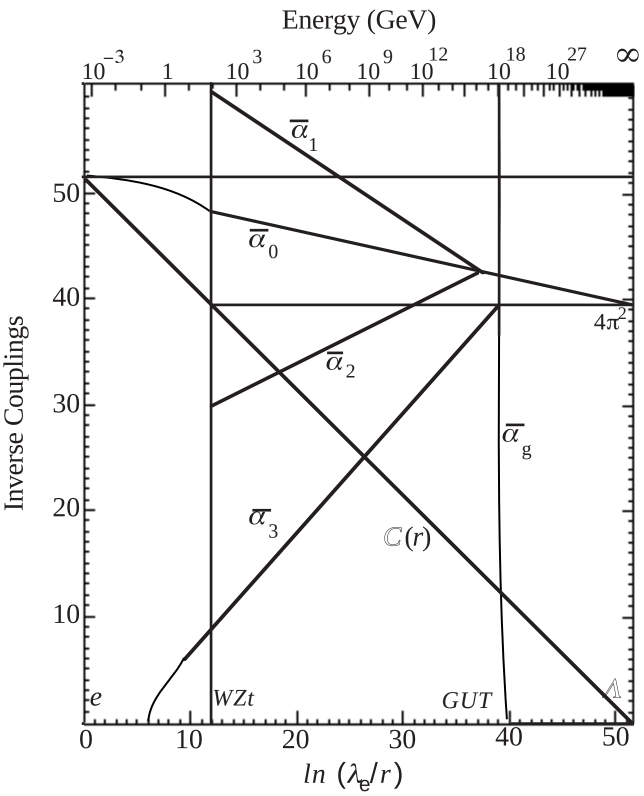

Figure 1. Inverse couplings versus logarithm of length-scale (bottom graduation), and mass-scale (top), in the scale-relativistic minimal standard model. (The inverse running coupling $\bar{\alpha}_0$ is defined as 3/8 of the electromagnetic inverse coupling and reaches the value (3/8)x137.036≈51.4 at the electron mass-scale). In scale relativity the Planck length plays the new role of an unpassable lowest length, invariant under dilatations (in the same way as the velocity of light is invariant under motion transformations). As a consequence of this new structure of space-time, the GUT scale becomes nothing but the Planck *mass-scale*. Several symmetries become apparent in this plane (see text).

5. On the top-quark mass.

In its present state, the scale-relativistic method is able to construct new fundamental scales, but without telling us which specific structure actually achieves them in the particle-field approach. So we are led to wonder whether there are structures in the standard model with which such a fundamental energy of 123.23 GeV could be identified. Two straighforward candidates come in mind.

The first is the *top quark*. Indeed, the experimental lower limit on the top mass is now larger than the *WZ* scale, $m_t > 89$ GeV [9], so that several authors begin to wonder whether the top quark could play a special role among elementary fermions. For example its has been proposed that the Higgs mechanism could be replaced with a dynamical symmetry breaking mechanism based on $t\bar{t}$ condensation [10,11]. The Higgs scalar would be replaced by a tightly bound $t\bar{t}$ state, and in all respects this scenario would look just like the minimal standard model. More generally, the fact that its mass is much closer from the *W* and *Z* boson masses than to other fermions suggests that there could exist some relationship between m_t and m_W [10,11]. These ideas allow one to predict top-quark masses $m_t \gtrsim 115$ GeV [11]. In another theoretical attempt, Osland and Wu [12] have predicted the Higgs scalar and top-quark masses by writing the finiteness of the one-loop correction in the electroweak theory. They find $m_t = (\frac{5}{2} m_Z^2 - m_W^2)^{1/2} \approx 120$ GeV. Different but compatible mass relations have been obtained by Blumhofer and Stech [13] from an analysis of the structure of the vacuum expectation value of the Higgs field. They find $m_t = 122$ GeV, in still better agreement with our prediction of Eq. (15).

From the experimental viewpoint, the top-quark mass is now fairly constrained by the precision electroweak experimental data. Amaldi, de Boer and Furstenau [5] or Marciano [10] find $m_t = 130 \pm 40$ GeV, and more recently del Aguila, Martinez and Quiros [14] have fitted precise electroweak data and obtain:

$$m_t = 122 \ ^{+25}_{-20} \ \text{GeV} \ , \qquad (16)$$

a value with which our prediction is in very good agreement.

6. The Electroweak Symmetry Breaking Scale and the U(1) and SU(2) Couplings

Another proposal (possibly not incompatible with the preceeding one) is that $2m_V$ yields the electroweak symmetry breaking scale. Under this conjecture we obtain

$$v = 246.456 \pm 0.016 \ \text{GeV} \ , \qquad (17)$$

where the uncertainty is mainly due to the badly known constant of gravitation *G*. The scale *v* is related to the *W* boson mass and to the SU(2) coupling by

$$v = \sqrt{\frac{\bar{\alpha}_2(m_W)}{\pi}} \ m_W \ . \qquad (18)$$

The value of m_W can be deduced from the precision LEP determination of $m_Z = 91.182(23)$ GeV [8,15] and from the value of $sin^2\theta_w = 1 - (m_W/m_Z)^2 = 0.2290(35)$ [5,15]. This gives $m_W = 80.05(19)$ GeV. We can then deduce the value of the SU(2) coupling at the *W* scale from Eq. (18) and our estimate for *v*. We find $\bar{\alpha}_2(m_W) = 29.78(15)$, so that:

$$\alpha_2(m_Z) = 0.03350(17) \ , \qquad (19)$$

in good agreement with the recent determination of Amaldi *et al* [5], $\alpha_2(m_Z) = 0.03322(25)$.

From our determination of $\bar{\alpha} = \bar{\alpha}_2 + \frac{5}{3}\bar{\alpha}_1$ of ref. [1]:

$$\bar{\alpha}(m_Z) = 32\pi^2/3 + \Delta\bar{\alpha}_{Z\infty} = 129.0 \pm 0.1 \qquad (20)$$

we can then deduce the value of $\alpha_1(m_Z)$. We find $\bar{\alpha}_1(m_Z) = 59.49(20)$, i.e:

$$\alpha_1(m_Z) = 0.01681(6) \ . \qquad (21)$$

Finally the crossing scale λ_{12} of the U(1) and su(2) couplings can be determined. It is given by the relation (scale-relativistic standard model, 1 Higgs doublet):

$$ln\,\frac{\lambda_Z}{\lambda_{12}} \;=\; \frac{30\pi}{109}\;\{\bar{\alpha}_1(m_Z) - \bar{\alpha}_2(m_Z)\} \qquad (22)$$

This yields $ln(\lambda_Z/\lambda_{12})$ = 25.63(23), i.e. m_{12} = 3.3 x 10^{16} GeV up to a factor of ≈2. This is a factor ≈50 smaller than the above determination of the GUT scale, $m_{GUT} = m_{\mathbb{P}}/2\pi$. This result is nothing but the scale-relativistic equivalent of the now well-known fact that the three coupling constants do not converge toward exactly the same scale [5, 17]. This has been considered by some authors as indicating possible new physics beyond the standard model, but one may remark with Ellis *et al*. [7] that there could be significant threshold effects at the unification scale. Moreover, in the frame of scale relativity, the question of unification is asked in a fundamentally different way, since it occurs at a scale where gravitation also plays a role (and become very quickly domimant for smaller length-scales). Gravitation may then both be essential in the symmetry breaking mechanism at the GUT scale (i.e. Planck mass-scale) and imply that the symmetries of the unified group (possibly SU(5)) be only approximate.

We may finally contemplate the possibility that *both* interpretations are correct, implying that the *W*, *Z*, *t* and *v* be related and of the same origin. This would mean that the running coupling parameter κ_t which relates the top mass to the scale *v* in the standard Higgs mechanism through the relation $m_t^2 = 2\pi\kappa_t(m_t)\, v^2$ [11] would be given by

$$\kappa_t(m_t) = \frac{1}{8\pi}\;. \qquad (23)$$

Now, among the solutions of the renormalization-group equation for the running κ_t, one self-consistent non trivial solution is singled out because it is an infrared-stable perturbative expansion about $\alpha_3 = 0$ [11]. It is given to two-loop in the case of the dynamical symmetry breaking mechanism by

$$\kappa_t \;=\; \tfrac{1}{3}\,\alpha_3 \;+ a\;\frac{\alpha_3}{\pi} \qquad (24)$$

where $a \approx 1$ [11]. Solving for α_3 this equation while assuming Eq. (23) to be true yields $\alpha_3(m_t) = 0.108$, i.e. $\alpha_3(m_Z) = 0.113$, in reasonable agreement with our above prediction 0.115 (Eq. 10) owing to the remaining uncertainties [11].

7. Summary and Conclusion

The aim of this letter was mainly to point out that well-defined structures are emerging in the plane (*log of scale, inverse couplings*) in the framework of the scale-relativistic minimal standard model. Concerning couplings, we have shown in Ref. [1], using its renormalization group equation, that the mean coupling $\bar{\alpha}_0 = \frac{3}{8}\bar{\alpha}_2 + \frac{5}{8}\,\bar{\alpha}_1 = \frac{3}{8}\,\bar{\alpha}$ was given to a very good approximation by $4\pi^2$ at the Planck length-scale, and here that the SU(3) inverse coupling crosses the gravitational inverse coupling at also the same value $\bar{\alpha}_3 = 4\pi^2$ at the Planck mass-scale. Concerning scales, we have argued that the electroweak / Planck scale ratio was also determined by the same number. Namely, the fundamental scale λ_V given by the relation $\mathbb{C}_V = 4\pi^2 = \bar{\alpha}_0(\infty)$ corresponds to a mass scale m_V = 123.23 GeV, which is very close to half the electroweak symmetry breaking scale (vacuum expectation value of the Higgs field), and also to recent theoretical predictions for the top-quak mass. A similar relation, applied to mass-scale rather than length-scale (recall that they are no longer directly inverse in scale relativity) yields a mass m_{WZ} = 87.393 GeV, closely connected to the *W* and *Z* boson masses. This leads us to the natural conjecture that, ultimately, the fundamental scales be determined by the fundamental couplings themselves.

These results, if confirmed by precision data, open two fundamental questions, which we shall try to answer in forthcoming works:

(i) Why is the value of the "bare" or critical charge $1/2\pi$? The answer certainly lies in gauge invariance, (the charges being the conservative quantities connected to the quantum phase transformations, which are themselves constrained to vary between 0 and 2π), and in an adequate description of physics at the unifying scale (recall that the quantization of charge is one of the results of GUTs, in particular if the unifying group is SU(5) [18]).

(ii) How does the charge determine the fundamental scales ? We shall suggest a renormalization group approach to this question in a forthcoming paper.

References

[1] L. Nottale, *Phys. Lett* **B** (1993) submitted

[2] L. Nottale, *Int. J. Mod. Phys.* **A7** (1991) 4899.
L. Nottale, *Int. J. Mod. Phys.* **A4** (1989) 5047.
L. Nottale, *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity* (World Scientific, 1992)

[3] W.J. Marciano, *Phys. Rev.* ***D*20** (1979) 274.
W.J. Marciano, *Phys. Rev. Lett.* **62** (1989) 2793.
U. Amaldi *et al., Phys. Rev.* ***D*36** (1987) 1385.

[4] D.R.T. Jones, *Phys. Rev*. **D25** (1982) 581.
K.S. Babu, & E. Ma, *Z. Phys.* **C 31** (1986) 451.

[5] U. Amaldi, W. de Boer, & H. Fürstenau, *Phys. Lett.* **B 260** (1991) 447.

[6] R. Becker-Szendy *et al., Phys. Rev*. **D42** (1990) 2974

[7] J. Ellis, S. Kelley & D.V. Nanopoulos, Phys. Lett. **B260** (1991) 131
A.D. Martin, R.G. Roberts, & W.J. Stirling, *Phys. Rev*. **D43** (1991) 3648.

[8] D.J. Miller, *Nature* **349** (1991) 379.

[9] K. Sliwa (CDF collaboration), in *Proc. XXVth Rencontre de Moriond*, ed. J. Tran Thanh Van (World Scientific, 1990).

[10] W.J. Marciano, *Ann. Rev. Nucl. Part. Sci.* **41** (1991) 469.

[11] W.J. Marciano, *Phys Rev. Lett*. **62** (1989) 2793.
J. Kubo, K. Sibold & W. Zimmermann, *Nucl. Phys.* **B259** (1985) 331.
W.J. Marciano, *Phys Rev.* **D41** (1990) 219.

[12] P. Osland,& T.T. Wu, *CERN Preprints* TH.6385-6387 (1992).

[13] A. Blumhofer & B. Stech, *Phys. Lett.* **B293** (1992) 137.

[14] F. del Aguila, M. Martinez & M. Quiros, *CERN preprint* TH.6389 (1992)

[15] P. Langacker, L. Mingxing,& A.K. Mann, *Rev. Mod. Phys.* **64** (1992) 87.

[16] F. Halzen & D.A. Morris, *Particle World* **2** (1991) 10.

[17] J. Ellis, S. Kelley & D.V. Nanopoulos, *Phys. Lett.* **B249** (1990) 441

[18] H. Georgi, & S.L. Glashow, *Phys. Rev. Lett.* **32** (1974) 438.